Scientific and Social Research, 2024, Volume 6, Issue 12
https://ojs.bbwpublisher.com/index.php/SSR
Online ISSN: 2981-9946
Print ISSN: 2661-4332

# Some Explanation of the Definition of Matrix Calculation and Rank of Matrix

**Haiyan Liu***

College of Jincheng, Nanjing University of Aeronautics and Astronautics, Nanjing, China

***Corresponding author:** Haiyan Liu, ziluolan7812@sina.com



**Abstract:** When studying linear algebra, students may feel very abstract. In most textbooks, the definition or theorem is given directly, so the students do not know the origin, meaning, or application of some knowledge. This study will explain some knowledge in the chapter of the matrix to help students understand the knowledge better and cultivate students' exploration ability by getting conclusions from observing and studying. The knowledge chosen is the definition of matrix calculation and the rank of a matrix,

**Keywords:** Matrix calculation; Rank of matrix; Exploration ability

**Online publication:** December 31, 2024

## 1. Introduction

Most linear algebra textbooks do not give the background of definition [1–3]. Most of the time, they will give definitions directly. The students will find it very abstract because they do not know the background and the meaning of the definition.

Many books and documents discuss the studying or teaching methods [4–7]. For linear algebra courses, few people discuss the background and the meaning of the definition. This study will give concrete explanations and examples for some definitions that the students find abstract in studying linear algebra. This study will choose some knowledge in the chapter on matrices, that is, the definition of matrix calculation and the rank of matrix.

## 2. The matrix calculation

It is known that there are relationships among things in the world. The matrix that stores information is not an exception. This study will give several examples to show that there will be some relationship among matrices. Then, the study will define the relationship among matrices as some matrix calculation. The matrix calculations discussed are matrix addition, multiplication between matrix and number, and matrix multiplication.

## 2.1. Matrix addition and multiplication between matrix and number

The common way to introduce matrix addition and multiplication between matrix and number are as follows.

Give two matrices *A* and *B* with *m* x *n*-dimensional, denote the sum of *A* and *B* as *A*+*B*, and set as follows.

$$A+B=\begin{pmatrix} a_{11}+b_{11} & a_{12}+b_{12} & \cdots & a_{1n}+b_{1n} \\ a_{21}+b_{21} & a_{22}+b_{22} & \cdots & a_{2n}+b_{2n} \\ \cdots & \cdots & \cdots & \cdots \\ a_{m1}+b_{m1} & a_{m2}+b_{m2} & \cdots & a_{mn}+b_{mn} \end{pmatrix}$$

The multiplication of number λ and matrix *A* is denoted by λA or Aλ and set as follows.

$$\lambda A = A\lambda = \begin{pmatrix} \lambda a_{11} & \lambda a_{12} & \cdots & \lambda a_{1n} \\ \lambda a_{21} & \lambda a_{22} & \cdots & \lambda a_{2n} \\ \cdots & \cdots & \cdots & \cdots \\ \lambda a_{m1} & \lambda a_{m1} & \cdots & \lambda a_{mn} \end{pmatrix}$$

For students, matrix addition and multiplication between matrix and number are not only abstract rules but they are produced from nowhere.

The study will give concrete application background for the two matrix calculations as follows.

Suppose there are four kinds of food put in storage by three stores in every quarter. Denote the four kinds of foods as $F_1$, $F_2$, $F_3$, $F_4$, and the three stores as $S_1$, $S_2$, $S_3$. The element $a_{ij}$ of matrix *A* stands for the quantity of $F_j$ put in storage by $S_i$.

$$A=\begin{pmatrix} 50 & 60 & 80 & 55 \\ 100 & 120 & 125 & 98 \\ 15 & 12 & 18 & 22 \end{pmatrix}$$

The quantity of $F_j$ ($j$ = 1, 2, 3, 4) put in storage by $S_i$ ($i$ = 1, 2, 3) in the second quarter is stored in Matrix *B*.

$$B=\begin{pmatrix} 45 & 52 & 60 & 58 \\ 96 & 105 & 100 & 110 \\ 18 & 17 & 13 & 20 \end{pmatrix}$$

Statistically, the quantity of $F_j$ ($j$ = 1, 2, 3, 4) put in storage by $S_i$ ($i$ = 1, 2, 3) in the first half of the year is stored in Matrix *C*.

$$C=\begin{pmatrix} 92 & 112 & 140 & 113 \\ 196 & 225 & 225 & 208 \\ 33 & 29 & 31 & 42 \end{pmatrix}$$

It can be seen that matrix *C* has a relationship with *A* and *B*. The element in matrix *C* is obtained from matrices *A* and *B*. The elements $c_{ij} = a_{ij} + b_{ij}$. The relationship among matric *A*, *B*, and *C* needs to be described. Then, the definition of matrix addition is given.

Suppose in the first quarter, the price of all the foods is 10 dollars, then can get the purchasing price for *Fi* (*i* = 1, 2, 3, 4) put in storage by S*j* (*j* = 1, 2, 3), and store the data in matrix *D*.

$$D=\begin{pmatrix} 500 & 600 & 800 & 550 \\ 1000 & 120 & 1250 & 980 \\ 150 & 120 & 180 & 220 \end{pmatrix}$$

It can be found that matrix *D* has a relationship with matrix *A*. The element in matrix *D* is obtained from matrix *A*. The elements $d_{ij} = 10 \times a_{ij}$. Then can define a matrix multiplied by a number.

If students are given a concrete application background, the definition of matrix addition and multiplication between matrix and number is no longer abstract. Besides, the students can find that there will be some relationship among matrices. The definition is not generated from nowhere, but according to their needs.

## 2.2. Matrix multiplication

The common background of matrix multiplication is as follows.

Suppose there are two linear transformations:

$$\begin{cases} y_1 = a_{11}x_1 + a_{12}x_2 + a_{13}x_3 \\ y_2 = a_{21}x_1 + a_{22}x_2 + a_{23}x_3 \end{cases} \quad (1)$$

$$\begin{cases} x_1 = b_{11}t_1 + b_{12}t_2 \\ x_2 = b_{21}t_1 + b_{22}t_2 \\ x_3 = b_{31}t_1 + b_{32}t_2 \end{cases} \quad (2)$$

Then the linear transformation from $t_1$, $t_2$ to $y_1$, $y_2$ is:

$$\begin{cases} y_1 = (a_{11}b_{11} + a_{12}b_{21} + a_{13}b_{31})t_1 + (a_{11}b_{12} + a_{12}b_{22} + a_{13}b_{32})t_2 \\ y_2 = (a_{21}b_{11} + a_{22}b_{21} + a_{23}b_{31})t_1 + (a_{21}b_{12} + a_{22}b_{22} + a_{23}b_{32})t_2 \end{cases} \quad (3)$$

Then can define that the matrix corresponding to Formula (3) is the multiplication of the matrices corresponding to Formulas (1) and (2), as follows.

$$\begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \end{pmatrix} \begin{pmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \\ b_{31} & b_{32} \end{pmatrix} = \quad (4)$$

$$\begin{pmatrix} a_{11}b_{11} + a_{12}b_{21} + a_{13}b_{31} & a_{11}b_{12} + a_{12}b_{22} + a_{13}b_{32} \\ a_{21}b_{11} + a_{22}b_{21} + a_{23}b_{31} & a_{21}b_{12} + a_{22}b_{22} + a_{23}b_{32} \end{pmatrix}$$

For the explanation of matrix multiplication, students find it very abstract. Because Formula (4) is a given rule just like the definition of matrix multiplication. The examples for matrix multiplication are given as follows.

The Matrix *A* occurred in section 2.1 is still used.

$$A = \begin{pmatrix} 50 & 60 & 65 & 49 \\ 100 & 120 & 125 & 98 \\ 15 & 12 & 18 & 22 \end{pmatrix}$$

Suppose for the first and the second quarter, the price of every food put in storage by every store is shown in matrix *E*. $e_{ij}$ stands for the price of food *i* in the *j*-th quarter ($i = 1, 2, 3, 4; j = 1, 2$).

$$E = \begin{pmatrix} 3 & 4 \\ 5 & 6 \\ 4 & 5 \\ 6 & 7 \end{pmatrix}$$

If one wants to count the price of every store spending for all four kinds of foods in every quarter. Suppose the storage quantity of every store for every food is not changed for the first and second quarters. The price information is stored in matrix *F*.

$$F = \begin{pmatrix} 50\times 3+60\times 5+65\times 4+49\times 6 & 50\times 4+60\times 6+65\times 5+49\times 7 \\ 100\times 3+120\times 5+125\times 4+98\times 6 & 100\times 4+120\times 6+125\times 5+98\times 7 \\ 15\times 3+12\times 5+18\times 4+22\times 6 & 15\times 4+12\times 6+18\times 5+22\times 7 \end{pmatrix} = \begin{pmatrix} 1004 & 1228 \\ 1988 & 2431 \\ 309 & 376 \end{pmatrix}$$

It is found that matrix *F* has a relationship with *A* and *E*.

$$f_{ij} = a_{i1}e_{1j} + a_{i2}e_{2j} + a_{i3}e_{3j} + a_{i4}b_{4j} (i = 1,2,3,4; j = 1,2)$$

Then, the definition of matrix multiplication can be given as follows.

Give two matrices *A* and *B* with m x s-dimensional and s x n-dimensional respectively, denote matrix multiplication of *A* and *B* as *AB*, and set as follows.

$$C = AB = \begin{bmatrix} \cdots & \cdots & \cdots & \cdots \\ a_{i1} & a_{i2} & \cdots & a_{is} \\ \cdots & \cdots & \cdots & \cdots \end{bmatrix}_{m\times s} \begin{bmatrix} \vdots & b_{1j} & \vdots \\ \vdots & b_{2j} & \vdots \\ \vdots & \vdots & \vdots \\ \vdots & b_{sj} & \vdots \end{bmatrix}_{s\times n} \quad (5)$$

$$c_{ij} = a_{i1}b_{1j} + a_{i2}b_{2j} + \cdots + a_{is}b_{sj} = \sum_{k=1}^{s} a_{ik}b_{kj} \, (i = 1,2,\cdots,m; j = 1,2,\cdots,n)$$

In this way, students can understand why the definition of matrix multiplication is given like the form in Formula (5).

Now, it is shown that a definition can be given according to the application. Since the calculation is just a rule according to the needs. Hence, other multiplications of the matrix can be given. Such as Hadamard product, another form of matrix multiplication.

Given two matrices *A* and *B* with m x n-dimensional. $A = (a_{ij})$, $B = (b_{ij})$. The Hadamard product of *A* and *B* is defined as $A*B = (a_{ij}*b_{ij})$.

Next, another definition the students find abstract in the chapter on the matrix will be discussed. It is the rank of a matrix.

## 3. Rank of matrix

The common definition of the rank of a matrix is the highest order of the subdeterminant which is not equal to zero.

The definition is very abstract, so the students cannot understand what is rank, and why the highest order of the subdeterminant which is not equal to zero is the rank of a matrix.

To explain the meaning of the rank of a matrix, it can begin by solving the linear system of equations.

A linear system of equations is given as follows.

$$\begin{cases} x_1 + x_2 + x_3 = 0 \\ x_1 + 2x_2 + 3x_3 = 0 \\ 2x_1 + 2x_2 + 2x_3 = 0 \\ 2x_1 + 3x_2 + 4x_3 = 0 \\ 4x_1 + 5x_2 + 6x_3 = 0 \end{cases} \tag{6}$$

Matrix *G* is used to represent a linear system of equations (6) and then the method of elimination is used to solve Formula (6).

After the row transformation during eliminating, *G* is changed to *H*.

$$G = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 3 \\ 2 & 2 & 2 \\ 2 & 2 & 4 \\ 4 & 4 & 6 \end{pmatrix} \rightarrow H = \begin{pmatrix} 1 & 1 & 1 \\ 0 & 0 & 2 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \tag{7}$$

From Formula (7), it can be found that the last three equations are redundant and can be substituted by the first two equations. This shows that the number of valid ranks of *G* which cannot substitute each other is two. The number of valid ranks of *G* is called a rank. Now, it can be seen that the rank of the matrix stands for the valid information of *G*, that is, the number of valid ranks. For example, there are two matrices, the order of one of them is two, and the other is 100, but both of them have two valid ranks, then the rank of the two matrices is the same.

From G, it can be seen that the valid ranks which one of the ranks of the valid ranks cannot be substituted by other ranks is two. Therefore, any third-order subdeterminant of G is equal to zero. Because the elements in the subdeterminant are part of G. There are only two valid ranks in G. So, there are also two valid ranks in the subdeterminant. Hence, the value of the subdetermiant is zero. The value of the subdeterminant whose order is over three if the susdeterminant exists is in the same way. For the second-order subdeterminants, it can be seen that there is a second-order matrix, $\begin{vmatrix} 1 & 1 \\ 1 & 3 \end{vmatrix}$ which is not equal to zero. That is, G has a second-order subdeterminant whose value is not equal to zero. And for any subdeterminant whose order is three or over three if the susdeterminant exists, its value is zero. So, the highest order of the subdeterminant not equal to zero is just equal to the rank of matrix *G*. So, the rank of a matrix can be defined by its subdeterminant. In this way, the students can understand the meaning of rank and why the highest order of the subdeterminant which is not equal to zero is the rank of matrix.

## 4. Conclusion

Most linear algebra does not pay attention to introducing the background and meaning of definitions, while only stating them directly. This can cause students to find linear algebra courses very abstract. This study chose some knowledge in the chapter on the matrix to elaborate on the definition. It will help students to understand the definition better and form the habit of studying and exploring.

## Disclosure statement

The author declares no conflict of interest.

## References

[1] School of Mathematical Sciences, Tongji University, 2023, Linear Algebra. Higher Education Press, Beijing.

[2] Lay DC, Lay SR, McDonald JJ, 2023, Linear Algebra and Its Application. China Machine Press, Beijing.

[3] Tian CH, Sun YB, 2022, Linear Algebra. People's Posts and Telecommunications Press, Beijing.

[4] Polya G, 2014, Maths and Speculation. Science Press, Beijing.

[5] Xu LZ, 2001, On Mathematical Methodology. Shandong Education Press, Jinan.

[6] Xiao JP, Du XC, 2024, Phytophysiology Blended Teaching Design and Exploration based on Rain-Classroom. Journal of Higher Education, 10(1): 103–106.

[7] Zhu Q, 2022, The Application of Blended Teaching for Linear Algebra. Computer Knowledge and Technology, 18(6): 175–177.

**Publisher's note**

Bio-Byword Scientific Publishing remains neutral with regard to jurisdictional claims in published maps and institutional affiliations.