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Designing a Mathematical Model to Determine the Heat Protective Performance of Special Clothing for High-Temperature Operation

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Abstract: This paper designs a mathematical model to determine the heat protective performance of special clothing for high-temperature operation. This is an important protective measure for high-temperature operation. Based on the known material parameters of special clothing, experimental data are obtained by simulating the actual situation and fitted with MATLAB software. A mathematical model to determine the heat protective performance of special clothing for high-temperature operation is established, which solves the issue of determining the optimal thickness of special clothing for high-temperature operation in certain conditions.

Keywords: Heat conduction; Heat protective clothing; MATLAB; Optimal thickness

1. Introduction

1.1. Significance and necessity of designing a mathematical model to determine the heat protective performance of special clothing for high-temperature operation

High-temperature operation generally refers to working in an environment with an average wet bulb globe temperature (WBGT) ≥ 25°C, such as fire rescue, aerospace, iron and steel metallurgy, and so on. In order to enable people to work normally under high-temperature conditions, people have come up with many heat protection and insulation methods, among which special clothing for high-temperature operation is a very convenient and practical protection method. We aim to reduce the thickness of special clothing in order to facilitate operations under high-temperature conditions. However, if it is too thin, the heat protective effect cannot be achieved. Therefore, the heat protective performance of special clothing for high-temperature operation directly affects people's health and safety [1].

1.2. Data source for establishing a mathematical model to determine the performance of special clothing for high-temperature operation

The special clothing for high-temperature operation is generally composed of three layers of fabrics, which are recorded as Layer I, II, and III. Layer I is in direct contact with the external environment, but there is a gap between layer III and the skin. This gap is recorded as layer IV. In order to ensure a reasonable design of heat protective clothing and the accuracy of research data as close to the actual situation as possible, a dummy is placed in a high-temperature environment whose temperature is controlled at 37°C in the laboratory. When determining the parameter values of the materials, that is the density, specific heat, thermal conductivity, and thickness of Layer I, II, III, and IV, the external ambient temperature and the temperature over the dummy's skin surface are measured and used as the experimental data.

2. Principles and methods of designing the mathematical model to determine the heat protective performance of special clothing for high-temperature operation

The data is preprocessed and coded in Python based on the experimental data that have been tested for 90 minutes. When the ambient temperature, Layer II thickness, and Layer IV thickness are all set to a given value, the change in human body temperature over time is recorded. The changing relationship between the dummy's skin temperature and time is observed. By using the heat conduction model from the heat protective clothing to the dummy's skin and the heat transfer model along with Fourier's heat transfer equation, the Gaussian function is fitted using MATLAB [2]; the correlation coefficient and fitting function are thus obtained. A mathematical model that determines the temperature change on the skin surface of the dummy is also established. Using the established mathematical model, the temperature distribution outside the dummy's skin is calculated; the calculated data are compared with the actual measured temperature data to test the rationality and accuracy of the mathematical model. The optimal thickness of the second layer of special clothing for high-temperature operation is obtained under the given initial conditions by establishing the mathematical model and heat conduction model [4].

3. Establishing a mathematical model to determine the heat protective performance of special clothing for high-temperature operation

First, the materials for the special clothing are selected. Only the parameter values of the materials are given here. The density of Layer I is 300 kg/m^3; the specific heat is 1,377 J/(kg°C); the thermal conductivity is 0.082 W/(m°C); the thickness is 0.6 mm. The density of Layer II is 862 kg/m^3; the specific heat is 2,100 J/(kg°C); the thermal conductivity is 0.37 W/(m°C); the thickness is 6 mm. The density of Layer III is 74.2 kg/m^3; the specific heat is 1,726 J/(kg°C); the thermal conductivity is 0.045 W/(m°C); the thickness is 3.6 mm. The density of Layer IV is 1.18 kg/m^3; the specific heat is 1,005 J/(kg°C); the thermal conductivity is 0.028 W/(m°C); the thickness is 5 mm. The experiment is carried out at an ambient temperature of 75°C, and the working time is 90 minutes. The temperature outside the dummy's skin is measured and assumed as the experimental data. The program is written in Python to import and analyze the experimental data, and the results are shown in **Figure 1**.

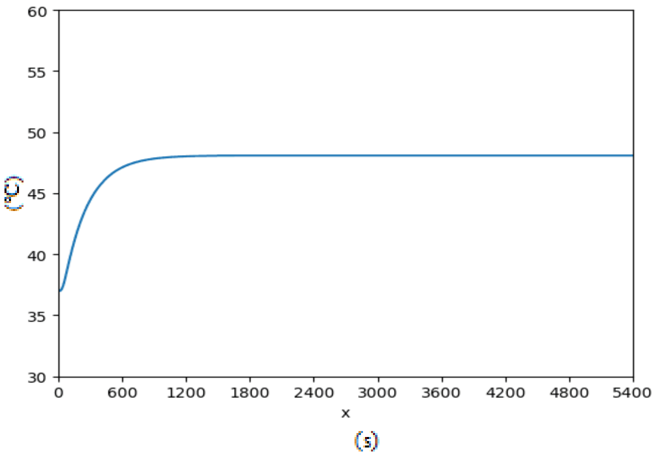

Figure 1. Variation of temperature with time

The abscissa represents time (s), and the ordinate represents temperature. It can be seen from **Figure 1** that the initial human body temperature 37°C. The increase in temperature outside the dummy's skin within

90 minutes is observed. Through observation, it is found that the human body temperature remains unchanged at a temperature of 48.08°C from 1745 s, so the data of 0 s and 1745 s are further fitted using MATLAB [2], as shown in **Figure 2**.

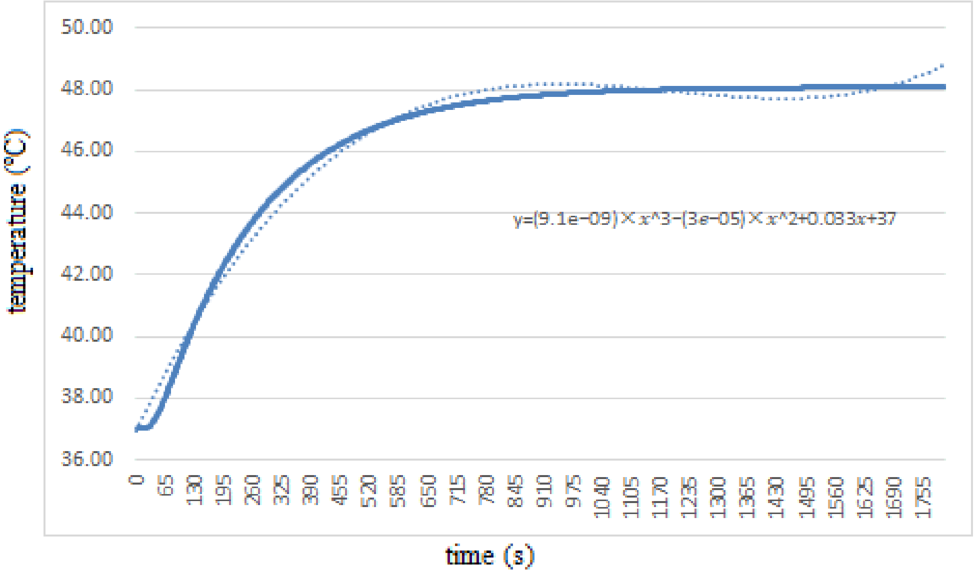

Figure 2. Fitted relationship between skin temperature and time

The fitted equation obtained is as follows:

$$y = (9.1e - 09) \times x^3 - (3e - 05) \times x^2 + 0.033x + 37 \tag{1}$$

The obtained fitted curve is used to calculate the temperature distribution on the outside of the dummy's skin, and the calculated data is compared with the actual measured temperature data, thus verifying that the fitted result is accurate and the established model is reliable.

The heat conduction model is established [3]. In this paper, due to different media, the established heat conduction model is divided into two parts: the heat conduction of different materials in the fabric layer of the heat protective clothing from the high-temperature working environment (i.e., the left side of Layer I) and the heat conduction model from the air partition layer (i.e., Layer IV) in the high-temperature protective clothing to the skin surface.

The heat conduction rate equation of one-dimensional plane is as follows:

$$-k\frac{\partial T}{\partial t}|_{x=0} = q \tag{2}$$

q represents the radiant energy from the surface of the material.

Second, the local differential equation of temperature distribution of multilayer composites is established based on Gibson and Torvi's pair theory.

$$\rho c_\rho \frac{\partial T}{\partial t} = \frac{\partial}{\partial x}\left(k\frac{\partial T}{\partial t}\right) \tag{3}$$

The first law of thermodynamics states that in all cases in which work is produced by the agency of heat, a quantity of heat that is proportional to the work done is consumed; conversely, by the expenditure of an equal quantity of work, an equal quantity of heat is produced. According to the first law of thermodynamics, the equation is as follows:

$$Q = w + \Delta E \tag{4}$$

Q represents the change of internal energy of the object; ΔE represents the increased in internal energy; W represents the change of work done. Since the external reverse conduction of human skin can be negligible in this paper, so W = 0; in that case, $Q = \Delta E$.

According to the law of conservation of energy, a differential equation is generated in the infinitesimal elements of the fabric:

$$\rho c \frac{\partial T}{\partial t} = \frac{\partial}{\partial x}\left(k \frac{\partial T}{\partial x}\right) - \frac{\partial}{\partial x}\left(q_{rad}(x = 0)\right) + G_{chem} \tag{5}$$

In the above equation, $\frac{\partial}{\partial x}\left(q_{rad}(x = 0)\right)$ represents the thermal radiation received by the object, which is simplified and negligible in this paper; G_{chem} denotes the external reverse conduction of human skin temperature, which is also negligible. Therefore, the following equation based on the law of conservation of energy is finally obtained:

$$\rho c \frac{\partial T}{\partial t} = k \frac{\partial^2 T}{\partial x^2} \tag{6}$$

The initial condition of heat protective clothing is as follows:

$$T(x, t = 0) = T_I(x) \tag{7}$$

The left boundary condition of the fabric layer is as follows:

$$-k \frac{\partial T}{\partial x}|_{x=0} = q \tag{8}$$

The right boundary condition of the fabric layer is as follows:

$$-k \frac{\partial T}{\partial x}|_{x=L} = q \tag{9}$$

The left and right boundaries of different materials are determined according to different values of L.

In the heat conduction model, since the distance between Layer IV and the dummy's skin is 0.6 mm to 6.4 mm, in which the thickness of the air layer is less than 6.4 mm, convection is not considered because the air gap is so small that the heat transfer of the air layer is dominated by conduction. According to Fourier's heat flow law, the one-dimensional plane heat conduction rate equation is as follows:

$$-k \frac{\partial T}{\partial t}|_{x=0} = q \tag{10}$$

The final regional distribution diagram of temperature is obtained using MATLAB simulation, as shown in **Figure 3**.

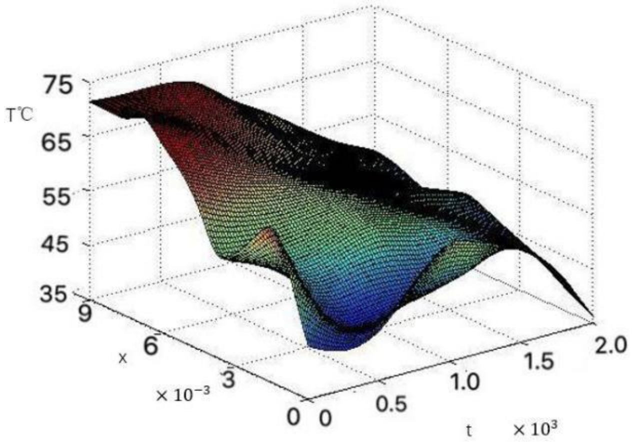

Figure 3. Regional distribution diagram of temperature

According to the established temperature distribution equation on the dummy's skin surface, when the ambient temperature is 65ºC, the thickness of Layer IV is 5.5 mm. When the parameter values of other materials remain unchanged, the optimal thickness of Layer II is determined, so as to ensure that the temperature on the outside of the skin does not exceed 47℃ and the time during which the temperature is above 44℃ is no more than 5 minutes when working for 60 minutes.

The following equation is established from the skin temperature and time fitted equation, the one-dimensional plane heat conduction rate equation, and the local differential equation of temperature distribution of multilayer composites:

$$\begin{cases} -k\frac{\partial T}{\partial t}|_{x=0}=q \\ \rho C_{\rho}\frac{\partial T}{\partial t}=\frac{\partial}{\partial x}(k\frac{\partial T}{\partial t}) \\ y=(9.1e-09)\times x^3-(3e-05)\times x^2+0.033x+37 \end{cases} \tag{11}$$

Let the thickness of the second layer be L, where $x_1 = 0.6\,\mathrm{mm}$, $x_2 = L\,\mathrm{mm}$, $x_3 = 3.6\,\mathrm{mm}$, $x_4 = 5.5\,\mathrm{mm}$.

The initial condition of Layer II is as follows:

$$T(x_2, 0) = TL \tag{12}$$

The left boundary condition of Layer II is as follows:

$$-k\frac{\partial T}{\partial x}|_{x=0} = (q'')|_{x=0.6} \tag{13}$$

The right boundary condition of Layer II is as follows:

$$-k\frac{\partial T}{\partial x}|_{x=0} = (q'')|_{x=L_2-0.6} \tag{14}$$

According to the question, the following results can be obtained:

$$\begin{gathered} T(x, t=3600) \leq 47℃ \\ T(x, t \leq 300) \leq 44℃ \end{gathered} \tag{15}$$

By solving the above equations, the following result can be obtained:

$$x_2 \geq 9\text{mm} \tag{16}$$

This concludes that the optimum thickness of the second layer is 9 mm.

4. Conclusion

In this paper, under the condition of simulating high-temperature operation, the data of human body temperature changing with time have been obtained, and the mathematical model of the human skin surface temperature changing with ambient temperature has been established: $y = (9.1e - 09) \times x^3 - (3e - 05) \times x^2 + 0.033x + 37$. Using the mathematical model of temperature change and thermal conduction model, the mathematical model of the thermal protective performance of special clothing for high-temperature operation has been established, so as to solve the issue of determining the optimal thickness of special clothing for high-temperature operation in certain conditions.

Project

This paper is one of the phased achievements of the general project “Research on the Construction Strategy of Dual Qualified and Dual Capable Teachers in Application-Oriented Universities” (Project Number: JG20DB248) approved by the 13th Five-Year Plan of Educational Science in Liaoning Province in 2020.

Disclosure statement

The author declares no conflict of interest.

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