Scientific and Social Research
2022, Volume 4, Issue 5

Mathematical Modeling of Oil Spill Dispersion in Marine Waters

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Abstract: During the extraction and transportation of oil in marine waters, oil spills may occur. It not only caused serious pollution in some sea areas but also have a grave impact on the marine environment. Studying the oil dispersion pattern is important for addressing oil spills accurately and timely. Therefore, through the mathematical knowledge, such as continuity equation and momentum equation, we establish a mathematical model for the dispersion of oil spills in marine water. The methods that are used include data statistics, construction of graphs and charts, example analogy, hierarchical analysis, and random calculation. The research and analysis are conducted for the oil spill process in marine oil fields.

Keywords: Fay formula; Dispersion; Oil spill; Mathematical model

1. Introduction

With the rapid economic development, oil is playing an increasingly important role in the industrialization process. However, marine oil resources are often exploited, leading to oil spills. For example, on June 4, 2011, the North China Sea Branch of the State Oceanic Administration received a report, which showed that a small amount of oil film of unknown origin was found on the sea surface in the northeast direction of Platform B of Penglai 19-3 Oilfield, owned by a subsidiary of a US-based company, ConocoPhillips, China. Additionally, another report was received on June 17, on North China Sea Branch, where large amount of spilled oil was found on Platform C and nearby waters. Later, the identification result confirmed that oil spills have also occurred in Penglai 19-3 Oilfield. The oil spills occurring during the extraction and transportation of marine oil have caused serious pollution in some of the sea areas and adversely impacted the marine environment. It not only increases the difficulty in treating water, but also has a long-term negative impact on the development of the current ecological environment. Therefore, the establishment of a mathematical model of the dispersion of marine oil spills and the study of oil dispersion patterns is of great importance to accurately and timely handle oil spills.

2. Research method of the mathematical model of marine oil spill dispersion

The oil spill accident that occurred at Penglai 19-3 Oilfield in Bohai Bay was used as an example in this study. Under the opinion that only one leakage point exists, and by using mathematical knowledge, such as continuity equations, momentum equations, and others, this paper managed to establish a mathematical model reflecting the dispersion pattern of oil spills by specifically studying and analyzing the oil spill process in the marine oil field. Multiple methods were used such as data statistics, construction of graphs and charts, example analogy, hierarchical analysis, and random calculation to develop the mathematical model. MATLAB software was then used to fit the variation of the expansion area of oil film as a function of time on the sea surface without monsoon and with zero current velocity, and the oil film area S as a function of time was obtained.

3. Establishment and study of the mathematical model of marine oil spill dispersion

Under the condition that there is only one leakage point, the submarine oil spill process is divided into two parts, which are the jet dispersion process of underwater oil spills and the dispersion process of the surface oil film, respectively. Subsequently, a model was built for each of the two processes.

3.1. Establishment of a model for the jet dispersion process of underwater oil spills

In this study, two-dimensional numerical simulation was used, where the horizontal axis is represented by the X-axis and the vertical axis by the Y-axis. To study two mutually immiscible fluids, VOF model was used. In the VOF model [1], α is introduced to represent the volume fraction of fluid in a calculated area unit. When $\alpha = 0$, the volume fraction of fluid in a calculated area unit is zero. In another word, there is no fluid in the area unit. Conversely, when $\alpha = 1$, the area unit is filled with the fluid. Gas, water, and oil are represented by $\alpha 1$, $\alpha 2$, and $\alpha 3$, respectively. The volume fraction equations (the continuity equation) are as follows:

$$\frac{\partial \alpha_i}{\partial t} + V \cdot \nabla \alpha_i = 0 \qquad i = 1,2,3 \tag{1}$$

The momentum equation is as follows:

$$\frac{\partial}{\partial t}(\rho V) + V \cdot \nabla(\rho V) = -\nabla p + \nabla \cdot [\mu(\nabla V)] + F \tag{2}$$

where α denotes volume fraction; V denotes unit average rate, m/s; ρ denotes density, kg/m^3; p denotes pressure, Pa; t denotes time, s; μ denotes dynamic viscosity, Pa·s; F denotes external force, N.

3.2. Establishment of a model for the dispersion process of surface oil film

The oil film on the sea surface is influenced by the monsoon and currents on the sea in one hand, and by gravity, inertia, surface tension, and sea surface turbulence on the other hand. Under an ideal condition of no monsoon and zero current velocity, an expansion model of the oil film was built by only considering the gravity, surface tension, and inertia, as the main decisive factors. In the end, a model of the oil film on the sea surface with a circular shape was obtained. According to Fay formula [2], the expansion diameter of the oil film was calculated in three stages.

In the inertial expansion stage, the expansion diameter was calculated as follows:

$$\mathrm{d}_1 = 2\mathrm{K}_1(\beta g V_{\mathrm{oil}})^{1/4} t^{1/2} \tag{3}$$

In the cohesive expansion stage, the expansion diameter was calculated as follows:

$$d_2 = 2K_2(\beta g V_{\mathrm{oil}}/\nu^{1/2})^{1/6} t^{1/4} \tag{4}$$

In the surface tension expansion stage, the expansion diameter was calculated as follows:

$$d_3 = 2K_3(\sigma/\rho_{water}\nu^{1/2})^{1/2}t^{3/4} \tag{5}$$

where t is the duration of oil spills.

$$\beta = 1 - \rho_{oil}/\rho_{water} \tag{6}$$

$$\sigma = \sigma_b - \sigma_a - \sigma_c \tag{7}$$

Under ideal conditions of no monsoon and zero current velocity, and by considering the main determinants, which are gravity, surface tension, inertia, and a right-angle coordinate system was established with the oil leakage point as the center, the x-axis pointing eastward, and the y-axis pointing northward.

$$\begin{cases} x = 10^{-3}\sqrt{10^5(Q_{oil}t)^{3/4}/\pi}\cos\theta \\ y = 10^{-3}\sqrt{10^5(Q_{oil}t)^{3/4}/\pi}\sin\theta \end{cases} \tag{8}$$

From the diagram generated using MATLAB, the following diagram pattern is obtained.

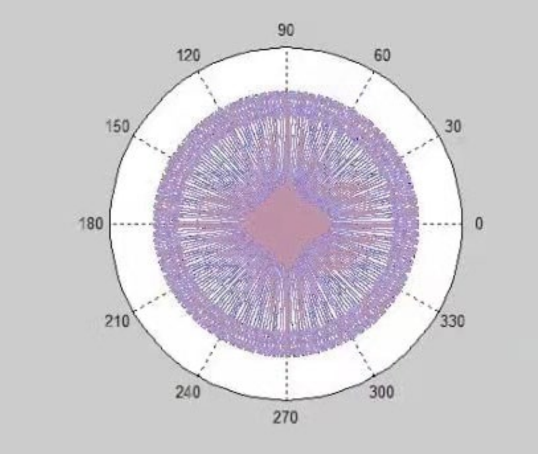

Figure 1. Shape simulation of oil film at the sea surface without monsoon and with zero current velocity

To promote the reliability of the study, it is assumed that the oil film on the sea surface consists of an infinite number of individual oil droplets in the process of simulating the expansion of the oil film on the sea surface. Therefore, the random dispersion and advection of the oil droplets were used to simulate the dispersion of the oil film on the sea surface [3].

The random numbers used for the dispersion of the oil droplets were derived using two random calculation methods, which are the uniform random number method and the normally distributed random number method. To calculate the turbulent dispersion of each oil droplet, the following equation was used.

$$\Delta\alpha = R\sqrt{2k_\alpha \Delta t} \tag{9}$$

where $\Delta\alpha$ denotes the turbulent dispersion distance in the α ($\alpha = x$ and $\alpha = y$) direction, R denotes a normally distributed random number with N in the range of (0, 1), kα denotes the turbulent dispersion coefficient in the α direction, and Δt is the time step.

$$V'(t) = \sqrt{2\,k_\alpha/\Delta t} \tag{10}$$

Therefore, the turbulent dispersion distance of the oil droplets can be obtained using the following formula.

$$x(t_i) = \mu'(t_i)\,\Delta t \qquad y(t_i) = V'(t_i)\,\Delta t \tag{11}$$

In practical circumstances, we have to take consideration to wind and currents to calculate the oil spill dispersion at the sea surface. Under this condition, the equation for the motion of each oil droplet was as follows.

$$\vec{V}(t) = \vec{V}_r(t) + \alpha \vec{V}_w(t) + V'(t) \tag{12}$$

In the above equation, the random walk method is used, where V'(t) represents the instantaneous velocity and the rest of the symbols have the same meaning as described above.

3.3. Model Solving

$$\frac{\partial \alpha_i}{\partial t} + V \cdot \nabla \alpha_i = 0 \qquad i = 1,2,3 \tag{13}$$

$$\frac{\partial}{\partial t}(\rho V) + V \cdot \nabla(\rho V) = -\nabla p + \nabla \cdot [\mu(\nabla V)] + F \tag{14}$$

The above equation is mainly used to calculate the volume fraction of fluid within a calculated unit cell, and the fluid distribution is shown by the superposition of multiple unit cells, thus reflecting the oil leakage condition [4].

Taking one of the unit cells as an example, α3 = 0.5, V = 5 m/s, the heavy crude oil density ρ is 0.90×1000kg/m3, the dynamic viscosity μ is 35 Pa·s, and F is 9 N.

When the above data is brought into equations (8) and (9), the volume share of the heavy crude oil in the unit cell can be obtained. Next, the volume share of the gas and seawater can also be obtained. The unit cells can be stacked and placed in a grid distribution diagram (see **Figure 1**) to obtain the jet dispersion diagram of underwater oil spills, see **Figure 2** and **Figure 3**.

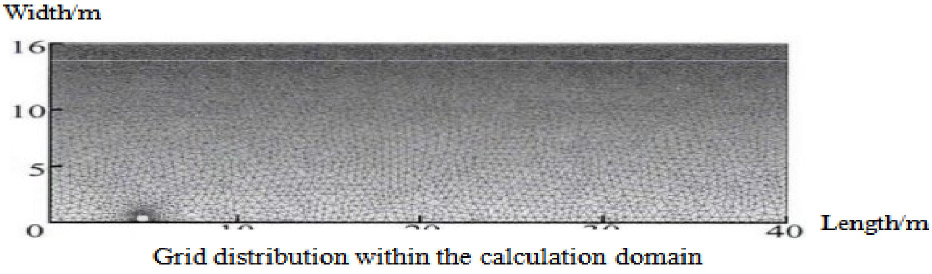

Figure 2. Grid distribution of changes in gas, water, and oil volume fractions

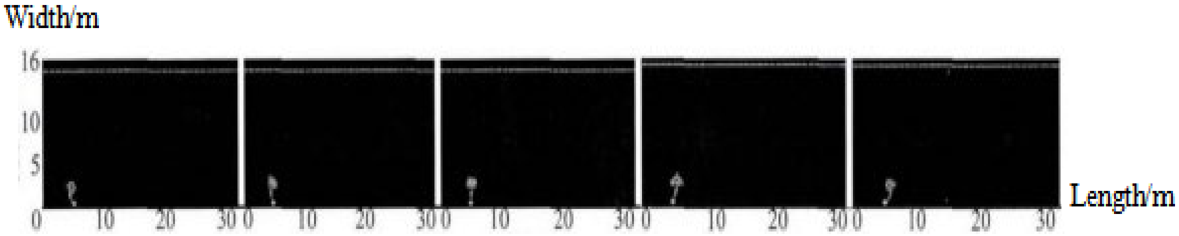

Figure 3. Jet diffusion of underwater oil spills

Finally, the MATLAB 6.1 software was used to fit the variation of the expansion area of oil film as a function of time on the sea surface with no monsoon and zero current velocity, as shown in **Figure 4**.

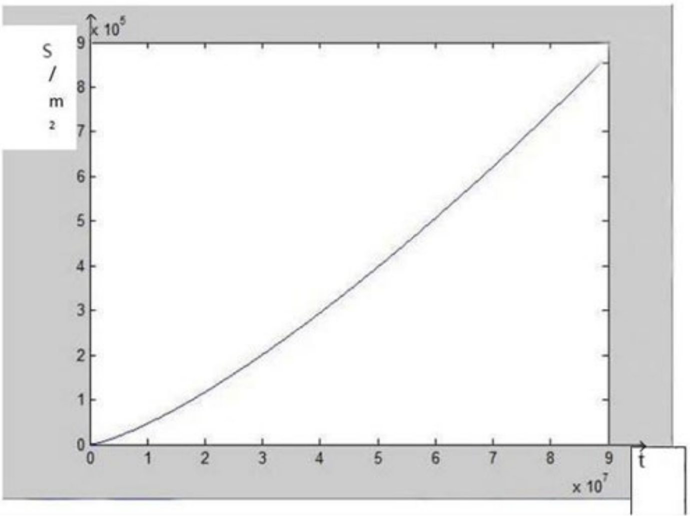

Figure 4. Variation of the expansion area of oil film as a function of time on the sea surface with no monsoon and zero current velocity

The variation pattern of the expansion area of oil film over time can be obtained as follows:

$$S = [(0.584t^{1/2}+0.735t^{1/4}+0.00015t^{3/4})/2]^2 \tag{15}$$

4. Conclusions

Under the condition that only one leakage point exists, this paper analyzed the process of oil spills from subsea pipelines, determined the jet dispersion process of underwater oil spills and the dispersion process of surface oil film, and lastly the VOF model and the turbulent dispersion model was developed to count the oil droplets.

By doing so, we obtained the unit-cell stacking graphs and charts of water, gas, and oil and the turbulent dispersion graphs and charts of oil droplets, presenting the oil spill pattern.

The MATLAB software was used to fit the variation of the expansion area of oil film as a function of time on the sea surface with no monsoon and zero current velocity, thus deriving the variation pattern of the expansion area of oil film with time.

Disclosure statement

The author declares no conflict of interest.

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