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Establishing a Mathematical Model for the Change of the Center of Gravity of Water Tanks

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Abstract: The subject of center of gravity is inseparable from our daily lives; hence, the research on center of gravity has very important practical significance. This paper mainly studies the change of the center of gravity of a cuboid containing two water tanks upon flipping. Using the idea of differential equations, Visio and RealFlow are used to simulate the entire process, and the influence on the distance between the centroid and the center of gravity of the cuboid (including the water tanks) caused by the proportion of water in the water tank is studied and the inclination angle of the water tank is calculated via integration. By establishing a model through differential equations, the change of the center of gravity is studied when the cuboid (including the water tanks) is flipped.

Keywords: Centroid; Center of gravity; Fluid; Differential equation

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1. Introduction

The center of gravity of an object refers to the balance point of the resultant force acting on each part of an object in a gravitational field. If the gravity of each part of the object is concentrated at a point, then apparently, this point is the center of gravity of the object. The center of gravity of an object is not necessarily on the object. The principle of center of gravity has a wide range of applications [1], such as in tumblers, electric fans, table lamps, and so on, which all use the principle of low center of gravity to increase stability. The low center of gravity of a truck makes it less likely to roll over when turning; gymnasts must always keep their body's center of gravity exactly on the balance beam while performing. This principle is widely applied in architecture, car design, etc. With the continuous advancement of science and technology, people have also designed many novel items using this principle, such as inclined speakers, flower pots, and so on. Therefore, the subject of center of gravity is inseparable from our daily lives, and the research on center of gravity has very important practical significance.

2. Mathematical modeling for the center of gravity of water tanks

Two water tanks are placed horizontally in a cuboid in fixed positions, and the mass of the cuboid (including the water tanks) is M_0+8/k, and the volume is always 240 m^3. The change of the center of gravity of the cuboid during flipping is studied. By using differential equations, Visio is used to draw the force analysis diagram of the water tanks, while RealFlow is used to simulate the motion process of fluid. By triple integration and Jacobian determinant, the weighted sum is converted into a static moment [2], and the influence of the proportion of water and the inclination angle of the water tank on the distance between the centroid and the center of gravity of the cuboid (including the water tanks) is calculated. A model is established through differential equations and is used for the calculation to analyze the change of the center of gravity of the cuboid (including the water tanks) during flipping at a certain angle.

3. Mathematical model establishment and solution for the center of gravity of water tanks

Two water tanks are horizontally placed in a cuboid in fixed positions. As shown in **Figure 1**, the lower left corner is water tank 1, and the upper right corner is water tank 2. When the cuboid is flipped at $\frac{\pi}{2}$, according to the shortest axis of the cuboid, the cuboid is laid vertically. The influence of the distance between the centroid and the center of gravity of the cuboid (including the water tanks) during the flipping is determined. It is assumed that the water filling ratio in the two tanks is the same.

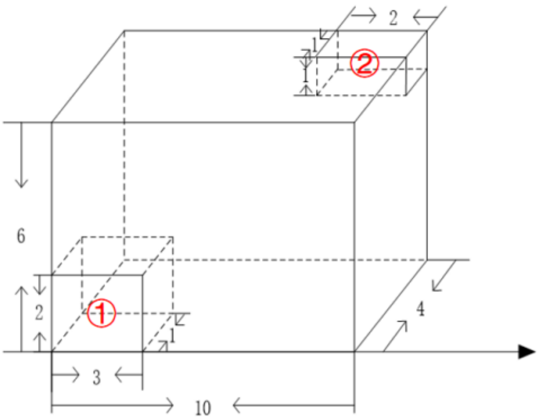

Figure 1. Water tanks laid flatly in a cuboid

In order to obtain the distance between the centroid and the center of gravity, it is first necessary to find the coordinates of the two points by triple integration. In the case of different water filling ratios, the above calculation is performed to obtain the distance between the two points. Then, the effects of water filling ratio on the distance between the centroid and the center of gravity of the cuboid during flipping are determined.

The force of the cuboid is analyzed from the initial state, the intermediate state, and the flipped state [3], as shown in **Figure 2**, **Figure 3**, and **Figure 4**, respectively. In the initial state, water tank 1 is subjected to gravity and the supporting force, whereas water tank 2 is subjected to gravity and tension. In the intermediate state, water tank 1 is subjected to gravity, the supporting force, and the friction force of the wooden block, whereas water tank 2 is subjected to gravity, tension, and the supporting force of the wooden block. In the flipped state, water tank 1 is subjected to gravity and the pulling force, whereas water tank 2 is subjected to gravity and the supporting force.

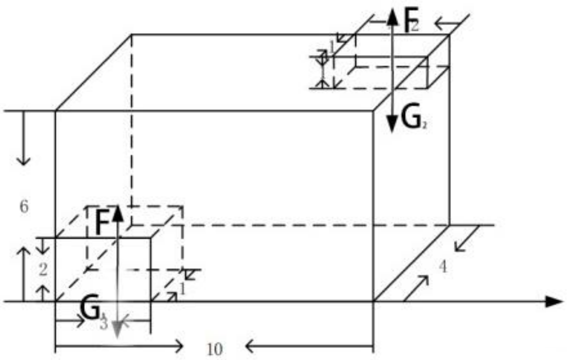

Figure 2. Force analysis diagram of the water tanks in horizontal position

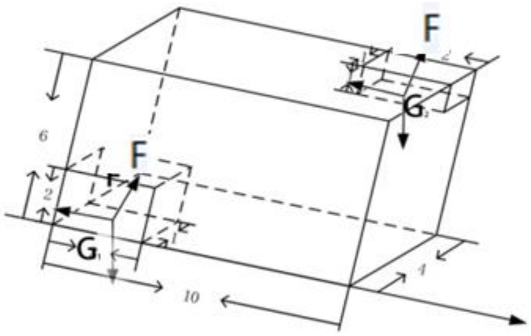

Figure 3. Force analysis diagram of the water tanks in the intermediate state

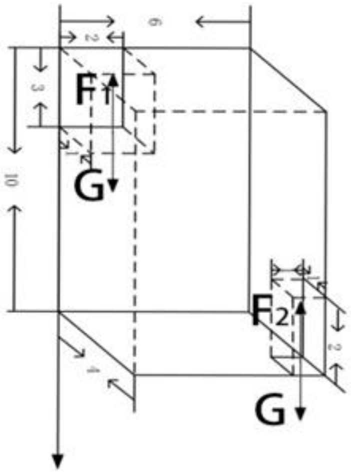

Figure 4. Force analysis of the water tanks in the flipped state with a flipping angle of $\frac{\pi}{2}$

The 3D (three-dimensional) fluid simulation software RealFlow is used to simulate fluid motion processes as shown in **Figure 5** and **Figure 6** [4].

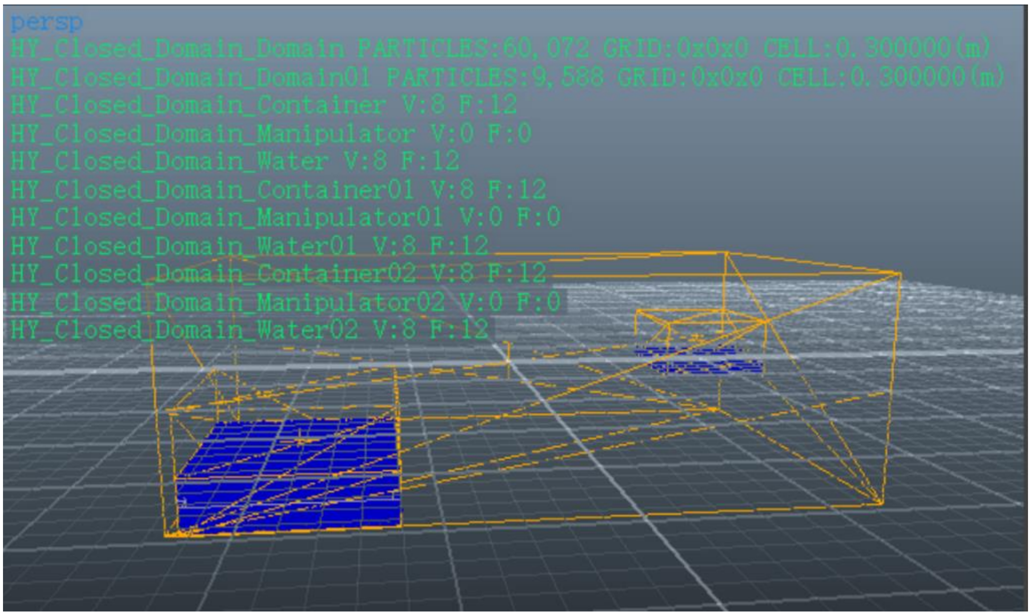

Figure 5. The motion process of the simulated fluid when the cuboid is laid flatly

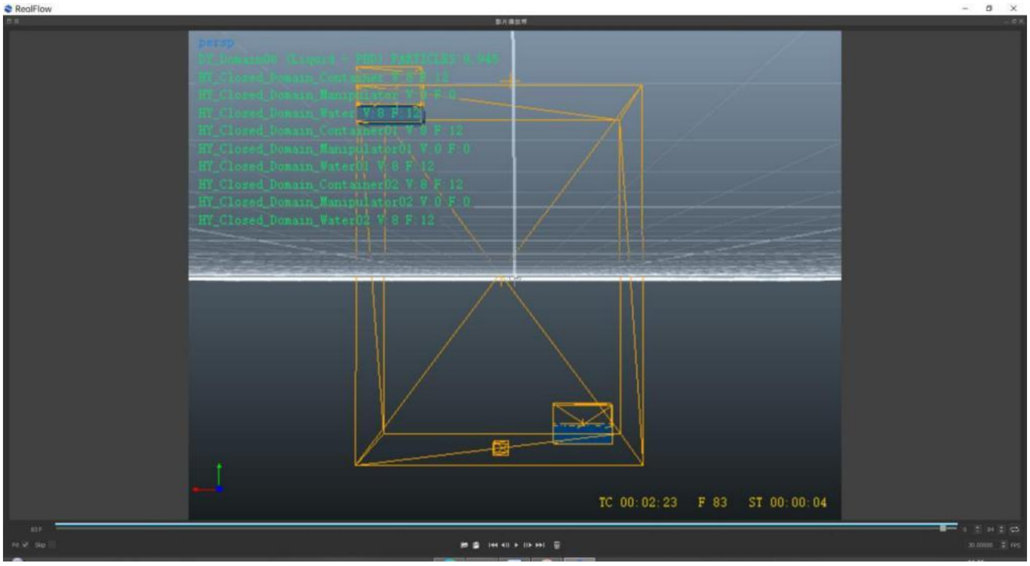

Figure 6. The motion process of the simulated fluid when the cuboid is laid vertically

The center of gravity is presented [5] as $(\bar{X}, \bar{Y}, \bar{Z})$, and the volume element is ΔV.

$$\begin{gathered} \Delta m_i = \rho \Delta V \\ m = \sum_i^{\infty} \rho \Delta V = \iiint_{\Omega} \rho dV \end{gathered} \tag{1}$$

The center of gravity needs to satisfy that the static moments about x, y, and z axes are the same as the sum of all the mass points in the cuboid and the static moments of x, y, and z.

$$\begin{cases} dM_x = x\rho dV \\ dM_y = y\rho dV \\ dM_z = z\rho dV \end{cases} \Rightarrow \begin{cases} M_x = \iiint_{\Omega} x\rho dV \\ M_y = \iiint_{\Omega} y\rho dV \\ M_z = \iiint_{\Omega} z\rho dV \end{cases} \tag{2}$$

Then, the following can be obtained:

$$\begin{cases} \bar{x} = \frac{\iiint_{\Omega} x\rho \mathrm{dv}}{m} = \frac{\iiint_{\Omega} x\rho dV}{\iiint_{\Omega} \rho dV} \\ \bar{y} = \frac{\iiint_{\Omega} y\rho \mathrm{dv}}{m} = \frac{\iiint_{\Omega} y\rho dV}{\iiint_{\Omega} \rho dV} \\ \bar{z} = \frac{\iiint_{\Omega} z\rho \mathrm{dv}}{m} = \frac{\iiint_{\Omega} z\rho dV}{\iiint_{\Omega} \rho dV} \end{cases} \tag{3}$$

The coordinates of the center of gravity can be obtained as $\left(\frac{\iiint_{\Omega} x\rho \mathrm{dV}}{\iiint_{\Omega} \rho \mathrm{dV}}, \frac{\iiint_{\Omega} y\rho \mathrm{dV}}{\iiint_{\Omega} \rho \mathrm{dV}}, \frac{\iiint_{\Omega} z\rho \mathrm{dV}}{\iiint_{\Omega} \rho \mathrm{dV}}\right)$.

Since the mass of the cuboid (including the water tanks) is $\mathrm{M_0} + \frac{8}{\mathrm{k}}$, and the volume is 240 m^3, then $\rho = \frac{\mathrm{M_0} + (\frac{8}{\mathrm{k}})}{240}$. The 3D rotation diagram and the corresponding front view are shown in **Figure 7** and **Figure 8**.

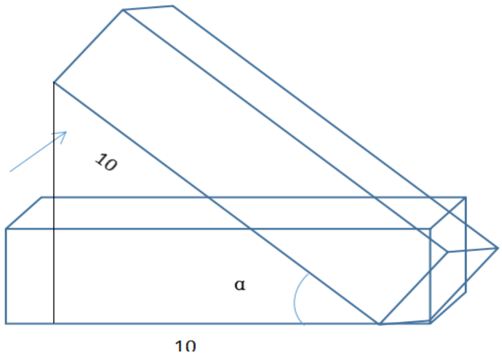

Figure 7. 3D rotation diagram

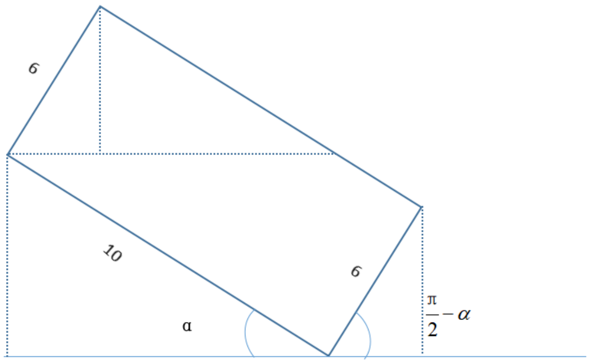

Figure 8. Front view of the 3D rotation diagram

Upon calculation, the integral range of Z is $0\sim10\sin\alpha+6\cos\alpha$, the integral range of y is $0\sim4$, and the integral range of x is $10-10\cos\alpha\sim10\cos\alpha+6\sin\alpha$; thus, the following can be obtained:

$$\bar{x}=\frac{\int_0^{10\sin\alpha+6\cos\alpha}\rho dZ\int_{10-10\cos\alpha}^{10\cos\alpha+6\sin\alpha}\mathrm{x}\mathrm{d}\mathrm{x}\int_0^4\mathrm{d}\mathrm{y}}{\int_0^{10\sin\alpha+6\cos\alpha}\rho dZ\int_{10-10\cos\alpha}^{10\cos\alpha+6\sin\alpha}\mathrm{d}\mathrm{x}\int_0^4\mathrm{d}\mathrm{y}} \tag{4}$$

In the calculation, $\left(2\mathrm{M}_0+\frac{16}{\mathrm{k}}\right)$ is eliminated. It can be seen that the coordinates of the center of gravity have nothing to do with the water filling ratio, so no matter what the water filling ratio is, the relative position of the center of gravity in the cuboid will not change [6].

The centroid is the point at which a force in any direction is applied and the bending moment is zero. The centroid of the surface is the geometric center of the cross-section surface, and the centroid is for abstract geometry. In this problem, the centroid is the geometric center of the cuboid. The difference between the centroid and the center of gravity is density. Hence, the coordinates of the centroid are as follows:

$$\left(\frac{\iiint_\Omega x\cdot\mathrm{dV}}{V},\ \frac{\iiint_\Omega y\cdot\mathrm{dV}}{V},\ \frac{\iiint_\Omega z\cdot\mathrm{dV}}{V}\right) \tag{5}$$

4. Conclusion

It can be concluded that the relative positions of the centroid and the center of gravity do not change during flipping, so the distance between them does not change with different water filling ratios. As long as the water filling ratios of the two water tanks are the same, no matter what the water filling ratio is, the center of gravity will not change. Hence, the distance between the centroid and the center of gravity remains unchanged. The ratio has no effect on the distance between the centroid and the center of gravity.

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Disclosure statement

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