

Risk Assessment of Local Government Debt based on KMV Model: A Case Study of Hangzhou, Zhejiang Province

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Abstract: The present government debt governance focuses on calculating, preventing, and controlling local government debt risk. The default probability of local government debt in Hangzhou, Zhejiang Province, is calculated using a modified “Kealhofer, McQuown, and Vasicek” (KMV) model. The findings reveal that Hangzhou’s debt risk in the next three years is usually manageable, but that debt risk will progressively emerge in the coming years when the debt payback cycle begins.

Keywords: Local government debt; Risk assessment; KMV model

Publication date: December 2021; **Online publication:** December 29, 2021

1. Introduction

In recent years, China has reinforced the standardized management of local government debt by “opening the front door and setting standards” in order to prevent and resolve the risk of local government debt. However, the rapid development of local government debt has been fueled by macroeconomic downward pressure and the centralized digestion and repayment of local platform stock debt. Following the outbreak of new coronavirus pneumonia in 2020, the volume of special local government bonds issued rose once more in order to maintain economic and social stability. Furthermore, illegal financing and guaranteeing of a few local governments still happens on occasion, and local government debt governance is complicated and unpredictable. In this environment, scientifically studying and judging the solvency of local governments, assessing their default risk, and diligently implementing the task of “preventing and resolving big risks” outlined in the report of the Communist Party of China’s 19th National Congress is critical. This paper conducts a case study using quantitative analysis, anticipates Hangzhou’s debt default likelihood over the next three years using the KMV model, and makes policy recommendations based on the prediction results, in the hopes of improving local government governance in the new era.

2. Application principle of KMV model

KMV model is a method established by KMV company in San Francisco in 1997 to estimate the default probability of money-borrowing enterprises. The reason why KMV model can be used to study government debt is that the basic ideas of studying local government debt risk and enterprise credit risk are the same. The local debt raising process is equivalent to temporarily “transferring” the financial rights owned by the government to creditors. When the debt matures and the local government pays off the total principal and interest of the debt, the financial rights can be “redeemed.” If the local government’s solvency income cannot cover the total principal and interest of maturing debt, default will occur ^[1]. Therefore, this paper

uses KMV model to evaluate the debt default risk of Hangzhou local government.

3. Risk assessment of Hangzhou local government debt based on KMV model

3.1. Model building

Assuming that local fiscal revenue follows Markov random process:

$$S_t = f(Q_t) \quad (1)$$

S_T is the guaranteed fiscal revenue of the local government at time t , $f(\cdot)$ is a specific function, and Q_T is a random variable.

Assuming that the local government bonds are due at time t , if the fiscal revenue S_T is less than the total government debt due B_T , the government will default in theory, that is, the default condition of the local government is $S_T < B_T$.

The probability of default is expressed in P , then

$$P = P(S_T < B_T) = P\{f(Q_T) < B_T\} = P\{Q_T < f^{-1}(B_T)\} \quad (2)$$

Let Q_T obey $N(0,1)$ distribution, then equation (2) is transformed into

$$P = N\{f^{-1}(B_T)\} \quad (3)$$

If the default distance is defined as $DD = -f^{-1}(B_T)$, then

$$P = N(-DD) \quad (4)$$

Because fiscal revenue of local government follows Markov random process, it can be expressed as

$$dS_t = \alpha S_t dt + \beta S_t dQ_t \quad (5)$$

Among it, α and β represents the fluctuation rate and growth rate of fiscal revenue available for debt repayment, dQ_t is the increment of Wiener process, $dQ_t = \varepsilon \sqrt{dt}$, ε obey $N(0,1)$ distribution.

Assuming that the beginning reimbursable fiscal revenue is S_0 , it can be obtained from the above formula that when $t > 0$, the local fiscal revenue is

$$S_t = S_0 \exp\left\{\left(\alpha - \frac{1}{2}\beta^2\right)t + \beta\varepsilon\sqrt{t}\right\} \quad (6)$$

At this time, the guaranteed fiscal revenue follows a lognormal distribution:

$$\ln S_t = \ln S_0 + \alpha t - \frac{1}{2}\beta^2 t + \beta\varepsilon\sqrt{t} \quad (7)$$

The mean and variance are:

$$E[\ln S_t] = \ln S_0 + \alpha t - \frac{1}{2}\beta^2 t \quad (8)$$

$$\text{Var}[\ln S_t] = \beta^2 t \quad (9)$$

It can be exported:

$$\alpha = \left[\frac{1}{n-1} \sum_{t=1}^{n-1} \ln \frac{S_{t+1}}{S_t} + \frac{1}{2}\beta^2 t \right] / t \quad (10)$$

$$\beta = \sqrt{\left[\frac{1}{n-2} \sum_{t=1}^{n-1} \left(\ln \frac{S_{t+1}}{S_t} - \frac{1}{n-1} \sum_{t=1}^{n-1} \ln \frac{S_{t+1}}{S_t} \right)^2 \right] / t} \quad (11)$$

Therefore, the default distance and default probability are respectively:

$$DD = \left[\ln\left(\frac{S_0}{B_t}\right) + \alpha t - \frac{1}{2}\beta^2 t \right] / \beta \sqrt{t} \quad (12)$$

$$P = N(-DD) \quad (13)$$

3.2. Financial revenue forecast of Hangzhou from 2020 to 2022

3.2.1. Data stationarity test

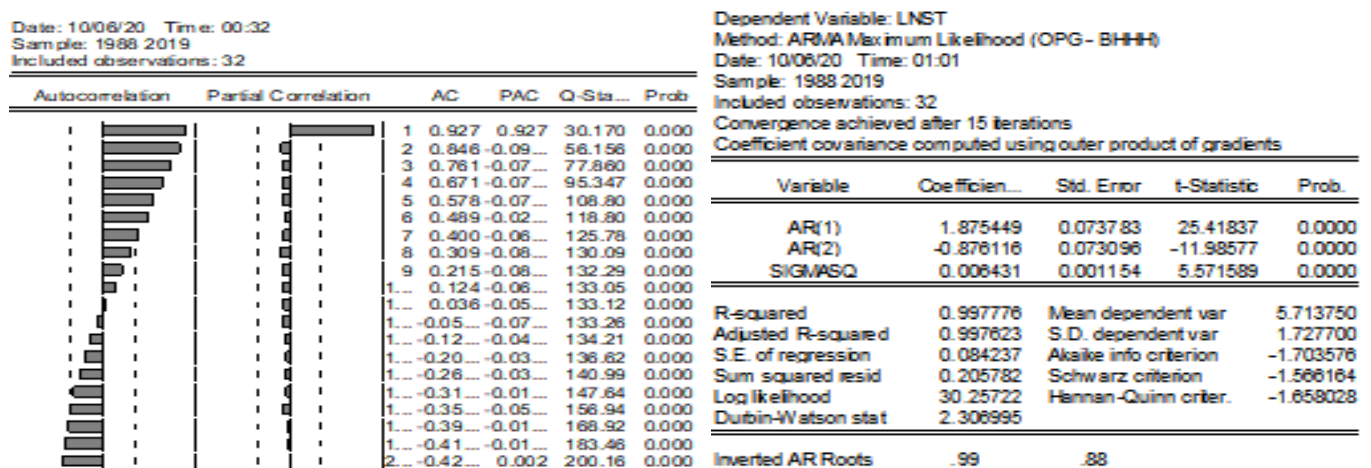
The total fiscal revenue of Hangzhou from 1988 to 2019 disclosed in Hangzhou Statistical Yearbook is selected as the original sample data and recorded as S_t . Due to the large growth of the original data, its logarithm $\text{Ln}S_t$ is taken to obtain smoother data. ADF test was used to test the stationarity of $\text{Ln}S_t$. The results are shown in the **Table 1** below. The ADF test (ADF test is also called unit root test, that is, to test whether there is a unit root in the time series. If there is a unit root, it is a non-stationary time series, on the contrary, it is a stationary time series) assumes that there is a unit root. The data of the first test shows that the statistical value of ADF is 0.193345, which is greater than -3.661661, -2.960411, -2.61916 at the critical points of 1%, 5% and 10%. Therefore, we accept the original hypothesis that there is a unit root in the sequence. Therefore, the sequence is tested again after the first-order difference processing. The data shows that the ADF statistical value is less than the value under the 5% critical value, indicating that the sequence is integrated at the next level of 5% significance. However, in order to make the results more accurate, the sequence is tested after the second-order difference to get the ADF statistical value which is less than the value under the 1% critical value, and the Prob. value is 0. It shows that the sequence coefficient after second-order difference processing is significant, and the result rejects the original hypothesis and is a stationary time series.

Table 1. $\text{Ln}S_t$ ADF test results

	Variate	ADF statistics	1% critical value	5% critical value	10% critical value	Prob.
First inspection	$\text{Ln}S_t$	0.193345	-3.661661	-2.960411	-2.619160	0.9293
Inspection after the first difference	$\text{Ln}S_t$	-3.100149	-3.670170	-2.963972	-2.621007	0.0373
Inspection after the second difference	$\text{Ln}S_t$	-7.102162	-3.679322	-2.967767	-2.622989	0

3.2.2. Construction and application of second-order autoregressive model of $\text{Ln}S_t$ sequence

The stationarity of the data has been tested. Next, the ARMA model of $\text{Ln}S_t$ time series is established by using EView9. Firstly, the autocorrelation function (ACF) and partial autocorrelation function (PACF) are tested for the series. The results are shown in the left figure (**Figure 1**). It can be seen from the figure that the ACF diagram has tailing phenomenon, while the PACF diagram has truncation phenomenon. Therefore, it is judged that the sequence has autocorrelation, and the AR (2) (ARMA (2,0)) model can be established. The results are shown in the right figure (**Figure 2**):



Data source: Export from EViews9 test result

Figure 1. ACF diagram, PACF diagram and ARMA (2,0) model results of $\text{Ln}S_t$ sequence

The second-order differential autoregressive model of LnS_t can be constructed from the above figure:
 $\text{LnS}_t = 1.8754\text{LnS}_{t-1} - 0.8761\text{LnS}_{t-2} + \varepsilon_t$

Based on the model, it is predicted that the fiscal revenue of Hangzhou from 2020 to 2022 will be 3789.54 billion yuan, 3904.95 billion yuan and 3983.83 billion yuan respectively.

3.3. Analysis of solvency in Hangzhou

The solvency of local governments can be reflected by the solvency revenue of local governments. Solvency revenue generally refers to the part of local fiscal revenue that can be used to repay local government debt after deducting rigid government expenditure. Government rigid expenditure can be divided into narrow sense and broad sense. The narrow sense of government rigid expenditure mainly includes five aspects: general public services, public security, health care, education, social security and employment. The broad sense of government rigid expenditure also includes science and technology, culture, tourism, sports and media, national defense, energy conservation and environmental protection, urban and rural communities, agriculture, forestry and water business on the basis of the narrow rigid expenditure. In order to evaluate the debt risk of the government to the greatest extent, this paper selects the government rigid expenditure in a broad caliber as the deduction of solvable fiscal revenue.

Table 2. Broad rigid expenditure and proportion of Hangzhou municipal government from 2010 to 2019 (100 million yuan)

Year Project	2010	2011	2012	2013	2014	2015	2016	2017	2018	2019
General public services	76.40	85.80	89.40	91.60	89.20	101.20	112.50	131.20	150.40	170.80
Public safety	41.60	50.70	53.30	55.40	62.30	71.30	92.60	94.40	104.10	119.00
National defense	0.90	1.00	0.90	0.90	1.10	0.90	1.00	0.90	1.10	1.00
Education	105.90	132.10	146.90	162.40	182.70	223.40	253.00	279.30	315.40	363.60
Science and technology	28.90	35.10	40.20	46.30	52.40	70.10	74.90	92.30	118.20	148.20
Cultural tourism, sports and media	15.40	18.50	20.80	22.40	24.20	44.90	28.40	30.90	33.20	39.00
Social security and employment	63.90	77.50	87.10	94.60	108.30	131.00	149.30	173.20	204.40	229.30
Health expenditure	41.70	51.30	55.90	62.20	71.70	76.60	97.00	112.30	110.80	130.00
Energy saving and environmental protection	13.90	14.10	16.20	19.70	30.00	44.60	40.90	37.80	36.00	47.20
Urban and rural communities	98.30	123.20	104.30	106.40	114.40	158.60	233.80	273.60	293.00	330.80
Agricultural, forestry and water	32.00	39.20	44.90	50.60	56.20	88.00	91.40	89.60	94.20	105.40
Housing security	3.20	6.00	6.50	7.70	9.40	12.30	22.00	28.60	22.70	22.90
Total	522.10	634.50	666.40	720.20	801.90	1022.90	1196.80	1344.10	1483.50	1707.20
Revenue	1245.43	1488.92	1627.89	1734.98	1920.11	2238.75	2558.41	2921.30	3457.50	3650.00
Proportion of rigid expenditure to income	41.92%	42.61%	40.94%	41.51%	41.76%	45.69%	46.78%	46.01%	42.91%	46.77%
Debt service revenue proportion	58.08%	57.39%	59.06%	58.49%	58.24%	54.31%	53.22%	53.99%	57.09%	53.23%

Data source: Hangzhou statistical yearbook

It can be seen from **Table 2** that in recent ten years, Hangzhou's fiscal revenue has increased significantly, and the amount of government expenditure in a broad sense has also shown an upward trend. After calculation, the proportion of solvent revenue shows a downward trend on the whole, although it

remains between 50% - 60%, there are differences between before and after. From 2010 to 2014, the proportion of debt service income remained at 58%, and in 2015, the proportion of debt service income decreased to 54%. Therefore, in order to facilitate the later calculation, this paper takes the average 54.37% of the proportion of Solvent revenue in recent five years as the proportion of debt service income in the next three years.

Based on the financial revenue predicted above, the formula is applied: Solvency income in 2020 = fiscal revenue in 2020 × debt-servicing ratio. It is concluded that the recoverable income in 2020 is 2060.37 billion yuan. By analogy, the debt service income of Hangzhou in 2021 and 2022 is 2123.12 billion yuan and 2166.01 billion yuan respectively.

3.4. Calculate the volatility and growth rate of solvent revenue

The volatility and growth rate of Hangzhou's solvency income in the next three years can be obtained by substituting the above predicted solvency financial income data and historical year data into formulas (10) and (11). The results show that the volatility α of solvent fiscal revenue in Hangzhou in the next three years remain at about 2%, with small fluctuation. As well as the growth rate β was no more than 10%, and the growth is relatively stable.

3.5. Calculate default distance DD and default probability P

Finally, the default distance and default probability of Hangzhou under different debt scales are calculated. In order to facilitate comparison, this paper calculates the debt scale, default distance and default probability when the debt scale accounts for 30%, 40%, 50%, 60%, 70%, 80% and 90% of fiscal revenue.

Table 3. Default distance and default probability under different debt scales in Hangzhou from 2020 to 2022

B_t/S_t	2020			2021			2022		
	B_t	DD	P	B_t	DD	P	B_t	DD	P
0.3	1136.86	60.8515	0	1171.49	9.9016	2.05E-23	1195.15	43.6312	0
0.4	1515.82	32.3713	3.48E-230	1561.98	4.8453	6.32E-07	1593.53	23.2270	1.22E-119
0.5	1894.77	10.2822	4.24E-25	1952.48	0.9227	0.1781	1991.92	7.4043	6.59E-14
0.6	2273.72	-7.7673	1	2342.97	-2.2812	0.9887	2390.30	-5.5248	0.9999
0.7	2652.68	-23.0347	1	2733.47	-4.9895	0.9999	2788.68	-19.8085	1
0.8	3031.61	-36.2525	1	3123.96	-7.3374	1	3187.06	-29.2837	1
0.9	3410.59	-47.9158	1	3514.46	-9.4060	1	3585.45	-37.6383	1

Moody's measured the relationship between bond credit rating and expected default probability: The company's debt with a credit rating above Moody's Baa3 rarely defaults, and the government debt must at least reach the rating of the company's bonds. Therefore, the critical value of the default probability of government debt is 0.4%. According to the standards of Moody's and other international rating agencies, the critical value of default probability of national municipal bonds is 0.4%. When the default probability is lower than 0.4%, the bonds issued can be considered highly repayable. According to **Table 3**, vertically, in the next three years, when B_T/S_T is 50% or less, the default probability of debt in Hangzhou is less than 0.4%, but when the debt accounts for more than 50% of income, the default probability is close to 1 or equal to 1, which proves that there will be a high default risk. In order to ensure that the probability of default is less than the warning line of 0.4%, the debt financing proportion of Hangzhou municipal government should be less than 50% of the fiscal revenue of the current year. Horizontally, from 2020 to

2022, the risk of local government debt tends to decrease gradually. However, this result is based on the assumption that the external conditions change little, and the actual external economic environment and policies will affect the sustainability of the government's debt paying income. For example, the special bonds issued in 2015 have reached the repayment period, and the principal of 7-year bonds issued in 2015 and 5-year bonds issued in 2017 will be repaid in 2022. Scholars such as Li Qian and Wenjing Feng predict that the maturity scale of special bonds in the next 5 years will reach the peak in 2023^[2]. Moreover, due to the expansion of the issuance scale of special bonds, the interest payable is also increasing. Besides, the tightening of the real estate market and the economic downturn caused by COVID-19 will have an unavoidable impact on the local government's debt risk. In conclusion, the debt risk of Hangzhou local government is generally controllable, but the potential risk of implicit debt and the impact of special debt repayment cycle cannot be ignored.

4. Countermeasures and suggestions for model results

Based on the measurement of the default risk of local government debt in Hangzhou in the next three years, combined with the current situation of local government debt in Hangzhou, this paper puts forward some suggestions on the default risk management of local government debt in Hangzhou from the perspectives of disclosure, repayment and early warning.

4.1. Regularly disclose the information of solvency income and implicit debt

The regular disclosure of the income available for debt repayment is conducive to the analysis of important information such as debt repayment ability and debt space, and effectively monitor the debt risk level. The "Government Accounting Standard No. 8 - Liabilities" implemented on January 1, 2019 has clearly included direct liabilities, explicit liabilities and some measurable contingent liabilities into the scope of accounting. At present, the debt data disclosed by local governments do not include implicit liabilities and some contingent liabilities that are difficult to measure, resulting in incomplete debt data and affecting the measurement of government default risk. Only by further defining the scope of local government implicit debt, counting its debt scale and making the implicit debt "explicit", can we obtain a reliable data basis for calculating the optimal debt scale of local government, so as to effectively monitor the debt risk level. However, due to the strong concealment of implicit debt and the lack of unified measurement caliber, it is difficult to disclose accurate data. It is necessary to make a breakthrough in government accounting, legal system and information technology^[3-4].

4.2. Withdraw debt service reserve fund or establish sinking fund

At the end of each year, the local government of Hangzhou can withdraw the debt service reserve fund according to a certain proportion in terms of the financial surplus of the current year, or directly make a detailed medium-term budget for the debt service in the next three years. Under the principle of ensuring financial sustainability, it can add a debt sinking fund, which will be implemented according to this budget after being approved by the central government. In addition, taking the debt service reserve fund or sinking fund as the content that must be accrued and disclosed at the end of each year in government debt accounting, and formulating medium and long-term stock debt resolution plans can make debt management more standardized and professional. The accrued debt service reserve fund or sinking fund is used to repay the stock debt. If there is a debt service crisis next year, it can also be used to make up for it, so as to reduce the debt service burden of the government and tide over the difficulties in time.

4.3. Establish a level III early warning mechanism

On the basis of comprehensive and reliable debt data and basically sound evaluation indicators, Hangzhou

local government should establish a three-level debt risk early warning mechanism to control the degree of debt risk in time. The level III early warning is a mild early warning. When the expected debt default rate is less than 0.4%, it proves that the pressure of the local government to repay the debt in one year is relatively small, and the financial funds of the local government are relatively sufficient to repay the debt. The level II early warning is a moderate early warning. The expected debt default rate is greater than or equal to 0.4% and less than 0.6%. At this time, if it is not controlled, the risk may change at any time. The level I early warning is a high-level early warning. When the expected debt default rate is greater than 0.6%, it can be considered that the local government has relatively large pressure to repay the debt after one year, which proves that there is a large risk of debt default. At this time, the local government departments should immediately reduce the debt scale, make an emergency plan for risk treatment, and report it to the Provincial Department of Finance and the province should report it to central government for approval ^[5]. If the risk status continues to rise, the provincial government shall announce that it is in a state of crisis and immediately start the plan to deal with the debt risk together with the local government.

Disclosure statement

The author declares no conflict of interest.

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