

# An Integral-based Study of Temperature Variation in Wildfire Safe Houses

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**Abstract:** In order to achieve the goal of protecting people from wildfire, we propose to build a safety house to reduce mortality. This paper mainly creates a mathematical model about the house's temperature change. Assuming an ideal heat balance model, we use the method of Joint Cube Systems and Self Iteration to simulate the whole process of heat radiation again.

**Keywords:** Radiation; Safety station; Integration

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## 1. Introduction

According to enormous accidents, forest fire has been received much publicity over last few years. The damage caused by the fire directly threaten the lives of plants, animals, and human and has bad influences on both economics and ecology.

People have done a lot to try to solve the problem. The current research about the wildfire focuses mainly on the prediction system, which monitors the data of climate in real time <sup>[1]</sup>. Mathematically, some experts had done experiments and computer simulation to model the moving pattern of heat radiation <sup>[2]</sup> and fire. Researches also analyze the climate data in different regions to find the relationship between climate and the frequency of wildfire <sup>[3]</sup>. In addition to the prevention of wildfire, we plan to build a safety station to directly protect people from wildfires. But the utility of the safety house should be estimated. Thus, the paper creates a mathematical model to accomplish such estimation.

## 2. The review of thermal theory

The paper focuses on the heat radiation and temperature change, so thermal theories are necessary.

### 2.1. Thermal radiation

Thermal radiation is electromagnetic radiation generated by the thermal motion of particles in matter. All matter with a temperature greater than absolute zero emits thermal radiation. Particle motion results in charge-acceleration or dipole oscillation which produces electromagnetic radiation.

### 2.2. Black body

A black body is an idealized physical body. Its behavior with regard to thermal radiation is characterized by its transmission rate  $\tau$ , absorption rate  $\alpha$ , and reflection rate  $\rho$ . For a black body,  $\tau = 0$ ,  $\alpha = 1$ , and  $\rho = 0$ . Planck offers a theoretical model for perfect black bodies, which he noted do not exist in nature: besides

their opaque interior, they have interfaces that are perfectly transmitting and non-reflective.

**2.3. Stefan-Boltzmann formula**

The Stefan–Boltzmann law describes the power radiated from a black body in terms of its temperature. Specifically, the Stefan–Boltzmann law states that the total energy radiated per unit surface area of a black body across all wavelengths per unit time (also known as the black-body radiant emittance) is directly proportional to the fourth power of the black body’s thermodynamic temperature T.

$$j = \epsilon\sigma T^4$$

In this equation,  $\epsilon$  is the radiation coefficient, as well as the absorption rate.  $\sigma$  is the Stefan-Boltzmann constant, about  $5.67 \times 10^8 \text{ W}/(\text{m}^2 \cdot \text{K}^{-4})$ .  $j$  represents the energy density which is the power of per unit area. The emitting power of an object is

$$P = j \times S = S \times \epsilon\sigma T^4$$

**2.4. The radiated power of fire**

Radiated power is the total radiant energy emitted per unit area of the surface of an object per unit time. Radiated power is a measure of the total power of all electromagnetic radiation passing through a certain area per unit time.

**3. Assumption**

**Assumption 1:** The safe house is considered as a black body so that the electromagnetic waves shining on the black body from the back are completely absorbed.

**Justification 1:** The model is easier to calculate. Also, the result considers safety margin because the safety house has reflectivity, but black body can absorb all energy without any reflection.

**Assumption 2:** The house in the model is regarded as the circumscribed sphere of the house itself.

**Justification 2:** The actual shape of house is difficult to treat mathematically, which makes the model difficult to calculate. But the sphere is easy to use formula. In addition, the sphere also considers safety margin.

**Assumption 3:** The shape of flame is ring-shaped and its height is fixed. internal surface plays the dominant role in radiating heat.

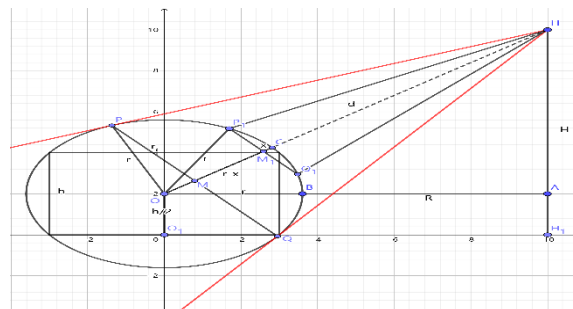
**Justification 3:** The actual distribution of fire is variable and unpredictable. So, we choose the average height and average radiant flux to simplify the model.

**Assumption 4:** The heat conduction in the house is ignored.

**Justification 4:** If the thermal conductivity is considered, it involves complex equations. However, the difference of whether considering it or not is very little, so this can make the model much easier. Also, it considers safety margin because the temperature of all parts of the house is instantaneously changing instead of rising slowly, so it is much larger than the actual one.

**4. Establishing mathematical model: safe house’s temperature change**

**4.1. Geometric model of safe house and fire source**



**Figure 1.** Geometric analysis of the model

As **Figure 1** shows, the safety station is considered into the model as a rectangle with height  $h$ . Then we use its circumscribed sphere to consider the mathematical model.  $r$  is the radius of the sphere.  $H$  represents a point heat source whose height is  $H$ , and  $R$  is the distance from the sphere to the point.  $d$  is the distance of  $OH$ , which intersects the circle at  $C$ . So  $r_c$  can be represented according to the following steps:

$$d = \sqrt{\left(H - \frac{h}{2}\right)^2 + (R + r)^2}$$

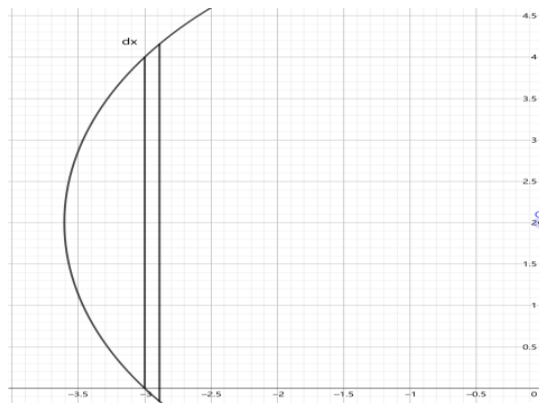
It is easy to prove that  $\triangle PHO \sim \triangle MPO$

$$\therefore \frac{OH}{OP} = \frac{PH}{MP}$$

$$\therefore MP = \frac{OP \times PH}{OH}$$

$$\therefore r_c = \frac{r \times \sqrt{d^2 - r^2}}{d}$$

In **Figure 1.**,  $AP$  and  $AQ$  are tangent to the circle at points  $P$  and  $Q$ , and it is obvious that all the radiations are involved into the tangent lines.  $P_1Q_1$  is the line parallel to  $PQ$ , which represents the movements of  $P$  and  $Q$ .  $PQ$  intersects  $HO$  at point  $M$ . We can draw a circle on the surface of the sphere which includes  $P_1$  and  $Q_1$ . Such circle contains all points on the sphere which have the same distance from point  $A$ . So we calculate the power acts on the circle through several procedures:



**Figure 2.** The explanation of differentiation

The differentiation of  $x$  is shown in **Figure 2**. Since  $x$  is very small, the differentiation of power is calculated, as shown below:

$$dE_p = E_b \frac{dA}{4\pi(d - r + x)^2} = E_b \frac{2\pi\sqrt{r^2 - (r - x)^2}}{4\pi(d - r + x)^2} dx = E_b \frac{\sqrt{r^2 - (r - x)^2}}{2(d - r + x)^2} dx$$

The largest distance that the radiation can reach is the circle which includes the tangent points, because the larger circle will have no intersection points with the lines pass through  $A$ .

The formula integrates the total differential circles to get the total power of one point:

$$E_p = \int_0^{\sqrt{r^2 - r_c^2}} E_b \frac{\sqrt{r^2 - (r - x)^2}}{2(d - r + x)^2} dx = \int_0^{\frac{r^2}{d}} E_b \frac{\sqrt{r^2 - (r - x)^2}}{2(d - r + x)^2} dx$$

The ring is divided into infinite lines. For a line,

$$E_p = \int_0^{\sqrt{r^2 - r_c^2}} E_b \frac{\sqrt{r^2 - (r - x)^2}}{2(d - r + x)^2} dx = \int_0^{\frac{r^2}{d}} E_b \frac{\sqrt{r^2 - (r - x)^2}}{2(d - r + x)^2} dx$$

And the total power of the ring is the integration of each  $E_l$ :

$$E = \int_0^{2\pi} E_l ds = \int_0^{2\pi} E_l h dl = \int_0^{2\pi} E_l R_{total} d\theta = 2\pi R E_l = 2\pi R_{total} \int_0^H \int_0^{\frac{r^2}{d}} E_b \frac{\sqrt{r^2 - (r - x)^2}}{2(d - r + x)^2} dx dh$$

Since the house itself has both absorbing and emitting power, the net change of power is the difference of the two. The emitting power is related to temperature, but the temperature changes as it receives heat, so we should make an additional assumption that T remains the same in a period of time. We can choose 1 min or less to substitute the differentiation of time.

Let E represents the emitting power of the fire, as well as the absorbing power of the house. S is the surface area of the sphere, and  $\sigma$  is the Stefan-Boltzmann constant. In the short period of time, the house's emitting power can be written as  $S \times \sigma \times T_{i-1}^4$ .

The net power of house's energy change is

$$P_{net} = E - S \times \sigma \times T_{i-1}^4$$

Then, the product of net power and time is the net energy change. But the house has reflectivity, so it should be considered. Then we can use the mass and specific heat of the house to calculate the change of temperature.

The following formula shows the process of calculating the change of temperature for natural numbers  $i$ :

$$T_0 = 22^\circ\text{C} = 295.15\text{K}$$

$$T_i = T_{i-1} + \frac{(1 - \alpha) \times P_{net} \times t}{c \times m}$$

## 4.2. Model simulation

Using the formula above, the computer simulates the actual change of temperature. Parameters of the simulation are based on the geographical data of Liangshan before the happening of wildfire during winter and spring. Portland cement is chosen as the construction material, and all parameters are labeled in **Table 1**.

**Table 1.** The parameters used for simulation of the elementary model

	Math Symbols	Numerical Value (or range)	Unit	Data Source
Absorption rate	$\alpha$	1	-	Theoretical value of black body
Width of isolation zone	R	10-20	m	Interview
Radius of Safe House	r	6-9	m	Field Research
Height of wildfire	H	30	m	Interview
Specific heat capacity of building materials	c	840	$\text{J} \cdot \text{kg}^{-1}\text{K}^{-1}$	Reference
Density of house	$\rho$	3075	$\text{kg} \cdot \text{m}^{-3}$	Reference
Initial temperature of environment	$T_0$	22	$^\circ\text{C}$	Interview

Rescuing time	$t$	24	Hours	Interview
Average flame emission Power	$E_b$	$2 \times 10^5$ <sup>[2]</sup>	$W \cdot m^{-2} \cdot k^{-4}$	Reference

#### 4.2.1. Notation description

These data are all related to our mathematics model: some of data are confirmed like density of house, while some of data have its value range according to the fireman's description, such as width of isolation zone, radius of safe house. Among these data, it is essential to explain the meaning of some unfamiliar concepts.

- (1) **The absorption rate:** The building's performance in reflecting or absorbing the radiation energy, where detailed explanation is on "The review of Thermal theory."
- (2) **Isolation zone:** Zone that all combustible is cleared out including woods, where always encircle the whole firing forest.
- (3) **Rescuing time:** The maximum time which safety house would endure burning. Because the rescuing time is totally different in each wildfire, thus we average the time to 24 hours.
- (4) **Initial temperature of environment:** The forest's temperature before the wildfire. The average temperature in spring and winter, when the wildfire always occurs, is chosen.

Such indicators are used to simulate and graph the approximate line to show the relationship between these indicators and temperature or energy of the safe house. Except for the range, we will determine a definite value under the range to simulate a hypothetical wildfire environment to test the safe house.

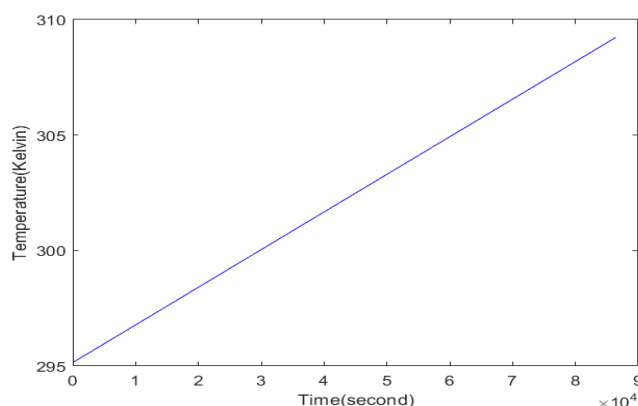
#### 4.2.2. Model simulation

After calculation of these data with simulation model, the relationship is approximately corresponding to expectation. The best way to reduce temperature for safe house is to maximize the radius of safety house and the width of isolation zone, size, as well as minimize the absorption rate. However, the final temperature exceeded our expectations. The method may be reasonable but the concrete result cannot be applied anymore. Therefore, we should aim at its method variation and model adjustment to get more accurate result and verify the method. **Table 2.** shows the data used for later simulation of the new model.

**Table 2.** The parameters used for simulation of the modified model

Mathematical Symbol	Numerical Value (or range)	Unit
$\alpha$	0.05	
$R$	6	meter
$r$	9	meter
$H$	30	meter
$c$	550	$J \cdot kg^{-1}K^{-1}$
$\rho$	7850	$kg \cdot m^{-3}$
$T_0$	295.15	Kelvin
$t$	24	Hour
$E_b$	$2 \times 10^5$	$W \cdot m^{-2}$
$h$	6	meter

**Figure 3** shows the relationship between time and temperature. After such safe house is burned in 24 hours, its temperature will reach less than 310K, approximately 37°C after 24 hours. So, the modified model ensures that safety house provides people with enough time to wait for rescue.



**Figure 3.** The change of temperature with time

## 5. Conclusion

Through research, we first know the factors that influence the efficiency of heat radiation. Then we propose a feasible plan to use a safety house to help people extend the survival time. But the safety house's temperature should be controlled in a safe range, so we also design a new experiment to verify the first model's conclusion about several relation. During that process, we also improve the structure of our safe house to enable it to have better fire-proof performance than its original design. Then, we realized that the reflection layer is necessary and the previous model overlooked the emitting power of the house itself, so we modified it to make the prediction more accurate and ensure the house to be much safer. Assuming an ideal heat balance model, we use the method of Joint Cube Systems and Self Iteration to re-simulate the radiation process and try to add the layer of radiation.

## Disclosure statement

The author declares no conflict of interest.

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