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The Influence of Nonlinear Damping on the Transport Properties of Brownian Particles

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Abstract: This paper studies the directed transport of Brownian particles in a flashing ratchet under a nonlinear damping environment. The research shows that the nonlinear relationship between the damping term and velocity can transform the system from a passive energy dissipation mode to a nonequilibrium process of actively acquiring energy from the environment. This study not only reveals the complex interaction mechanism between nonlinear damping and system energy dissipation but also breaks the limitations of linear theory in describing nonequilibrium energy conversion. It provides a new theoretical perspective for in-depth understanding of the dynamic behavior of micro-nonequilibrium systems, and has important theoretical significance and application value.

Keywords: Nonlinear damping; Brownian particles; Flashing ratchet; Directed transport

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1. Introduction

Biomolecular motors (such as ATP synthase and myosin) serve as core "machines" for energy conversion in living systems, capable of converting chemical energy into mechanical energy at the nanoscale with nearly 100% efficiency [1–3]. For example, ATP synthase on the inner mitochondrial membrane, as a sophisticated rotary molecular motor, can use the proton transmembrane gradient to drive the rotation of its "rotor" component, thereby efficiently catalyzing the synthesis of ATP, which is the core mechanism of biological energy conversion [4–6]. To understand the microdynamics and energy conversion of biomolecular motors in organisms from a physical perspective, models such as the "Brownian ratchet" have been proposed based on Brownian particle theory [7]. Brownian particles refer to microscopic particles suspended in a fluid medium that undergo irregular motion due to random collisions with surrounding molecules, acting as a bridge connecting the microscopic world and macroscopically observable phenomena [8–9]. In fact, the working environment of Brownian particles is usually a medium with viscosity much greater than inertia, such as liquids or gases [10]. Due to the irregular thermal motion of molecules in the environment, particles are continuously subjected to random forces, resulting in random and uncertain motion. Linear damping, as a classic ideal model describing energy dissipation in such systems, has a damping force proportional to the first power of the motion velocity, with the mathematical expression $F = -\gamma_0 v$ (where γ_0 is the damping coefficient and v is the instantaneous velocity of the object). As a universal model for passive energy dissipation, this model can be used to describe the conversion and dissipation of system mechanical energy into thermal energy or other forms of energy [11–12].

However, the classic linear damping model has limitations in describing the complex motion of real molecular motors. In this regard, this paper introduces a new form of damping—Rayleigh-Helmholtz damping (referred to as RH damping) to overcome the shortcomings of linear damping, thereby generating richer dynamic behaviors [13]. According to the fluctuation-dissipation theorem, the energy exchange between particles and the surrounding medium is usually determined by both damping forces and random forces [14]. In a linear damping environment, the system's energy is mainly passively dissipated through friction or viscous resistance from the external environment, leading to relatively low dissipation efficiency. The introduction of nonlinear damping can establish a dynamic balance between energy input and dissipation, preventing the system from becoming unstable due to over-driving, thereby maintaining stable operation of the system in a nonequilibrium environment [15]. For example, Klimontovich's nonlinear damping theory reveals that atoms in a laser cooling field convert external light energy into internal momentum through a nonlinear dissipation mechanism, realizing the transformation from passive to active energy absorption, and providing a key theoretical framework for energy conversion in active matter.

Therefore, this paper establishes a stochastic dynamic model of Brownian particles in a flashing ratchet and studies the influence of nonlinear damping on their directed transport. From the perspective of energy dissipation, this study reveals that nonlinear damping breaks detailed balance, drives the system away from equilibrium, and achieves directed flow, providing new ideas for understanding the directed transport of Brownian particles in nonequilibrium statistical physics.

2. Model

This paper mainly studies the transport properties of Brownian particles in a flashing ratchet under a nonlinear damping environment, and their dynamic behavior can be described by the following motion equation:

$$\dot{x} = v,\quad \dot{v} = -\gamma(v)v - \frac{\partial V(x)}{\partial x} Z(t) + \xi(t) \tag{1}$$

where x represents the position coordinate of the particle, v represents the velocity of the particle, $\gamma(v)$ denotes the Rayleigh-Helmholtz damping function, whose expression is:

$$\gamma(v) = \gamma_0 (v^2 - v_0^2) \tag{2}$$

Here, γ_0 is the intensity parameter of the damping coefficient, and v_0 is the characteristic velocity.

This paper considers Brownian particles moving in a flashing ratchet, so the specific form of $V(x)$ is:

$$V(x) = \begin{cases} \dfrac{V_0}{\lambda} x, (0 < x \le \lambda), \\ \dfrac{V_0}{L-\lambda}[L - x], (\lambda < x \le L), \end{cases} \tag{3}$$

where V_0 represents the barrier height, L is the spatial period of the flashing potential, and λ is the asymmetry coefficient.

The flashing switch $Z(t)$ form of the potential field is $Z(t)=\begin{cases}1, n\tau < t \le n\tau+\tau_{on} \\ 0, n\tau+\tau_{on} < t \le (n+1)\tau\end{cases}$, where τ_{on} denotes the time the potential field remains "on", τ_{off} denotes the time the potential field is "off", and its period is $\tau=\tau_{on}+\tau_{off}$, where $n=0,1,2,\cdots$.

In Equation (1), $\xi(t)$ represents Gaussian white noise, which satisfies the following relations: $<\xi(t)>=0$, $<\xi(t)\xi(t')>=2D\delta(t-t')$, where $D=k_B T$ is the noise intensity, k_B is Boltzmann's constant, and T is the external temperature.

This paper uses the second-order Runge-Kutta method to numerically calculate the average velocity of Brownian particles, whose specific form is as follows:

$$<\dot{x}>=\lim_{t\to\infty}\frac{<x(t)-x(0)>}{t} \quad (4)$$

To obtain a stable ensemble average, this paper selects a time step $h=1\times10^{-3}$, simulates 1×10^{5} trajectories, and each trajectory evolves for 10^6 steps. Other parameters: $\gamma_0=3.0, v_0=1.0, V_0=1.0, D=0.1, L=2.0, \tau_{on}=\tau_{off}=2.0$.

3. Results and discussion

This paper discusses the dependence of the transport properties of Brownian particles on the asymmetry coefficient λ. The results show that in a linear damping environment, the average velocity $<v>$ of Brownian particles changes non-monotonically with the asymmetry coefficient. It is not difficult to see from Equation (3) that the potential field is symmetrically distributed when $\lambda=1.0$, and particles cannot form an obvious directed flow at this time. However, when $\lambda\neq1.0$, the potential field has obvious asymmetry, and the system can generate a net directed flow under time-periodic regulation. Specifically, when the potential field is in the "on" state (potential field is "activated"), Brownian particles are confined in potential wells because the external environment cannot provide sufficient energy to cross the potential barrier, making it difficult to form directed motion. After a period of time, when the potential field switches to the "off" state (potential field is "deactivated"), thermal noise drives Brownian particles to move freely, resulting in their uniform distribution in space. When the potential field is "turned on" again, due to the spatial asymmetry of the potential field, the probability of particles "sliding" into the potential wells from both sides of the barrier top is different, thereby generating a "net" directed flow. In other words, when $\lambda<1.0$, the spatial size of the left side of the barrier is significantly smaller than that of the right side, so the probability of Brownian particles sliding from the barrier top to the left side of the potential well is lower than that to the right side. Conversely, when $\lambda>1.0$, the spatial size of the left side of the barrier is larger, so particles will generate a negative directed flow. Its essence is the combined effect of the spatiotemporal symmetry breaking of the potential field and thermal noise, which breaks the detailed balance and time-reversal symmetry of the system, thereby inducing a "net" directed flow of Brownian particles.

However, in the RH damping environment, Brownian particles exhibit different transport behaviors (as shown in **Figure 1(b)**). This is due to the nonlinear relationship between the RH damping term and the particle velocity—even when Brownian particles are not driven by external energy sources, RH damping can continuously provide energy for the particles. More importantly, when $\lambda\neq1.0$, even though the flashing ratchet periodically resets the spatial distribution of Brownian particles, RH damping can inject sufficient energy into the particles to cross the potential barrier. For example, when $\lambda > 1.0$, the average velocity of Brownian particles in the linear damping environment is negative, while Brownian particles in the nonlinear damping environment can obtain sufficient energy to cross the potential barrier, showing positive transport characteristics (average velocity greater than zero). This further confirms that RH damping can provide sufficient energy for particles to cross the potential barrier, thereby realizing directed transport.

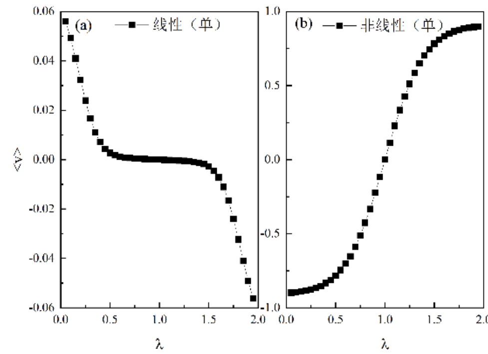

Figure 1. Curves of the average velocity $< v >$ of Brownian particles varying with the asymmetry coefficient λ

4. Conclusions and prospects

By establishing a stochastic dynamic model, this paper systematically studies the transport behavior of Brownian particles in a flashing ratchet and explains the mechanism of RH damping on the directed transport of Brownian particles. Its physical essence lies in the fact that RH damping breaks the spatiotemporal reversal symmetry and detailed balance conditions of the flashing ratchet, thereby inducing particles to form a directed flow. From the perspective of energy dissipation, this study shows that nonlinear dissipation, as an externally tunable parameter, can effectively control the intensity and direction of the directed flow. This theory provides a new perspective for understanding the energy conversion of biomolecular motors in organisms and developing the dissipation-fluctuation coordination theory in nonequilibrium statistical physics. On the one hand, this study reveals how molecular motors realize directed motion using chemical energy (such as ATP hydrolysis) in a nonequilibrium state, a process that is essentially “controlled” nonlinear dissipation. On the other hand, this model can describe the adaptive behavior of molecular motors in complex cellular environments (such as viscous media, periodic potential fields, and non-conservative force fields), explaining the mechanism of maintaining directed motion and efficient energy conversion under external disturbances. This research not only deepens people’s understanding of the energy conversion mechanism in living systems but also provides an important theoretical basis for the design optimization of artificial molecular motors and biophysical regulation strategies based on nonequilibrium thermodynamics. In addition, the important bridge established by the nonlinear damping model between stochastic dynamics and active matter motion broadens the application scope of fluctuation theorems in nonequilibrium systems and provides a theoretical basis for describing the entropy production dynamics and response characteristics of different nonequilibrium systems. In the future, research in this direction will play a key role in connecting basic theories with interdisciplinary fields such as life sciences and materials, further deepening people’s understanding of random fluctuations and complex system dynamics.

Disclosure statement

The authors declare no conflict of interest.

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