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Research on Teaching Methods of Probability Theory and Mathematical Statistics Based on Modeling Ideas

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Abstract: This paper delves into teaching methods for probability theory and mathematical statistics grounded in modeling concepts, to augment students' comprehension and application of these theories through practical modeling cases. By dissecting the significance of modeling in these fields, the paper puts forth teaching approaches centered on modeling ideas, demonstrating their efficacy through practical illustrations. Additionally, the paper emphasizes the integration of hands-on modeling tasks. It encourages students to collect and analyze data independently, fostering their problem-solving skills and enhancing their engagement and participation in learning.

Keywords: Probability Theory and Mathematical Statistics; Mathematical modeling; Teaching methods; Case study analysis

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1. Introduction

Probability Theory and Mathematical Statistics are important branches of mathematics. They study the regularities of random phenomena and statistical data analysis methods [1]. Probability Theory mainly studies the probability laws of random phenomena, including the probability of random events, the distribution of random variables and probability distribution functions. Mathematical statistics, on the other hand, is concerned with inferring population characteristics and making decisions based on sample data, including parameter estimation, hypothesis testing, analysis of variance, and so on [2].

In practical applications, Probability Theory and Mathematical Statistics are widely used in various fields such as natural sciences, social sciences and engineering [3]. For example, in natural sciences, probability theory and mathematical statistics are used to analyze experimental data and study natural laws [4–7]. In social sciences, they are used to analyze social phenomena and forecast trends [8]. In engineering, they are used to design experiments and optimize production processes [9].

Traditional teaching methods often focus on theoretical derivation and formula memorization, making it difficult for students to connect theoretical knowledge with practical problems, leading to a lack of deep understanding [10]. Some students have expressed that they can solve homework and test questions, but they struggle to apply the learned knowledge to solve real-world problems. Essentially, this is due to a lack of modeling training, or in other words, a lack of modeling mindset. The modeling mindset refers to the way of thinking that involves transforming real-world problems into mathematical language and abstracting, simplifying, and idealizing them. Therefore, integrating the theory of Probability Theory and Mathematical Statistics with real-world problems through the use of modeling ideas is an effective way to improve teaching effectiveness.

By introducing the modeling mindset, students can better understand abstract mathematical concepts and apply their knowledge to solve real-world problems [11]. For example, in teaching, real-world cases can be introduced, allowing students to solve practical problems by establishing mathematical models, thereby cultivating their modeling and problem-solving abilities. This approach can enhance students' understanding and application abilities of probability theory and mathematical statistics, thereby improving teaching effectiveness.

2. The application of modeling thinking in teaching

The application of modeling thinking in teaching refers to using modeling methods and thought processes to guide students in learning and solving real-world problems [12–14]. Teaching methods based on modeling thinking emphasize introducing Probability Theory and Mathematical Statistics through practical problems, cultivating students' modeling abilities and problem-solving skills. For example, when introducing discrete random variables, a "coin toss" experiment can be used to allow students to calculate the probability of the coin landing heads up based on experimental data, thus understanding the concept and calculation of discrete random variables.

Specifically, the application of modeling thinking in teaching includes the following aspects:

(1) Problem Abstraction and Modeling:

Transform real-world problems into mathematical problems and establish corresponding mathematical models. Teachers can guide students to analyze the essence and key factors of problems, thereby cultivating students' abilities in abstraction and modeling.

(2) Model Assumptions and Solution:

When building models, it is necessary to make certain assumptions and simplifications of the problem, and then use mathematical tools for solving it. Through this process, students can learn to simplify problems with reasonable assumptions and use mathematical methods to solve them.

(3) Model Evaluation and Optimization:

After establishing the model, it needs to be evaluated to see if it fits the actual situation and then optimized. This helps cultivate students' critical thinking and innovation abilities.

(4) Result Interpretation and Application:

Finally, students need to interpret the results of the model as solutions to real-world problems and explore the practical value of these results. This helps students combine theoretical knowledge with practical problems, enhancing their problem-solving abilities.

Overall, the application of modeling thinking in teaching can stimulate students' interest in learning, cultivate their practical problem-solving skills and innovative spirit, and improve their mathematical modeling and practical abilities. This approach makes teaching more lively, practical and challenging.

3. Case study analysis

This section will explore specific examples of how modeling thinking can be effectively applied in teaching probability theory and mathematical statistics. Through these examples, it will be demonstrated how integrating practical problems and modeling exercises can enhance students' understanding and application of theoretical concepts.

The mathematical expectation of a random variable is the weighted average of its possible values. In other words, it is the sum of the products of each possible value of the random variable and their respective probabilities in repeated experiments. The mathematical expectation reflects the average tendency or central location of the random variable. During specific teaching sessions, the application of mathematical expectation in real life can be illustrated as follows.

3.1. Example 1: Expected number of students on a randomly selected bus

A class of 120 students is taking three buses to a symphony concert. One bus has 36 students, another bus has 40 students, and the third bus has 44 students. When the buses arrive, a student is randomly selected from these 120 students. Let X represent the number of students on the bus from which the randomly selected student comes and the expected value of X, the number of students on the bus, are to be found.

Solution:

(1) Define the random variable: X is the number of students on the bus that the randomly selected student is on.

(2) Possible values and their probabilities: There are 36 students on the first bus. There are 40 students on the second bus. There are 44 students on the third bus. The probability of selecting a student from each bus is proportional to the number of students on that bus.

(3) Calculate the probabilities: Since each student has an equal chance of being selected, the probability that the selected student is from a particular bus is proportional to the number of students on that bus. Probability p_1 that the student is from the first bus:

$$p_1 = P\{X = 36\} = \frac{36}{100},$$

Probability p_2 that the student is from the second bus:

$$p_2 = P\{X = 40\} = \frac{40}{120},$$

Probability p_3 that the student is from the third bus:

$$p_3 = P\{X = 44\} = \frac{44}{120},$$

(4) Expected value calculation. The expected value E[X] is calculated as the weighted average of the number of students on each bus, using the probabilities p_1, p_2, and p_3 as weights.

$$E[X] = 36p_1 + 40p_2 + 44p_3 = 40.2667.$$

Table 1. Probability distribution of random variable

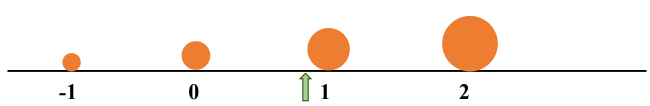
Number of students on the bus *X*	36	40	44
Probability, *p*	36/100	40/120	44/120

The expected number of students on the bus from which a randomly selected student comes is approximately. Although the average number of students per bus is 120/3 = 40, the expected value of the number of students on the bus from which a randomly selected student comes is 40.2667. This expected value is higher than the simple average, and this discrepancy can be explained by considering the probabilities of selecting students from each bus. The higher expected value occurs because a student is more likely to be chosen from a bus with more students. This phenomenon is a general principle: the more students a bus has, the higher the probability that a randomly selected student will be from that bus. Consequently, buses with more students contribute more heavily to the expected value, effectively weighting the expected value towards buses with larger numbers of students.

One of the concepts explained in the class is the Mathematical Expectation and the Concept of Center of Mass. The concept of mathematical expectation can be likened to the physical concept of the center of mass in mass distribution. Considering a discrete random variable X with a probability distribution, it can be visualized that a massless rod with small masses placed at points x_i, where each mass corresponds to the probability $p(X = x_i),\ i \geq 1$. **Figure 1** shows a schematic diagram of mass distribution. Imagine placing masses at points x_i on a rod. The point at which the rod balances is called the center of mass. For those familiar with basic mechanics, it is straightforward to prove that this balance point is indeed E[X], the expected value of X. This schematic diagram of mass distribution provides a clear visual understanding of the concept of the expected value. It shows how the combination of probability masses and their corresponding positions determines the expected value. The concept of the center of mass calculates the expected value more intuitive and easier to grasp. This approach helps students connect abstract mathematical concepts with real-world physical phenomena, thereby enhancing their understanding of probability and statistics.

The expected value E[X] being greater than the simple average number of students per bus illustrates an important characteristic of probability distributions: larger groups (or higher values with higher probabilities) have a greater influence on the expected value. This concept is crucial in understanding the behavior of random variables and their practical applications, emphasizing the importance of weighting in calculating expectations.

By considering the analogy to the center of mass, students familiar with basic mechanics can intuitively grasp the concept of expected value, further solidifying their understanding of probability and statistics.



Center of Mass 0.9

$p(-1) = 0.10,\ p(0) = 0.25,\ p(1) = 0.30,\ p(2) = 0.35$

Figure 1. Schematic diagram of mass distribution.

When dealing with problem sets, one should not only focus on problems with sufficient conditions but also introduce some real-world problems with insufficient conditions. Students should be encouraged to collect, analyze data and build models. By actively engaging students in understanding, abstracting and solving problems, their interest and ability can be enhanced. In addition, regular thematic discussions should be held to encourage students to dare to ask questions and express their opinions, thus strengthening their ability to communicate and learn from each other. For example, when teaching the binomial distribution, to deepen students' understanding of the knowledge, a specific example can be used to illustrate the practical application background and mode of the binomial distribution.

3.2. Example 2: Impact of new policy on students' participation in extracurricular activities

Suppose a university student union wants to evaluate the impact of a new policy on students' participation in extracurricular activities. This policy requires students to participate in at least one student union-organized activity per semester to earn extracurricular activity points.

Problem statement: The student union would like to understand the willingness of students to participate in student organization activities before and after the implementation of the new policy.

Problem analysis: Participation in extracurricular activities can be divided into two categories: (1) before the implementation of the new policy and (2) after the implementation of the new policy. First, guide students to conduct research and collect data to estimate the frequency of student participation in activities before and after the policy. Assuming that before the new policy was implemented, an average of 30% of students participated in student organization activities. The binomial distribution can be used to calculate the probabilities of different numbers of participants. Assuming that after the new policy is implemented, the percentage of students participating in student organization activities increases to 40%. The binomial distribution can be used again to calculate the probabilities of different numbers of participants.

Approximation: The probabilities can be calculated using the approximation of the normal distribution. When the total number of students is sufficiently large, the binomial distribution can be approximated by the normal distribution. In this case, it is convenient to calculate the probability of having n or more students participating in student organization activities. For example, if the total number of students is known to be 10,000 and after the new policy is implemented, the number of participants follows a binomial distribution X–$B(10000,0.4)$, where X is approximately normally distributed with a mean of 4000 and a variance of 2400. Therefore, the probability of 2000 or more students participating in student organization activities can be approximated by:

$$
\begin{aligned}
& P\{X \geq 2000\} \\
& = P\left\{\frac{X-4000}{\sqrt{2400}} \geq \frac{2000-4000}{\sqrt{2400}}\right\} \\
& = 1-\Phi\left(\frac{2000-4000}{\sqrt{2400}}\right) \\
& \approx 1
\end{aligned}
$$

Figure 2 shows the probability distribution of the number of students participating in activities. By solving this problem, students not only learn how to collect, analyze data, build models and solve practical problems but also experience the fun of this process. At the same time, this learning method also enables students to easily grasp probability knowledge, enhancing their enthusiasm and initiative in learning probability. Therefore, this problem-solving approach not only effectively improves students' ability to apply mathematics but also opens up broader learning and thinking space for them.

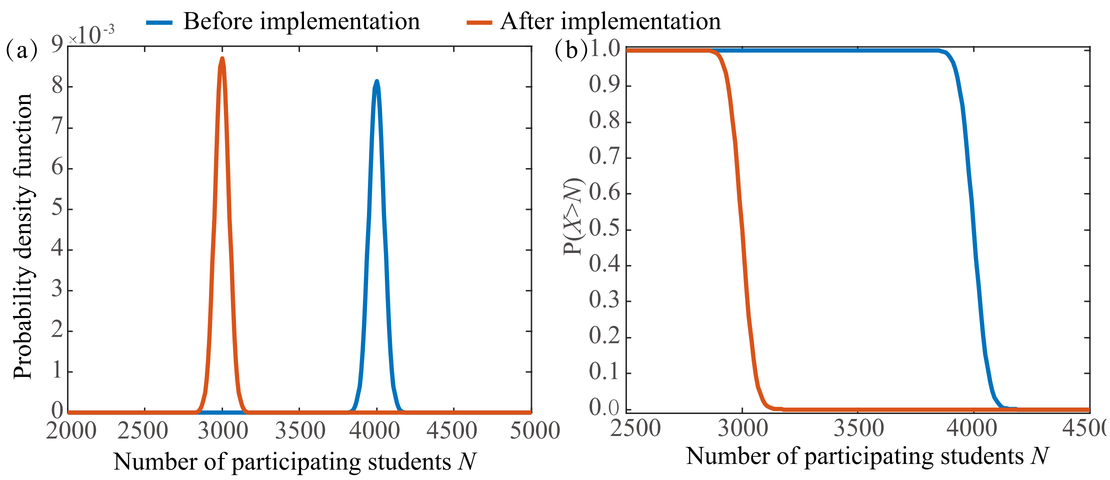


Figure 2. Probability distribution of student participation in activities.

4. Conclusion

In the teaching process of probability theory and mathematical statistics, there are many ways to cultivate students' innovation ability. However, for students to have the innovative ability and the courage to innovate, it is most important to require teachers to improve their teaching methods, guide students to use their brains and hands more, be courageous in exploration, dare to propose new questions and be adept at solving new problems [15]. Teaching methods based on modeling thinking can enhance students' interest and participation in learning, help them integrate theoretical knowledge with practical problems and cultivate their ability to solve practical problems. Therefore, it is necessary to introduce modeling thinking more in teaching to make it more vivid and practical.

In addition, modeling-based teaching methods can also cultivate students' teamwork and communication skills. In the modeling process, students need to cooperate, discuss and communicate with each other, which helps them better play their roles in the team and improve their overall quality. Introducing modeling thinking in teaching can deepen students' understanding of theoretical knowledge through various forms of practical activities, such as case analysis, group discussions and project research.

To better implement the modeling-based teaching method, teachers need to constantly update their knowledge structure and teaching philosophy, actively participate in teaching reform research and explore teaching models that are suitable for students' characteristics. At the same time, schools should also provide necessary support and resource guarantees, such as equipping advanced teaching equipment and tools and organizing teacher training and seminar activities, to create a good environment for the smooth implementation of teaching reform.

In conclusion, the modeling-based teaching method is not only a beneficial supplement to traditional teaching methods but also an important way to improve teaching effectiveness and student innovation ability. In future teaching practices, it is important to continue to explore and improve this teaching method, summarize experiences and achievements and strive to contribute to the cultivation of more high-quality talents with innovative and practical abilities.

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