Education Reform and Development, 2024, Volume 6, Issue 6
https://ojs.bbwpublisher.com/index.php/ERD
Online ISSN: 2652-5372
Print ISSN: 2652-5364

The Application of Markov Chain in Middle School Mathematics Teaching Evaluation

Zhanhuai Ji*, Yixue Yang

Xi'an University of Science and Technology, Xi'an 710054, Shaanxi Province, China

***Corresponding author:** Zhanhuai Ji, specialyyx@163.com

Abstract: This paper discusses the application of Markov Chain model to evaluate middle school mathematics teaching. First, the paper introduces the basic concept of the Markov Chain and proposes how to divide student grades into different levels and construct the initial state vector and the transition probability matrix. Then, the paper demonstrates how to use the Markov Chain model to analyze the changes in mathematics grades of two classes, calculate the degree of progress and efficiency, and evaluate the teaching effectiveness of different teachers through actual cases. The results show that Class One has a better teaching effect, with higher degrees of progress and efficiency. The research proves that using the Markov Chain model can more objectively analyze teaching effects and guide teachers to improve their teaching methods. This model provides a new approach to evaluating the quality of middle school mathematics teaching and has practical application value.

Keywords: Markov Chain; Mathematics teaching; Teaching quality evaluation

Online publication: July 12, 2024

1. Introduction

Mathematics, as a field that studies quantitative relationships and spatial forms, plays a very important role in the formation of human rational thinking, the spirit of research, and the promotion of individual intellectual development [1]. The evaluation of mathematics classroom teaching plays an extremely important role in the teaching process. Teaching evaluation is an essential means to improve the quality of teaching and promote the comprehensive development of students [2].

The Markov Chain, a mathematical model that describes the stochastic changes of a system, was first proposed by the Russian mathematician Andrey Markov [3]. Markov Chain has a wide range of applications in various fields, including computational mathematics, finance and economics, bioengineering and education. Through the Markov Chain model, educators can better understand the learning process of students, predict their future performance, and thus improve teaching methods and strategies to enhance the quality of teaching [4].

2. Presentation of the problem

Our traditional teaching evaluations focus more on the teaching status of the teacher and the situation of the students in a certain lesson. However, teaching is a dynamic process, and students' initial learning foundation and literacy also affect the evaluation of teaching quality [5]. These initial differentiation characteristics of students will lead to a large difference in the original level of students, so the results may be unfair [6].

In middle school mathematics teaching, the evaluation of teaching quality heavily relies on the analysis of student grades. Currently, student performance is often considered based on data such as average scores, standard deviations, and ranges [7]. While these data have certain reference values, they may not fully reflect the actual teaching effectiveness. The Markov Chain, is a mathematical model built on a random process in which the probability of each state transitioning to another depends only on the current state and is independent of the previous sequence of states [8].

3. Introduction of the Markov Chain model

3.1. The definition of a Markov Chain

Let $\{Xn\}$ be a random vector, State space is $E=\{i\}$. If for any positive integer n and any state $i, j, i_0, i_1, \ldots, i_{n-1}$, in E, it holds that:

$$P\{X_{n+1}=j \mid X_0=i_0 \mid X_1=i_1, \ldots X_{n-1}=i_{n-1}\, X_n=i\}=P\{X_{n+1}=j \mid X_n=i\}$$

Then we called $\{Xn\}$ a Markov Chain.

We usually consider n as the "present," n+1 as the "future," and the moments 0, 1, …, n-1 as the "past." From the definition, it is easy to observe that the "next state" in a Markov Chain depends solely on the "current state" and is independent of the "past states." This property is the absence of after-effect of Markov chains [9].

3.2. Transition probability

Let $\{Xn\}$ be a Markov Chain defined on the state space E, then there has:

$$P_{ij}^{(k)}(m) = P\{X_{m+k}=j \mid X_m=j\}, i, j \in E$$

In the formula, $P_{ij}^{(k)}(m)$ is referred to as the transition probability of $\{Xn\}$ starting from state i at a time, m and reaching state j after k steps. The matrix $P^{(k)}(m) = P_{ij}^{(k)}(m)$ is called the k-step transition probability matrix of $\{Xn\}$ starting from time, m. The probability of transitioning from state i to state j in one step is known as the one-step transition probability of the Markov Chain, and $P(m)=(P_{ij}(m))$ is called the one-step transition matrix [10]. In this paper, we mainly discuss the Markov Chain model with a one-step transition matrix, so the next step is to provide the one-step transition matrix [11].

$$P = \begin{bmatrix} p_{11} & \cdots & p_{1r} \\ \vdots & \ddots & \vdots \\ p_{r1} & \cdots & p_{rr} \end{bmatrix}$$ [12]

3.3. Steps to Establish a Markov Chain Model

3.3.1. Categorizing into levels and establishing a rational indicator system

To choose a suitable sample, you need to determine the possible states of your Markov Chain. For example, in our application in teaching evaluation, we divide student test scores into n levels, which are n states [13]. By calculating the proportion of all levels in the selected sample, you can obtain the initial state vector, represented as $\pi_0 = \left[\frac{n_1}{n}, \frac{n_2}{n}, \cdots, \frac{n_n}{n}\right]$

3.3.2. Constructing the transition probability matrix

Based on the changes in each level of the sample, construct the one-step transition probability matrix p, which is represented as:

$$P = \begin{bmatrix} \frac{n_{11}}{n_1} & \cdots & \frac{n_{1n}}{n_1} \\ \vdots & \ddots & \vdots \\ \frac{n_{n1}}{n_n} & \cdots & \frac{n_{nn}}{n_n} \end{bmatrix}$$

Where n_i represents the total number of instances in the "previous moment" that were in the i-th level, and n_{ij} represents the total number of transitions from level i to level j in the "previous moment" to the "next moment." Based on the initial state π_0 and $P^{(n)}$, $\pi_n = \pi_0 P^{(n)}$ is calculated, and the teaching effect evaluation is further given.

4. An example application of the Markov Model in middle school mathematics

This paper selects twenty students from Class One and Class Four of the eighth grade in Middle School E in City X as the research subjects. These two classes are taught by two different mathematics teachers. The math scores of the two classes in the first monthly exam and the final exam of the first semester are recorded. By constructing a comprehensive evaluation grade for student performance and a state transition matrix, the daily teaching of the two teachers is evaluated [14].

4.1. Construct the initial state vector

Based on the analysis of the scores, the grades are divided into five levels:

A (102–120 points), B (84–102 points), C (72–84 points), D (48–72 points), and E (below 48 points). The following table shows the math scores of the two classes in the first monthly exam. From this, the initial state vectors for the scores of the two classes can be obtained.

$$T_1 = \left(\frac{3}{10}, \frac{2}{5}, \frac{1}{10}, \frac{1}{20}, 0\right) \quad T_4 = \left(\frac{7}{20}, \frac{7}{20}, \frac{3}{20}, \frac{1}{10}, \frac{1}{20}\right)$$

Table 1. The first monthly math exam score table for the two classes

Class	A	B	C	D	E
Class One	6	8	3	2	1
Class Four	7	7	3	2	1

4.2. Constructing the state transition matrix

After one semester of teaching, the final exam scores of the two classes were re-tabulated as follows:

Table 2. The final exam math score table for the two classes

Class	A	B	C	D	E
Class One	7	7	3	3	0
Class Four	8	6	2	2	2

By comparing the scores from the two exams, we can obtain the comprehensive grade transition for mathematics performance for each class. For example, in Class One, there were 6 students in grade A on the first exam, and by the final exam, 5 remained in grade A, with 1 transitioning to grade B. The specific situations are shown in the following table:

Table 3. Class One Mathematics comprehensive grade transition situation

	Number of students in final exam grade level						
	Level	**T1**	**T2**	**T3**	**T4**	**T5**	**Total**
Number of students in first monthly math exam grade level	T1	5	1	0	0	0	6
	T2	2	5	1	0	0	8
	T3	0	1	2	0	0	3
	T4	0	0	0	2	0	2
	T5	0	0	0	1	0	1

Table 4. Class Four Mathematics comprehensive grade transition situation

	Number of students in final exam grade level						
	Level	**T1**	**T2**	**T3**	**T4**	**T5**	**Total**
Number of students in first monthly math exam grade level	T1	5	2	0	0	0	7
	T2	3	2	2	0	0	7
	T3	0	2	0	0	1	3
	T4	0	0	0	1	1	2
	T5	0	0	0	1	0	1

Based on the contents shown in the table, the probability transition matrices for Class One and Class Four can be calculated respectively.

$$P_{class1} = \begin{pmatrix} \frac{5}{6} & \frac{1}{6} & 0 & 0 & 0 \\ \frac{1}{4} & \frac{5}{8} & \frac{1}{8} & 0 & 0 \\ 0 & \frac{1}{3} & \frac{2}{3} & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{pmatrix} \quad P_{class4} = \begin{pmatrix} \frac{5}{7} & \frac{2}{7} & 0 & 0 & 0 \\ \frac{3}{7} & \frac{2}{7} & \frac{2}{7} & 0 & 0 \\ 0 & \frac{2}{3} & 0 & 0 & \frac{1}{3} \\ 0 & 0 & 0 & \frac{1}{2} & \frac{1}{2} \\ 0 & 0 & 0 & 1 & 0 \end{pmatrix}$$

Through calculation, it can be obtained that

$T^{(1)}_{class1} = T^{0}_{class1} P_{class1} = \left(\frac{7}{20}, \frac{1}{3}, \frac{7}{60}, \frac{1}{20}, 0\right), T^{(1)}_{class4} = T^{0}_{class4} P_{class4} = \left(\frac{2}{5}, \frac{3}{10}, \frac{1}{10}, \frac{1}{10}, \frac{1}{10}\right)$ [15]

Based on the aforementioned grade classification, let the score weights be W = (110, 90, 70, 50, 30). Using software like MATLAB, the results for the progress degrees of the two classes can be calculated as follows:

$E^{1}_{m} \approx 5.413, E^{4}_{m} \approx -0.946$

5. The application conclusions of the Markov Chain model

For Class One and Class Four, taught by two different teachers, the initial average score for Class One was 86.3, which improved to 87.5 in the second assessment. Class Four's initial average score was 89.1, which slightly decreased to 87.1 in the second assessment. Overall, Class Four had a slightly higher average score than Class One. However, in actual teaching, there are some factors such as individual differences among students. Looking at the changes in scores, Class One's average score has improved, while Class Four's average score has slightly decreased. Furthermore, from the progress matrix and efficiency values, it can be seen that the teaching effect of the teacher in Class One is better than that in Class Four. This indicates that by using the Markov Chain model, the influence of students' difference factors has been reduced to a certain extent, providing a more accurate evaluation of teaching effectiveness.

In this study, by utilizing the Markov Chain model, a practical and effective model for evaluating teaching effectiveness has been established. This model can help schools analyze teaching quality more objectively can remind teachers to take certain intervention measures. Under the current high school entrance examination reform, Mathematics, as a subject with a significant weight in the total score, requires us to control the quality of Mathematics classes in time. Therefore, through the establishment of this model, a reasonable data analysis can be provided for each teaching class, and teaching suggestions can be made based on this trend, facilitating teachers in carrying out subsequent teaching work.

Disclosure statement

The authors declare no conflict of interest.

References

[1] Xu XM, 2011, The Core of Mathematics Classroom Teaching: Process, Problem-solving, and Subjectivity. Primary and Secondary Education Reference, 2011(22): 5 + 33–37.

[2] Roger AH, Charles RJ, 2013, Matrix Analysis. Cambridge University Press, England, 456–458.

[3] Dong QH, Wang CW, 2018, The Application of Markov Chains in the Evaluation of Teaching Effectiveness in Advanced Mathematics. Practice and Cognition in Mathematics, 48(8): 314–320.

[4] Ding J, 2022, Research on the Application of Markov Chain in Teaching Quality Evaluation. Chinese Journal of Multimedia and Online Teaching (First Ten Days), 2022(11): 209–212.

[5] Wang F, 2022, Research on the Application of Markov Chain in Professional Quality Evaluation. Lucheng Journal, 2022(1): 86–90.

[6] Cui Y, 2020, Evaluation of Developmental Teaching Effectiveness Based on Markov Chain. Knowledge Economy, 2020(3): 150–151 + 154.

[7] Yin Y, 2020, Development and Application of Markov Chain. Strategic Emerging Industries in China, 2020(12): 78–79.

[8] Baheguli H, 2020, Application of Stratified Teaching Method in Junior Middle School Mathematics Teaching. New Education Era Electronic Magazine (Teacher Edition), 2020(6): 95.

[9] Dong X, 2022, Where to Find the Essence of Mathematics? There are Models in the Sea of Questions: A Type of Probability Test Research Based on Markov Chains. Friends of Mathematics, 36(18): 80–81.

[10] Wang X, 2019, Evaluation of the Effectiveness of Tibetan Chinese Bilingual Teaching in Middle School Mathematics Based on Markov Chains, thesis, Northwest University for Nationalities.

[11] Zheng K, 2023, Application of Consistency in Teaching Evaluation in Middle School Mathematics Teaching. Test Questions and Research, 2023(29): 4–6.

[12] Jiang L, 2022, Exploring the Application Path of Integrated Teaching Evaluation Model in Junior High School Mathematics Teaching. Chinese Science and Technology Journal Database (Full Text Edition) Education Science, 2022(11): 253–256.

[13] Zhao D, 2019, Discussion on the Application of Combining Numbers and Forms in Middle School Mathematics Teaching. Examination and Evaluation, 2019(7): 157–157.

[14] Zhao Y, 2022, Research on the Application of Process Evaluation in High School Mathematics Teaching. Development of Education Science, 4(8): 116–118.

[15] Yao B, 2019, Application of Virtual Reality Technology Based on Fuzzy Mathematics Evaluation in Junior High School Chemistry Classroom. Western Quality Education, 5(21): 136–138.