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Research on Teaching Function Continuity in the GeoGebra Environment

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Abstract: This study focuses on exploring how visualization technology can effectively alleviate the cognitive challenges students encounter when understanding the abstract mathematical concept of function continuity. Given that function continuity often appears elusive to students, this research introduces advanced visualization tools such as GeoGebra, which strive to transform complex mathematical operations into intuitive and perceptible graphics or animations. The aim is to provide students with a more direct and vivid learning experience, helping them gradually build a profound understanding of the essence of function continuity and thereby enhancing their grasp of fundamental mathematical theory systems. The results demonstrate that integrating visualization tools into teaching practice not only significantly improves students' depth of comprehension of abstract concepts like function continuity but also greatly stimulates their interest in learning, promoting the development of spatial imagination and logical thinking abilities. Based on these findings, we recommend widely adopting visualization tools as powerful instructional aids in future teaching, especially when teaching subjects that involve abstract concepts such as function continuity. This will not only enhance the effectiveness and appeal of classroom teaching but also substantially improve students' learning outcomes and overall abilities.

Keywords: Function continuity; Visualization tools; GeoGebra

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1. Introduction

Higher mathematics is often perceived as difficult to understand by students due to its abstractness and complexity [1]. However, the introduction of mathematical visualization technology—by transforming complex mathematical objects (including concepts, structures, and methods) into easily understandable graphics, images, or animations—has greatly facilitated students' comprehension and cognition. In classroom teaching, teachers use information technology means such as diagrams, colors, and animations from software like GeoGebra to realize the organic combination of shapes and numbers, building a bridge from visualization to abstract concepts, eliminating barriers to understanding, improving the efficiency of knowledge delivery, and thereby enhancing students' mathematical literacy [2]. As a powerful mathematical visualization tool, GeoGebra not only enhances the flexibility and depth of teaching and learning, stimulates students' enthusiasm for exploration, but also becomes a key tool for facilitating teaching innovation and students' independent learning. Its mobile application further expands learning scenarios, making mathematics learning more vivid and interesting. Especially when teaching function continuity, GeoGebra can dynamically demonstrate the changes in function values as the independent variable approaches a specific value, helping students intuitively grasp the concept of continuity. Students can directly observe the continuity or discontinuity of function graphs at specific points by adjusting function expressions or parameters. Using the "tracking" function, they can real-time observation of the changes in function values with the independent variable, thereby accurately judging whether a function is continuous at a certain point and identifying the type of discontinuity. This interactive exploration method deepens students' understanding of the concepts of continuity and discontinuity. For example, encouraging students to use GeoGebra to verify hypotheses or explore the boundary conditions of function properties by changing parameters can stimulate their curiosity and creativity, fostering a spirit of scientific inquiry.

The in-depth direction of our current research lies in moving beyond the superficial description of tool application to the level of empirical research and theoretical construction. Future explorations should focus on: developing and systematizing specific teaching cases and activity sequences for function continuity supported by GeoGebra based on the design research paradigm; empirically testing the specific impact mechanism of this visualization inquiry model on students' conceptual understanding, mathematical reasoning ability, and affective factors (such as self-confidence and interest) through rigorous quasi-experimental research or learning analytics technology; combining learning science theories such as constructivism, cognitive load theory, and embodied cognition to deeply analyze how visualization interaction promotes the formation of students' mathematical concepts and the development of cognitive structures; exploring how to effectively integrate in-class and out-of-class learning activities with GeoGebra in blended or flipped classroom models to form a coherent learning experience [3]. Conducting teaching research on function continuity in the GeoGebra environment responds to the urgent requirement of the educational digitalization strategic action to deepen the integration of subject teaching and information technology. It is not merely to make teaching more "appealing" or "interesting", but its fundamental purpose is to reveal the inherent process of the occurrence and development of mathematical knowledge through technological empowerment, transforming learning from passive acceptance to active meaning construction, thereby effectively enhancing students' mathematical abstract thinking, intuitive imagination ability, and inquiry skills and innovative capacity. This study aims to provide valuable paths and cases for theoretical development and practical innovation in this field through systematic teaching design and empirical discussion [4].

2. Function continuity

Generally speaking, phenomena involving continuous change are related to a specific mathematical fact: for a function, when its independent variable undergoes an extremely small change, the change in the function value is similarly small. That is, if the increment of the independent variable approaches zero, the corresponding increment of the function also approaches zero. Therefore, continuity can be defined as follows: Let a function $y = f(x)$ be defined in a neighborhood of a point x_0. If $\lim_{\Delta x \to 0} \Delta y = 0$ (or $\lim_{\Delta x \to 0} \left[f(x_0 + \Delta x) - f(x_0) \right] = 0$, or $\lim_{x \to x_0} f(x) = f(x_0)$), then the function $f(x)$ is said to be continuous at the point x_0. If $\lim_{x \to x_0^-} f(x)$ exists and equals $f(x_0)$, i.e., $\lim_{x \to x_0^-} f(x) = f(x_0)$, the function $f(x)$ is left-continuous at x_0; $\lim_{x \to x_0^+} f(x)$ exists and equals

$f(x_0)$, i.e., $\lim_{x \to x_0^+} f(x) = f(x_0)$, the function $f(x)$ is right-continuous at x_0. From the necessary and sufficient condition for the existence of a limit, the left-continuity and right-continuity of a function $f(x)$ at a point x_0 are the necessary and sufficient conditions for the function $f(x)$ to be continuous at that point x_0. This conclusion is often used to judge the continuity of piecewise functions.

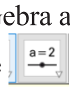
For example, determine the value of a in the function $f(x) = \begin{cases} \cos x, x > 0 \\ a + x, x \le 0 \end{cases}$ such that the function $f(x)$ is continuous at x=0.

Solution: Use the necessary and sufficient condition (left-continuity = right-continuity = function $f(x)$ value at the point x=0) to determine a. $f(0^-) = \lim_{x \to 0^-} f(x) = \lim_{x \to 0^-} a + x = a$, $f(0^+) = \lim_{x \to 0^+} f(x) = \lim_{x \to 0^+} \cos x = 1$, and $f(0) = a$, to ensure $f(x)$ continuity at x=0, we must have $f(0^-) = f(0^+) = f(0) = a$, so a=1.

For students, understanding the necessary and sufficient conditions for the continuity of functions with parameters is an important and complex task. They may only master the theoretical knowledge but lack an intuitive understanding of its underlying significance. However, using GeoGebra's dynamic demonstration function, we can transform these abstract concepts into intuitive visualization experiences. Geometrically, the graph of a univariate continuous function is a smooth curve without discontinuities—this intuitive representation is crucial for understanding function continuity. To this end, we can draw the graph of a function $f(x) = \begin{cases} \cos x, x > 0 \\ a + x, x \le 0 \end{cases}$ with parameters and dynamically adjust the parameter values, allowing students to observe how the graph changes with parameters and how these changes affect the function's continuity at specific points x=0, thereby deepening their understanding.

3. Dynamic demonstration of the detailed production process using GeoGebra

Launch GeoGebra and open the Algebra View and Graphics View, respectively.

Click the 

on the toolbar to create a slider in the Graphics View. For easy observation, set it to integer type in the properties window (**Figure 1**) with a default range of -10 to 10. Note: This slider controls the function expression and graph when $x \le 0$.

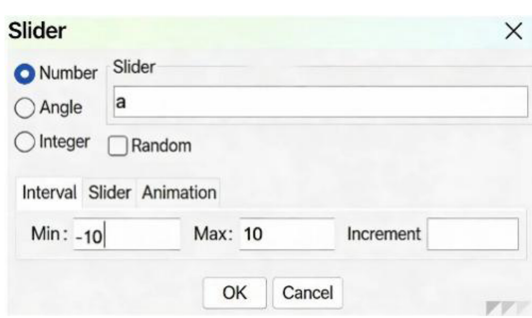


Figure 1. Properties of Slider .

In English input mode, enter the command in the Input Bar 

: If( ). Note: Draw the piecewise function $f(x) = \begin{cases} \cos x, x > 0 \\ a + x, x \le 0 \end{cases}$ curve (**Figure 2**).

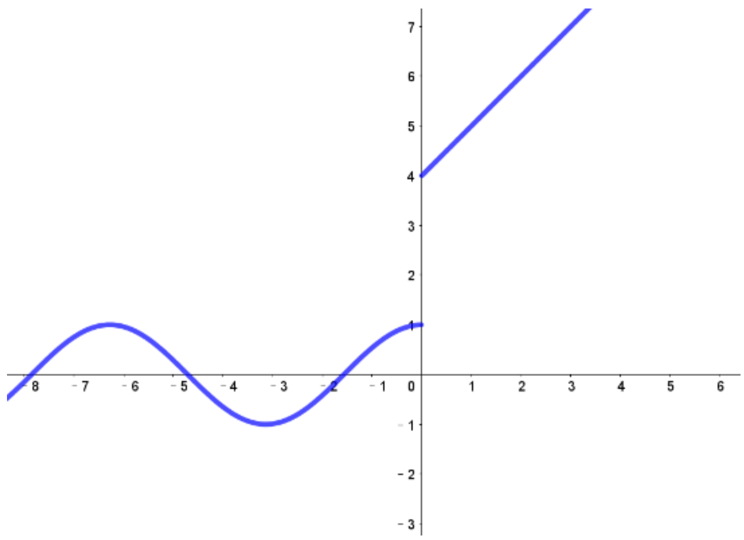


Figure 2. Graph of the piecewise function.

After completing the above steps, students can dynamically adjust the value of a via the slider to observe changes in the function graph. This interactive design allows them to intuitively see how the graph transitions from disconnected segments to a continuous curve as it changes. When the graph becomes a smooth curve without breakpoints (**Figure 3**), the parameter a ensures the function's continuity at x = 0.

This in-depth interaction based on dynamic demonstration fundamentally changes students' understanding of function continuity. Traditional text and static images only present the "result state" of knowledge, while GeoGebra reveals its "generation process" [5]. Instead of passively accepting the definition of "continuity", students become active observers and experimenters [6]. Controlling the independent variable to approach a point via the slider transforms the abstract logic of "infinite approach" into a visible continuous path—this "process visualization" converts the intuitive ideas behind the formal language into a perceptible experience, establishing a solid "conceptual imagery" in students' minds that is far more profound than memorizing symbols.

Furthermore, parameter adjustment elevates learning from mastering a single concept to exploring the laws of function families. By modifying functions or parameters in real-time, students can immediately observe coordinated changes in graph shape and continuity. For example, when exploring the continuity of a piecewise function with parameters, the slider allows them to witness how the graph transitions between "continuity" and "jump discontinuity" when the parameter crosses a critical value [7]. This "hypothesis-regulation-observation-induction" cycle simulates the basic paradigm of mathematical research [8], cultivating students' ability to grasp invariant laws amid changes and perceive general essence from specifics—this high-level mathematical literacy is more valuable than memorizing isolated conclusions.

This learning journey ignites intrinsic motivation, allowing students to experience the joy of intellectual creation. When they can manipulate mathematical objects and witness laws manifest, mathematics transforms from cold symbols into a vibrant world [9]. GeoGebra acts not just as a teaching aid, but as a partner for students to extend their thinking and make discoveries, helping them build solid mathematical intuition, rigorous reasoning, and a spirit of scientific exploration.

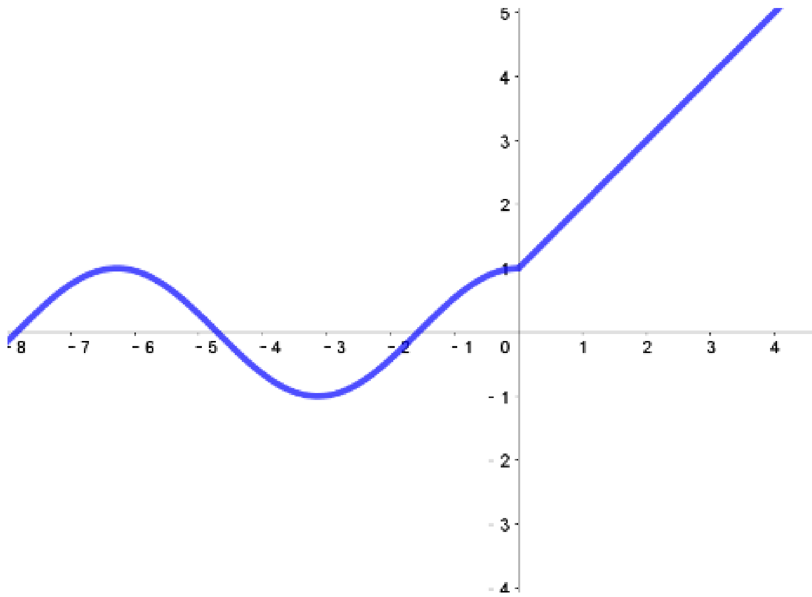


Figure 3. Function continuous at x = 0.

4. Conclusion

In summary, advanced educational technology tools like GeoGebra successfully transform abstract mathematical concepts (function continuity and parameter changes) into vivid visual experiences. Students not only witness dynamic changes behind formulas but also experience the charm of mathematics through hands-on interaction [10]. This immersive exploration aligns with the cognitive principle of progressing from concrete experience to abstract concepts [11], as students manipulate parameters and witness the "critical transition" between continuity and discontinuity, they gain a robust "conceptual imagery" that underpins a deep understanding of formal definitions, reducing cognitive load and freeing up mental resources for higher-order thinking [12].

Students' ability growth is multi-dimensional: their intuitive imagination and spatial reasoning are enhanced, and their exploration and induction skills are systematically cultivated through scientific inquiry [13]. The successful application of GeoGebra in teaching function continuity demonstrates a clear pathway for technology-empowered mathematics education transformation—deeply integrating technology into curriculum design can restructure teaching, release student agency, and foster a classroom culture of "interaction, inquiry, understanding, and creation" [14].

Looking forward, this exploration transcends optimizing single-knowledge-point teaching. It provides a replicable paradigm for reconstructing advanced mathematics curricula and cultivating innovative talents with solid mathematical intuition, rigorous logic, and excellent inquiry abilities in the era of big data and artificial intelligence [15]. Ultimately, students gain not just a foundation for academic research, but an enduring curiosity and rational confidence—making the journey of learning mathematics a fulfilling intellectual adventure.

Disclosure statement

The authors declare no conflict of interest.

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