

Research on the Construction and Practice of Primary School Mathematics Large-Unit Teaching Model Based on Conceptual Understanding: Taking the Unit “Parallelograms and Trapezoids” as an Example

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Abstract: Under the guidance of core competencies, the reform of primary school mathematics teaching urgently needs to go beyond the superficial teaching of knowledge and skills, and construct a new teaching paradigm aimed at promoting students' conceptual understanding and endogenous literacy. Addressing the long-standing problems in the field of graphics and geometry, such as fragmented knowledge, superficial thinking, and vague literacy goals, this study takes Erickson's “concept-based” curriculum and teaching theory as the meta-framework, integrates domestic research on big idea teaching, and constructs a three-dimensional collaborative large-unit teaching model of “Knowing—Doing—Understanding” (KUC). Taking the unit “Parallelograms and Trapezoids” in primary school mathematics (People's Education Press edition) as an empirical carrier, the paper systematically elaborates the complete design path from “extracting subject big ideas” and “establishing conceptual perspectives” to “designing hierarchical guiding questions,” “constructing performance evaluation” and “sequencing iterative learning activities,” and develops an anchoring performance task of “campus transformation designer”. Practice shows that this model can effectively drive students' thinking to leap from “factual memory” to “concept construction,” providing a theoretical reference and practical model for the transformation of competency-based classrooms.

Keywords: Big idea teaching; Concept-based; Graphics and geometry; Core competencies; Large-unit design; Consistency of teaching, learning and evaluation

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1. Problem proposal: Practical dilemmas and theoretical demands of primary school mathematics teaching in the literacy era

Competency-oriented curriculum reform aims to cultivate students' mental flexibility when facing various

complex and open situations^[1]. One of the keys to achieving this goal is to attach importance to and promote the development of students' conceptual understanding^[2]. The Compulsory Education Mathematics Curriculum Standard (2022 Edition) has established curriculum goals oriented by the core competencies of "Three Abilities", marking the official transition of primary school mathematics education from "double basics" to "competency-based"^[3]. However, examining the current teaching scene, there are still three interwoven deep-seated dilemmas:

Firstly, the lack of a knowledge structure leads to weak cognitive schemas and insufficient transfer ability. Restricted by the traditional linear class hour logic, contents related to graphics and geometry are often treated atomically. Students acquire mostly fragmented "concept labels" and "property conclusions," making it difficult to construct a systematic and networked cognition of the internal genealogy of relevant graphic systems and the core subject idea that spatial relationships determine graphic properties. This fragmented knowledge is difficult to transform into an adaptive cognitive framework for coping with complex situations, resulting in knowledge inertia and transfer failure.

Secondly, the superficialization of thinking training inhibits the development of higher-order geometric thinking and core subject competencies. In current classrooms, the simplified inquiry model of "observation-instruction-verification" is still prevalent. Students' cognitive activities are confined to factual memory and shallow operations, depriving them of the opportunity to conduct in-depth questioning on essential subject issues such as the construction logic of geometric concept systems and the inherent connections between graphic properties, thereby hindering the development of higher-order geometric thinking such as conjecture, argumentation and systematization^[4].

Thirdly, the vagueness of literacy goals leads to structural disconnection in the consistency of teaching, learning and evaluation. Although the new curriculum standard clearly describes literacy dimensions such as spatial awareness and geometric intuition, the evaluation focus dominating daily teaching still stubbornly points to the mechanical memory of definitions, accurate repetition of properties and rapid identification of standard graphics. This evaluation culture conflicts with the comprehensive and situational characteristics of literacy goals, resulting in the systematic narrowing or even neglect of higher-order literacy goals in key links of teaching and evaluation^[5].

The "concept-based" theory advocated by international curriculum experts H. Lynn Erickson and Lois A. Lanning provides a powerful theoretical fulcrum for solving the above dilemmas. The theory points out that curriculum design in the literacy era must upgrade from the two-dimensional model of "knowledge + skills" to the three-dimensional model of "knowledge—process—conceptual understanding"^[6]. In this model, conceptual understanding is at the core, serving as the cognitive anchor for students to achieve knowledge integration and far transfer. The value of conceptual understanding lies in realizing transferability within a wider range by forming general ideas about the meaning of things^[7]. Based on this and deeply integrating domestic research on big idea teaching, this study is committed to constructing a KUC model of primary school mathematics large-unit teaching with conceptual understanding as the core, and conducts systematic design explanation and empirical exploration with the unit "Parallelograms and Trapezoids" as an example.

2. Theoretical basis: Research evolution and subject adaptation of big idea teaching

In recent years, around the implementation of core competencies, research on big idea teaching in China's

educational circle has moved from concept introduction to in-depth subject construction, providing rich theoretical nourishment for the model construction of this paper.

2.1. Connotation distinction between conceptual understanding and subject big ideas

The transformation from everyday concepts to scientific concepts, and from naive theories guiding daily actions to scientific theories based on disciplinary practice norms, neither occurs inevitably nor transitions naturally^[8]. “Conceptual understanding” emphasizes the meaningful construction and flexible application of core big ideas in disciplines. Liu (2022)^[9] pointed out in Big Idea Teaching: Competency-Oriented Unit Overall Design that big ideas are “located at the center of curriculum learning, which can not only reveal the laws behind subject knowledge but also connect the real world, with extensive transfer value”. They endow factual knowledge with meaning and promote its structuring. From the perspective of children’s mathematics education, Wu (2022)^[10] emphasized that big ideas are “core clues running through the entire process of primary school mathematics learning”, which help children “build ‘load-bearing walls’ and break through ‘partition walls’,” realizing the integration of knowledge and positive transfer of learning.

2.2. Extraction path and teaching research of mathematics big ideas

How to extract big ideas from mathematics curriculum standards and subject essence is the key to practice. Ma (2022) proposed the idea of extracting big ideas based on “learning themes”. Liu (2022) systematically elaborated the extraction strategy combining “top-down” (based on curriculum standards and subject essence) and “bottom-up” (based on students’ cognitive difficulties and life reality). In the field of graphics and geometry, core competencies such as “spatial awareness,” “geometric intuition,” and “model thinking” provide a solid basis for the accurate positioning of big ideas in this unit.

2.3. Research on design and evaluation of large-unit teaching

In terms of implementation, large-unit teaching is regarded as an ideal carrier for implementing big idea teaching. Cui (2019) emphasized that large-unit design must be organized around core problems arising in real situations to achieve “consistency of teaching, learning and evaluation.” Based on decades of practice, Wu (2022) proposed that the overall unit teaching of primary school mathematics should follow the principles of “establishing structure in connection, deepening understanding in comparison, and improving accomplishment in application.” In terms of evaluation, performance evaluation is highly respected because it can directly assess students’ understanding and transfer level of big ideas.

3. Model construction: KUC three-dimensional collaborative framework of primary school mathematics large-unit teaching

Disciplinary conceptual knowledge can only be transformed into individual personal knowledge through the generation of personal understanding^[11]. Based on Erickson’s theory and deeply integrating domestic research results, we have constructed the “KUC” three-dimensional model of primary school mathematics large-unit teaching (**Table 1**). The model emphasizes that Knowledge (K) and Skills (D) are the cornerstones and carriers for constructing Conceptual Understanding (U), while Conceptual Understanding (U) injects soul and direction into the acquisition of Knowledge (K) and Skills (D). The three are interdependent and mutually promoting, forming a cognitive development closed loop of “factual perception—skill practice—concept formation—

reflection and application,” ultimately pointing to the generation of core competencies.

Table 1. Constituent elements of the KUC three-dimensional model for the large unit “Parallelograms and Trapezoids”

Dimensions	Core connotation in primary school mathematics	Specific manifestations and theoretical analysis of this unit
Knowledge Dimension (Knowing)	Stable declarative knowledge forming the foundation of mathematics, including mathematical facts, terms, symbols, definitions, axioms, etc.	Precise definitions of parallelism and perpendicularity: “Two straight lines in the same plane that do not intersect are called parallel lines”; “If two straight lines intersect at a right angle, we say the two straight lines are perpendicular to each other”. These are the logical cornerstones and starting points of geometric reasoning.
Skill Dimension (Doing)	Procedural knowledge and cognitive strategies executed by students to explore mathematical meanings and solve problems, including calculation, drawing, reasoning, modeling, communication, etc.	Operational skills in geometric drawing: proficiently using tools such as set squares, straightedges, and protractors to standardly draw parallel lines, perpendicular lines, and specified parallelograms and trapezoids. Higher-order skills in mathematical modeling and problem-solving: transforming spatial problems in the real world into geometric models, and comprehensively using learned knowledge and strategies to seek creative solutions.
Conceptual Understanding Dimension (Understanding)	Profound, lasting and transferable conceptual cognition of core big ideas in mathematics. Composed of “concepts” (abstract ideas) and “generalizations” (propositions expressing fundamental relationships between concepts). This is the ultimate pursuit and deep goal of teaching.	Core concepts: Spatial relationship, invariance, classification, dimension. Core generalization: Humans have constructed the classification and reasoning system of geometry by defining and exploring stable spatial relationships between graphic elements (such as parallelism and perpendicularity); understanding and mastering these relationships is the key for us to interpret spatial order, conduct creative design and solve problems.

4. Practical empiricism: Application of the KUC model in the unit “Parallelograms and Trapezoids”

Taking the unit “Parallelograms and Trapezoids” (Grade 4, Volume 1, People’s Education Press edition) as an example, this section details the complete teaching design path based on the KUC model.

4.1. Accurate positioning of big ideas and core generalizations

The conceptual perspective established for this unit is “spatial relationships,” and the core generalization is: “Humans have constructed the classification and reasoning system of geometry by defining and exploring stable spatial relationships between graphic elements (such as parallelism and perpendicularity); understanding and mastering these relationships is the key for us to interpret spatial order, conduct creative design and solve problems.” This generalization serves as the “conceptual anchor” integrating the entire unit’s learning.

4.2. Design of hierarchical guiding question chains

To propose mathematical problems means the following for students:

- (1) Students can put forward mathematical problems, including mathematical expressions and mathematical graphs based on existing contexts;
- (2) Students can add reasonable information to reconstruct the original problems ^[12]. As a cognitive activity for students to exert creative thinking ^[13], integrating problem posing as a teaching method into actual classroom teaching by teachers can promote students’ conceptual understanding ^[14,15].

A problem chain is the core engine that drives students' cognition to climb from specific facts to abstract concepts. We have designed three levels of problems advocated by Erickson:

- (1) Factual questions (F): Questions pointing to the Knowledge Dimension (K), such as inquiries about definitions of relevant concepts and characteristics of graphics.
- (2) Conceptual questions (C): Questions connecting the Knowledge/Skill Dimensions and the Conceptual Understanding Dimension, such as inquiries about the reasons for graphic classification criteria and the invariant properties of graphics.
- (3) Philosophical/transfer questions (P): Questions pointing to the transfer and application of the Conceptual Understanding Dimension, such as inquiries about the application of graphic spatial relationships in real life and creative design based on such relationships.

4.3. Construction of the “Campus Transformation Designer” performance evaluation system

4.3.1. Task scenario

The school plans to transform an idle area into a “creative activity park” and solicits design proposals from students.

4.3.2. Core requirements

- (1) Functional area planning: The design plan must include specific graphic functional areas, with clear marking and written explanations of graphic characteristics.
- (2) Road system design: The road system must include specific spatial relationship designs, with standard symbol marking and explanations of design reasons.
- (3) Design proposal demonstration: Prepare an oral report to explain design ideas, focusing on the application of spatial relationships and the advantages of the plan.

4.3.3. Evaluation rubric

Comprehensive evaluation is conducted from three dimensions, knowledge understanding, skill application, and concept transfer, with four levels set to ensure the consistency of teaching, learning and evaluation.

4.4. Sequencing of iterative learning activities

The entire learning process is designed as three spiral and iterative stages, taking about 6 class hours:

- (1) Stage 1: Initial exploration of facts and concepts (about 2 class hours): Abstract geometric graphics from life examples and learn standardized mathematical definitions and symbolic expressions.
- (2) Stage 2: Skill and concept deepening (about 2 class hours): Operate deformable teaching tools to experience graphic invariance, verify graphic properties through multiple methods, and construct a structured knowledge network.
- (3) Stage 3: Understanding transfer and achievement creation (about 2 class hours): Fully implement the “campus transformation designer” project, with students completing design drawings, making models and preparing reports to solidify and sublimate conceptual understanding.

5. Discussion and reflection: Practical boundaries and optimization paths of the model

The effective implementation of the KUC model places higher requirements on teachers' curriculum understanding and design capabilities, requiring careful handling of the following relationships in practice.

5.1. Balance between conceptual rigor and children's cognition

Graphics and geometry concepts are highly abstract. Teaching should adhere to the principle of intuition, allowing students to approach the essence of concepts through intuitive materials, counterexample comparison and embodied operations.

5.2. Guarantee of reliability, validity and efficiency of performance evaluation

In large classes, performance evaluation faces challenges of being time-consuming and subjective. Countermeasures include developing detailed evaluation rubrics, implementing multi-subject evaluation mechanisms, and exploring the use of information technology to improve efficiency.

5.3. Construction of a support system for teachers' professional development

The model requires teachers to shift from "teaching textbooks" to "using textbooks to teach", requiring solid subject content knowledge and curriculum design capabilities. It is necessary to construct a "research-training-practice-reflection integrated professional learning community" to realize continuous professional empowerment of teachers.

6. Research conclusions and future prospects

6.1. Research conclusions

The KUC three-dimensional large-unit teaching model constructed and practiced in this study provides an embodied path for the transformation of primary school mathematics classrooms towards competency orientation through big idea integration, question chain driving and performance evaluation anchoring. Practice shows that this model can effectively solve knowledge fragmentation, deepen thinking levels, and ensure the consistency of teaching, learning and evaluation.

6.2. Future prospects

Future research can further expand the application scenarios of the model, optimize it based on student differences, integrate digital technology, and verify its long-term effects, so as to provide more comprehensive support for the in-depth development of primary school mathematics teaching reform.

Disclosure statement

The authors declare no conflict of interest.

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