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Exploration of the Multi-axis Model for Sandy Soil

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Abstract: In soil mechanics, multi-axis generalization is the key to the wide application of sandy soil models. The multi-axis model for sandy soil has important applications in geotechnical engineering (including earthquake engineering), numerical simulation, and other relevant fields. Although the traditional triaxial model has made remarkable progress in studying the basic behavior of sandy soil, it still has limitations when simulating the behavior under complex stress conditions. Multi-axis generalization can better simulate the complex stress states commonly encountered in practical engineering, thereby improving the applicability and accuracy of the model. This paper reviews the development history of the sandy soil multi-axis model, analyzes the influence of different yield criteria, hardening rules, and flow rules on the stress-strain relationship of sandy soil, introduces common experimental methods and numerical simulation techniques, and explores its application in engineering practice and future development trends.

Keywords: Multi-axis model; Sandy soil; Yield criterion; Hardening rule

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1. Introduction

As a common engineering material, sandy soil plays an important role in civil engineering. However, the behavior of sandy soil under compound loads is complex, involving the coupling effects of shear and volumetric deformation. Therefore, establishing a model that can accurately describe the behavior of sandy soil is crucial for engineering design and safety assessment [1,2]. The multi-axis mechanical behavior of sandy soil is complex and variable due to its particle characteristics, strain localization, and anisotropic properties. Research on the multi-axis model of sandy soil is crucial for predicting engineering problems such as foundation bearing capacity, tunnel stability, liquefaction during earthquakes, and seabed evolution. In recent years, multi-axis models based on experiments, theories, and numerical simulations have been continuously developed, enabling researchers to more accurately describe the deformation and failure characteristics of sandy soil under complex stress conditions [3–8].

2. Basic characteristics of sandy soil behavior

Sandy soil exhibits complex deformation behavior when subjected to monotonic or cyclic shear loads. This behavior is mainly affected by the coupling effects of shear and volumetric deformation. The deformation characteristics of sandy soil include shear deformation and volumetric deformation. Shear deformation is mainly caused by shear stress, while volumetric deformation is mainly caused by the combined effects of shear stress and normal stress.

The deformation behavior of sandy soil is also affected by its initial density and structure. Initial density refers to the porosity of sandy soil in an undisturbed state, while structure involves the arrangement of sandy soil particles. Studies have shown that initial density and structure have a significant impact on the deformation behavior of sandy soil. For example, high-density sandy soil usually exhibits higher strength and smaller deformation, while the opposite is true for low-density sandy soil [9,10].

In addition, the deformation behavior of sandy soil is also affected by the initial normal stress and shear stress. Normal stress refers to the stress perpendicular to the shear plane, while shear stress is the stress parallel to the shear plane. A multitude of studies have demonstrated that the synergetic effect of normal stress and shear stress exerts a significant influence on the deformation behavior of sandy soil. Especially under extreme conditions such as earthquakes, the liquefaction behavior of sandy soil is closely related to the relationship between normal stress and shear stress [11–13].

3. Development history of multi-axis model for the sandy soil

The multi-axis model is an important tool for describing the behavior of sandy soil. Early multi-axis models were mainly based on simple elastic theory and plastic theory, such as the Mohr-Coulomb criterion and the Drucker-Prager criterion. These models can describe the basic deformation behavior of sandy soil to a certain extent, but they are still insufficient in complex loads.

With the deepening of research, researchers began to focus on the stress-strain relationship of sandy soil and proposed various improved multi-axis models. Among them, the model based on the concept of state has shown excellent performance in describing the behavior of sandy soil. The concept of state was first proposed by Roscoe *et al.* in 1963. They emphasized the comprehensive influence of density and normal stress on soil behavior and proposed the critical state as a reference to measure the influence of the initial state on soil behavior.

On the above basis, Been *et al.* (1985), Bolton (1986), Ishihara (1993), and Verdugo (1992) and other researchers proposed various indices and parameters for describing the behavior of sandy soil. These studies have laid the foundation for the development of subsequent models. The multi-axis model for sandy soil has evolved from classical soil mechanics theory and experimental equipment to modern computational simulation. In the future, it will become more intelligent and accurate and be applied to practical engineering in combination with AI and big data technologies.

(1) Classical Soil Mechanics Stage: Coulomb and Mohr Theories

- Coulomb (1773) proposed the shear failure criterion, laying the foundation for soil mechanics.
- Mohr (1900) proposed the concept of the stress circle, providing theoretical support for multi-axis stress analysis.
- Research on Soil Shear Characteristics
- In the 1900s, Terzaghi proposed the effective stress principle, explaining the mechanical behavior of saturated sand.
- Direct shear tests and triaxial tests are used to study the strength and deformation characteristics of

soil.

(2) Development of Multi-axis Testing Equipment and Constitutive Models

(A) Development of Triaxial Tests

- In the 1930s, the standard triaxial test was widely used and could measure the stress-strain relationship of sand under different confining pressures.
- In the 1950s, static and dynamic triaxial tests (Dynamic Triaxial Test) were used to study the response of sand under earthquake loads.

(B) Multi-axis Testing and Anisotropy Research

- In the 1970s, multi-axis testing equipment (such as true triaxial testing apparatus) was developed, capable of applying independent stresses in different directions.
- From the 1980s to the 1990s, research on the anisotropic properties of soil emerged, and several advanced constitutive models were developed.

(3) Modern computational models

(A) Elastoplastic constitutive models

- Cambridge Model (Cam-Clay Model, 1968): Applied to clay and later extended to sand.
- Drucker-Prager Model (1971): A multi-axis yielding model based on the Mohr-Coulomb criterion.
- Sand Hardening Constitutive Model (1990s): Considers the dilatancy and strain softening of sand.

(B) Numerical calculation methods

- Finite Element Method (FEM): Widely used in the multi-axis stress analysis of sand, such as in geotechnical engineering simulations.
- Discrete Element Method (DEM): Simulates the interaction between particles and is particularly suitable for analyzing quicksand and collapse.
- Coupled Method (FEM + DEM): Combines continuum mechanics and particle mechanics to improve simulation accuracy.

(C) AI and big data development

- Machine learning is used to predict the mechanical behavior of sand under different working conditions, improving the efficiency of engineering analysis [10,11].
- Combined with digital twin technology, real-time monitoring and optimization of sandy soil engineering can be achieved.

4. Basic equations of multi-axis generalization of sandy soil models

The multi-axis generalization of sandy soil models involves stress-strain relationships (elastic, elastoplastic models), yield criteria (Mohr-Coulomb, Drucker-Prager), flow rules (associated/non-associated flow), hardening rules (isotropic/anisotropic), and modern numerical models (Cam-Clay, Nor-Sand, PM4Sand), providing a solid theoretical foundation in rock and soil mechanics, soil mechanics, and computational mechanics. These equations and models provide a solid theoretical foundation for geological engineering, geotechnical analysis, and numerical simulation. The following are the basic equations of the sandy soil model in the multi-axis stress state.

4.1. Stress-strain relationship

During the loading process, the failure state is judged according to the Mohr-Coulomb criterion. When the failure state is not reached, the stress-strain relationship is usually obtained through experimental data fitting and can generally be approximately expressed as a linear elastic relationship $\Delta\sigma_{ij} = D_{ijkl}\Delta\varepsilon_{kl}$, where D_{ijkl} is the elastic stiffness tensor, and $\Delta\sigma_{ij}$ and $\Delta\varepsilon_{kl}$ are the stress increment and strain increment, respectively. When the failure state is reached, the strain will suddenly increase, the stress may remain unchanged (strain softening stage), or continue to increase but at a changed rate. For nonlinear sandy soil materials, elastoplastic constitutive relationships can be adopted, such as the Cambridge model or the Drucker-Prager model.

4.2. Yield criterion

The yield criterion of sandy soil determines when the material enters the plastic deformation stage under multi-axis stress, and the common ones are as follows:

(1) Mohr-Coulomb yield criterion

$$f = \tau - \sigma tan\phi - c = 0$$

Or in tensor form:

$$f = \sqrt{J_2} + \sigma_m sin\phi - c cos\phi = 0$$

where:

τ is the shear stress;

σ is the normal stress;

c is the cohesion;

ϕ is the internal friction angle;

$J_2 = \frac{1}{2}S_{ij}S_{ij}$ is the second invariant (deviatoric stress invariant);

$\sigma_m = \frac{1}{3}(\sigma_1 + \sigma_2 + \sigma_3)$ is the mean stress.

(2) Drucker-Prager yield criterion

$$f = \alpha I_1 + \sqrt{J_2} - k = 0$$

where:

$I_1 = \sigma_1 + \sigma_2 + \sigma_3$ is the first stress invariant;

α and k are material parameters related to the internal friction angle Φ and cohesion c.

The Drucker-Prager criterion is a smooth approximation of the Mohr-Coulomb criterion and is more suitable for finite element calculations.

4.3. Flow rule

The flow rule determines the relationship between the direction and magnitude of the plastic strain increment when sandy soil undergoes plastic deformation under multi-axis stress. It is based on plastic theory and is an indispensable part of multi-axis stress analysis, which is crucial for understanding the mechanical behavior of sandy soil under complex loading conditions.

(1) Associated Flow Rule: Assumes that the direction of the plastic strain increment is consistent with the gradient direction of the plastic potential surface.

$$\varepsilon_{ij}^{p} = \lambda \frac{\partial f}{\partial \sigma_{ij}}$$

Where λ is the plastic multiplier.

(2) Non-associated Flow Rule: Assumes that the direction of the plastic strain increment of sandy soil does

not strictly follow the direction determined by the associated flow rule. The non-associated flow rule considers the influence of factors such as the internal structure change and anisotropy of the soil body on the direction of plastic deformation.

$$\varepsilon_{ij}^{p} = \lambda \frac{\partial g}{\partial \sigma_{ij}}$$

where *g* is the plastic potential function, which is not necessarily equal to the yield function *f*.

4.4. Hardening rule

It describes the evolution of the yield surface of sandy soil during loading, which can be divided into isotropic hardening and anisotropic hardening:

(1) Isotropic hardening: The yield surface expands uniformly with the increase in plastic strain:

$$k = k_0 + H \cdot p^p;$$

where k_0 is the initial yield strength, *H* is the hardening modulus, and p^p is the cumulative plastic strain.

(2) Anisotropic hardening: The yield surface expands or rotates in specific directions to describe the strain softening and dilatancy characteristics of sandy soil.

4.5. Generalization of constitutive models

Widely used sandy soil multi-axis models in numerical calculations include:

(1) Cam-Clay Model (applicable to clay and can also be extended to sand)

(2) Drucker-Prager Model (applicable to the isotropic behavior of sand)

(3) Nor-Sand Model (a sandy soil model considering strain softening and dilatancy)

(4) PM4Sand Model (a constitutive model for sandy soil in earthquake dynamic analysis)

In multi-axis stress space, the generalization of models primarily concentrates on the following aspects:

(1) Generalization of Elastic Modulus: In the triaxial model, the elastic modulus G and K are considered functions of pressure p and the current porosity e. In multi-axis generalization, these moduli still depend on pressure and porosity, but their calculations need to consider a more complex stress state.

(2) Generalization of Yield Surface: In the triaxial model, the yield surface is defined as a function of the stress ratio. In multi-axis generalization, the yield surface is generalized into a cone shape to adapt to a wider range of stress states.

(3) Generalization of Plastic Modulus: In the triaxial model, the plastic modulus is related to the boundary stress ratio. In multi-axis generalization, the calculation of the plastic modulus needs to consider a more complex stress state and hysteresis behavior.

5. Future outlook

The future of the sandy soil multi-axis model will rely on multi-disciplinary cross-fertilization and technological innovation, shifting from pure mechanics research to more intelligent and environmentally adaptive systematic tools. Its development will not only serve traditional civil engineering but may also extend to emerging fields such as planetary geology (e.g., simulation of lunar/Martian soil) and new energy development.

(1) Multi-disciplinary integration and multi-scale modeling

Micro-macro Coupling: Combine the Discrete Element Method (DEM), Molecular Dynamics (MD), or Grain Flow Theory to deduce macroscopic mechanical behavior from microscopic scales (such as particle breakage, contact friction) and establish a more physically-based multi-axis constitutive model.

Cross-disciplinary Integration: Integrate materials science, geophysics, and artificial intelligence technologies to reveal the complex responses of sandy soil in different environments (such as humidity, temperature, chemical action).

(2) Data-driven constitutive models

Machine Learning and AI-assisted Modeling: Use deep learning, neural networks, or reinforcement learning techniques to automatically discover constitutive relationships from experimental data or numerical simulations, replacing traditional empirical formulas and improving the generalization ability of the model for different working conditions.

Digital Twin and Real-time Feedback: Combine sensor data and real-time monitoring to dynamically correct model parameters and achieve real-time prediction and risk warning at engineering sites.

(3) Dynamic response under complex working conditions

Dynamic Loading and Cyclic Loading: Improve the description of the model for dynamic multi-axis loading conditions such as earthquakes and wave loads, such as the plastic flow and hysteresis behavior under non-proportional loading paths.

Extreme Condition Simulation: Study the mechanical properties of sandy soil in extreme environments such as high temperature, high pressure (e.g., deep geotechnical engineering), and high-speed impact (e.g., explosions or meteorite impacts).

(4) Integration of environmental and climatic factors

Humidity-Temperature Coupling Effect: Quantify the impact of climate change (such as freeze-thaw cycles, dry-wet alternation) on the strength, deformation, and stability of sandy soil and develop multi-field coupling (water-heat-force-chemical) constitutive models.

Ecological Geotechnical Engineering: Combine the influence of biological factors such as vegetation roots and microbial activities on the mechanical properties of sandy soil to support applications such as ecological slope protection and desertification control.

(5) Computational efficiency and engineering application optimization

High-performance Computing and Parallelization: Develop model algorithms suitable for GPU acceleration or distributed computing to meet the rapid simulation needs of large-scale projects (such as land reclamation, tunnel engineering).

Simplification and Standardization: Propose simplified models that are easy to obtain parameters, have low computational costs, and controllable accuracy for engineering practice needs and promote the standardization process.

(6) Sustainability and environmental friendliness

Resource Utilization and Circular Economy: Study the mechanical behavior of sandy soil in waste landfilling and after mixing with recycled materials (such as construction waste) to support sustainable engineering practices.

Carbon Footprint Assessment: Combine the mechanical analysis of the sandy soil model with carbon emission calculations to optimize engineering design and reduce the environmental load.

(7) Future challenges

Theoretical Breakthroughs: How to uniformly describe the phase transition behavior (such as liquefaction, dilatancy-densification transition) and non-local effects of granular materials.

Multi-physics Field Coupling: Solve the problem of constitutive modeling under multi-field interactions such as water-force-chemical-thermal interactions.

Universality: Balance the complexity and practicality of the model to make it applicable to sandy soil types with different geographical locations and particle size distributions.

6. Conclusion

The multi-axial model of sandy soil plays a crucial role in geotechnical engineering, earthquake engineering, and related fields. While traditional triaxial models have advanced the understanding of sandy soil behavior, multi-axial extensions provide a more accurate representation of complex stress states in real-world engineering, enhancing both applicability and precision. The mechanical behavior of sandy soil under multi-axial loading is influenced by factors such as particle characteristics, strain localization, and anisotropy. Its deformation includes both shear and volumetric components, which are governed by initial density, structural properties, and stress conditions. The evolution of multi-axial models has progressed from classical soil mechanics and experimental advancements to modern computational simulations, achieving significant milestones along the way. Future developments will emphasize intelligence and precision, integrating AI and big data technologies to enhance practical applications. Extending sandy soil models into the multi-axial domain involves key theoretical components, including stress-strain relationships, yield criteria, flow rules, hardening laws, and constitutive model expansions, providing a robust foundation for research and application. In the future, multi-axial sandy soil models will increasingly rely on interdisciplinary collaboration and technological innovation, evolving from purely mechanical frameworks into intelligent, adaptive tools. Beyond traditional civil engineering, their applications will extend to emerging fields such as planetary geology and new energy development. However, challenges remain, including theoretical breakthroughs, multi-physical field coupling, and achieving broad applicability.

Disclosure statement

The author declares no conflict of interest.

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