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Influence of Ambient Temperature on the Thermal Safety of Fireworks Products in Civil Aviation Transportation

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Abstract: Fireworks products are energy-containing materials and are hazardous during production, storage, transportation, and use. By analyzing the range of civil aviation ground ambient temperature and civil aviation cabin ambient temperature in storage and ground operation as well as establishing a spontaneous combustion mathematical model for cylindrical fireworks products based on the spontaneous combustion theory, we identified the critical temperature for spontaneous combustion of a single spray and analyzed the thermal safety of fireworks products under the civil aviation ambient temperature by example to provide theoretical support for the feasibility study of transporting fireworks products by civil aviation.

Keywords: Civil aviation ambient temperature; Fireworks; Spontaneous combustion theory; Thermal safety

1. Introduction

Fireworks and firecrackers are crafts made from pyrotechnic powder and produce sound, light, color, smoke, and other effects through combustion (explosion) reactions. Fireworks are colorful and festive, and now they have become an indispensable item during festivals and major celebrations. As the largest producer and exporter of fireworks and firecrackers, China has a huge demand for domestic and international transportation of fireworks and firecrackers. However, fireworks and firecrackers are energy-containing materials and are hazardous during production, storage, transportation, and use. Fireworks and firecrackers are mainly transported via land and sea. The low transportation efficiency and long transportation time heighten the uncertainty and risk of danger in the transportation process. The use of air transport, with high speed and high efficiency, would enhance the economic benefits of certain industries. In this paper, we analyze the civil aviation ambient temperature limit, establish a spontaneous combustion mathematical model for fireworks products based on the theory of spontaneous combustion, as well as calculate and analyze the thermal safety of fireworks products under the civil aviation ambient temperature.

2. Introduction to civil aviation transportation of fireworks products

According to the International Air Transport Association's "Dangerous Goods Regulations," fireworks products are assigned to five transport numbers UN0333–UN0337, of which the dangerous goods category and explosives assembly group of UN0336 include 1.4 G explosives, which can be transported by civil aviation all-cargo aircraft. 1.4 G fireworks and firecrackers mainly include fireworks shells, spit beads, rockets, bouquets, spray flowers, sparklers, Bengal sticks, runners, whip firecrackers, and other types of fireworks [1]. In this study, the cylindrical spray product was selected for analysis.

3. Analysis of ambient temperature in civil aviation

The theory of spontaneous combustion provides an assumption that the environment in which the studied object is in is at a constant temperature, and the temperature of each point inside the system is equal to the ambient temperature at the beginning. As the reaction proceeds, the internal temperature exceeds the ambient temperature, and the geometric center of the system is the most unfavorable for heat dissipation. Thermal autoignition occurs if the rate of heat generation by the reaction is greater than the rate of heat loss to the environment. Fireworks and firecrackers are typical exothermic systems. When transported by civil aviation, it is necessary to consider the influence of civil aviation ground ambient temperature and civil aviation flight cabin ambient temperature on the thermal safety of fireworks and firecrackers during transportation.

3.1. Civil aviation ground ambient temperature

3.1.1. Extreme surface atmospheric temperature records

The surface atmospheric high temperature record is 56.7°C (Death Valley, California, USA on July 10, 1913), while the surface atmospheric low temperature record is -89.2°C (Antarctic on July 21, 1983). Civil aviation aircrafts performing international flight missions hardly encounter harsh environment at high or low temperature. If the influence of thermal radiation on ambient temperature is not considered, after a long time, the temperature of each part of the aircraft will be consistent with the ambient temperature. However, in actual engineering design, the influence of thermal radiation on temperature needs to be considered, and this influence is significant. Other than that, heat capacity, color temperature, and structural form also affect the actual temperature in different cabins and structures.

3.1.2. Effect of thermal radiation on temperature

For the extreme high temperature of aircrafts, it is necessary to consider the impact of solar radiation when the extreme high temperature appears. According to a document from the United States, the internal temperature of an aircraft when parked outside is 15–30°C higher than the ambient temperature. According to our aircraft temperature measurements data in the 1970s, the temperature of the internal equipment compartment of an aircraft under direct sunlight is 15–20°C higher than the ambient temperature. Based on actual measurement results, GJB1060.2-91 stipulates that the open-air parts shall conform to the high temperature extremes corresponding to the time the equipment is exposed to the site plus the rise in temperature generated by 1110 W/m^2 solar radiation (equivalent to 17°C). Although the thermal characteristics of different parts of the aircraft vary, after a long time, the maximum temperature of each part of the aircraft will reach about 74°C [2].

Radiation also affects low temperature extremes. During winter, the external environment of the aircraft is mainly white. When there is no solar radiation or wind at night and the color temperature of the aircraft is higher than that of the surrounding environment, the amount of outward radiation is greater than the amount of radiation it absorbs, thus causing the temperature of the aircraft to be lower than that of the surrounding environment. Foreign data and actual measurement results in China have demonstrated that the temperature in the cabin of an aircraft parked outside is 3–5°C lower than the ambience temperature; that is to say, the low extreme storage temperature of aircraft performing international flights should be around -95°C.

As mentioned above, the extreme ambient temperature that fireworks products may encounter when loading, unloading, and staying on the ground is the atmospheric temperature when the aircraft is on the ground; the maximum temperature is about 74°C, while the minimum is about -95°C.

3.2. Civil aviation cabin ambient temperature

During flight, the passenger cabin and cargo compartment of civil aviation airliners will heat and pressurize at the same time. The air pressure and temperature in the cabin are about 0.8 standard pressure and 24°C, respectively. The ambient temperature of the cabin and cargo compartment is basically the same. For all-cargo aircrafts, although the temperature of the cargo compartment of different types of cargo aircrafts is not exactly the same, the temperature range of the cargo compartment can be controlled between 0°C and 30°C. For example, the temperature control range of the main cabin, front cabin, rear cabin, and bulk cabin of B777-200F is 4–27°C. The minimum temperature range of the main cabin of B767-300F is 2–22°C, while the maximum temperature range is 18–30°C; the temperature range of the front cabin and the rear cabin is 4.5–10°C; the minimum temperature range of the bulk cabin is 4.5–10°C, while the maximum temperature range is 15.5–23.8°C.

Dangerous goods such as fireworks products are transported by all-cargo aircrafts, and the ambient temperature during the flight is the same as the temperature inside the cargo of the aircraft. Even if the specific flight conditions change, the changes in temperature range in the cargo compartment during flight will not exceed the range of extreme high and low temperatures on the ground, of which the high extreme temperature is about 74°C, while the low extreme temperature is about -95°C.

4. Spontaneous combustion model for fireworks products

Based on the thermal explosion theory, a single cylindrical firework is regarded as a chemical exothermic reaction system, and on the basis of the law of energy conservation, the following basic assumptions are made [3]: (i) in addition to the heat from chemical reaction, there is no other volumetric heat sources; the heat exchange between the substance and the environment occurs only on the surface of the system, and Newton's cooling law is satisfied; (ii) when ignition occurs, the conversion rate of the reaction substance is very low, so it is assumed that the change in the initial concentration of the reaction substance during the whole process from self-heating to explosion is negligible and the chemical reaction kinetics can be described by the so-called "zero-order reaction," obeying the Arrhenius law; (iii) the respective isotropy is within the substance; the physical properties of the substance (thermal conductivity, density, and specific heat capacity), the chemical properties (activation energy, pre-exponential factor, and heat of reaction), and the heat transfer conditions between the substance and the environment (surface heat transfer coefficient, ambient medium temperature, as well as the shape and size of the reactants) remain constant throughout the process from self-heating to explosion; (iv) since heat transfer inside the substance is caused by heat conduction and there is no relative motion of the reacting substances, there is no convective heat transfer; (v) there is close contact between the chemical inside the fireworks and the shell, *i.e.*, the contact thermal resistance is negligible.

4.1. Spontaneous combustion model for cylindrical fireworks

As shown in **Figure 1**, for a cylindrical exothermic system with a radius of a_0 and an aspect ratio of H in the cylindrical coordinate system, assuming that the central maximum temperature point O is the coordinate origin, the energy conservation equation of the cylindrical model is as follows:

$$\sigma c_v \frac{\partial T}{\partial t} = k\left(\frac{\partial^2 T}{\partial R_1^2} + \frac{1}{R_1}\frac{\partial T}{\partial R_1} + \frac{\partial^2 T}{\partial R_2^2}\right) + QA\exp\left(-\frac{E}{RT}\right) \tag{1}$$

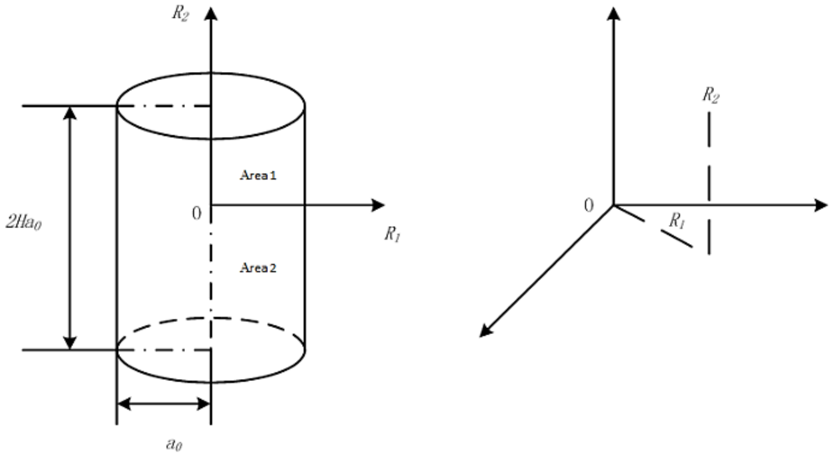

Figure 1. Schematic diagram of a finite length cylinder

In the steady state, $\frac{\partial T}{\partial t}=0$; hence, the energy conservation differential equation of the exothermic reaction in the steady state is as follows:

$$k\left(\frac{\partial^2 T}{\partial R_1^2}+\frac{1}{R_1}\frac{\partial T}{\partial R_1}+\frac{\partial^2 T}{\partial R_2^2}\right)+QA\exp\left(-\frac{E}{RT}\right)=0 \tag{2}$$

The upper and lower parts of point *O* satisfy the following two sets of boundary conditions, respectively. The boundary conditions of area 1 are as follows

$$R_1=0,\frac{\partial T}{\partial R_1}=0,\left(0\le R_2\le X\right) \tag{3}$$

$$R_2=0,\frac{\partial T}{\partial R_2}=0,\left(0\le R_1\le a_0\right) \tag{4}$$

$$R_1=a_0,k\frac{\partial T}{\partial R_1}+\chi\left(T-T_a\right)=0,\left(0\le R_2\le X\right) \tag{5}$$

$$R_2=X,k\frac{\partial T}{\partial R_2}+\chi\left(T-T_a\right)=0,\left(0\le R_1\le a_0\right) \tag{6}$$

The boundary conditions of area 2 are as follows:

$$R_1=0,\frac{\partial T}{\partial R_1}=0,\left(0\le R_2\le X-2Ha_0\right) \tag{7}$$

$$R_1=0,\frac{\partial T}{\partial R_1}=0,\left(0\le R_2\le X-2Ha_0\right) \tag{8}$$

$$R_1 = a_0, k\frac{\partial T}{\partial R_1} + \chi\left(T - T_a\right) = 0, \left(0 \le R_2 \le X - 2Ha_0\right) \tag{9}$$

$$R_2 = X - 2Ha_0, k\frac{\partial T}{\partial R_2} + \chi\left(T - T_a\right) = 0, \left(0 \le R_1 \le a_0\right) \tag{10}$$

where X is the distance from the ignition point to the upper end surface; T and T_a are the temperature in the system and the ambient temperature, respectively; k and σ are thermal conductivity and density, respectively; Q , E, and A are the heat of reaction, activation energy, and pre-exponential factor, respectively; R is the universal gas constant; and χ is the heat transfer coefficient.

For convenience of research, the following parameters with a dimension of 1 are introduced:

dimensionless coordinates $\rho_1 = R_1 / a_0$ (11)

dimensionless coordinates $\rho_2 = R_2 / Ha_0$ (12)

dimensionless temperature $\theta = \left(T - T_a\right) / \left(\mathrm{R}T_a^2 / E\right)$ (13)

dimensionless activation energy $\varepsilon = \mathrm{R}T_a / E$ (14)

Frank-Kamenetskii parameter $\delta = a_0^2 QE\sigma A\exp\left(-E / \mathrm{R}T_a\right) / k\mathrm{R}T_a^2$ (15)

Biot number $Bi = \chi a_0 / k$ (16)

The dimensionless thermal explosion equation and boundary conditions of cylindrical fireworks can be written as follows

$$\frac{\partial^2\theta}{\partial\rho_1^{\;2}} + \frac{1}{\rho_1}\frac{\partial\theta}{\partial\rho_1} + \frac{1}{H^2}\frac{\partial^2\theta}{\partial\rho_2^{\;2}} + \delta\exp\left(\frac{\theta}{1+\varepsilon\theta}\right) = 0 \tag{17}$$

The dimensionless form of the boundary condition for region 1 is as follows:

$$\rho_1 = 0, \frac{\partial\theta}{\partial\rho_1} = 0, (0 \le \rho_2 \le x) \tag{18}$$

$$\rho_2 = 0, \frac{\partial\theta}{\partial\rho_2} = 0, (0 \le \rho_1 \le 1) \tag{19}$$

$$\rho_1 = 1, \frac{\partial\theta}{\partial\rho_1} + Bi_r\theta = 0, (0 \le \rho_2 \le x) \tag{20}$$

$$\rho_2 = x, \frac{\partial\theta}{\partial\rho_2} + HBi_{z1}\theta = 0, (0 \le \rho_1 \le 1) \tag{21}$$

The dimensionless form of boundary conditions for region 2 is as follows:

$$\rho_1 = 0, \frac{\partial \theta}{\partial \rho_1} = 0, (x-2 \le \rho_2 \le 0) \tag{22}$$

$$\rho_2 = 0, \frac{\partial \theta}{\partial \rho_2} = 0, (0 \le \rho_1 \le 1) \tag{23}$$

$$\rho_1 = 1, \frac{\partial \theta}{\partial \rho_1} + Bi_r \theta = 0, (x-2 \le \rho_2 \le 0) \tag{24}$$

$$\rho_2 = x-2, \frac{\partial \theta}{\partial \rho_2} + HBi_{z2} \theta = 0, (0 \le \rho_1 \le 1) \tag{25}$$

where x is the dimensionless value of the distance from the ignition point to the upper end surface, and the range of x is from 0 to 2; ρ_1 and ρ_2 are dimensionless coordinate measurements; θ is the dimensionless temperature rise; ε is the dimensionless activation energy; δ is the Frank-Kamenetskii parameter; Bi_r, Bi_{z1}, and Bi_{z2} are the *Biot* numbers of the side surface, upper end surface, and lower end surface, respectively.

4.2. Boundary condition processing

Since fireworks and firecrackers have relatively complex shell conditions, the influence of the shell and the surrounding air on the heat dissipation of pyrotechnic agents is considered, and the concept of equivalent *Biot* number is proposed, which reasonably simplifies the boundary conditions. The calculation process can be simplified. The boundary condition treatment of spray flowers is taken as an example in this paper. The shell condition of the side surface is a layer of kraft paper shell, and the shell condition of the upper and lower surfaces is a layer of sealing powder and a layer of mud bottom, respectively.

4.2.1. Treatment of side surface boundary conditions

The shell of the side surface of cylindrical fireworks is a single or multi-layer cylindrical paper shell. The heat transfer at the boundary of the side surface involves three processes: (i) heat transfer between the pyrotechnic agent and the paper shell, which mainly entails the problem of contact thermal resistance; (ii) heat transfer from the high temperature side to the low temperature side of the paper shell wall, whereby the heat conduction thermal resistance of the paper shell is involved in the heat conduction of the paper shell wall; (iii) heat transfer between the low temperature side of the paper shell wall and the external environment. The heat transfer process of the side surface shell is shown in **Figure 2**.

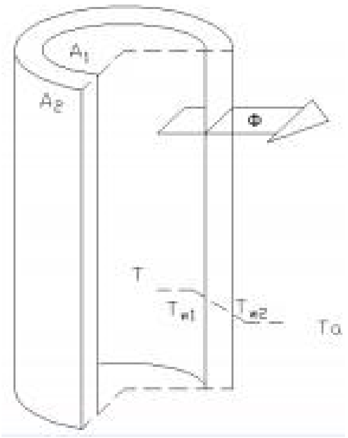

Figure 2. Schematic diagram of the heat transfer process of the side surface shell

According to the law of steady-state heat flow [4], the boundary conditions can be expressed as follows:

$$r = a_0, kA_1 \frac{dT}{dr} + \frac{T - T_a}{R_r + R_c + \frac{1}{\chi A_2}} = 0 \tag{26}$$

where A1 is the inner surface area of the paper shell, and k is the thermal conductivity of the pyrotechnic agent.

Introducing dimensionless parameters ρ_1 and θ, the above formula can be written as follows:

$$\rho_1 = 1, \frac{d\theta}{d\rho_1} + \frac{a_0}{A_1 k} \times \frac{1}{R_r + R_c + \frac{1}{\chi A_2}} \theta = 0 \tag{27}$$

The equivalent *Biot* number is defined as follows:

$$Bi_r^{'} = \frac{a_0}{kA_1} \times \frac{1}{R_r + R_c + \frac{1}{\chi A_2}} \tag{28}$$

The corresponding boundary conditions are then transformed into:

$$\rho_1 = 1, \frac{d\theta}{d\rho} + Bi_r^{'} \theta = 0 \tag{29}$$

From formula (28), we can see that when defining the equivalent *Biot* number, $a_0/(kA1)$ is the thermally conductive thermal resistance of the pyrotechnic agent. $R_r + R_c + 1/(\chi A_2)$ denotes the thermal conductivity thermal resistance of the pyrotechnic agent at the boundary through the paper shell and the air convection heat transfer thermal resistance. When the equivalent *Biot* number is small, the internal heat conduction resistance is far smaller than the convective heat transfer heat resistance at the shell, and the temperature drop is mainly manifested at the shell; when the equivalent *Biot* number is large, the temperature drop is mainly manifested in the pyrotechnic powder agent inside.

4.2.2. Treatment of upper and lower surface boundary conditions

The upper surface of the spray flower is a round cake-shaped sealing powder, and the treatment method for its boundary condition is the same as that of the lower surface mud bottom. The heat transfer process is connected in series with three links: (i) heat transfer between the pyrotechnic agent and the end surface, in which the main issue is contact thermal resistance; (ii) heat transfer from the low temperature side to the high temperature side of the end face, *i.e.*, heat conduction across the end face, involving the thermal resistance of the end face; and (iii) heat transfer between the low temperature side of the end face and the external environment (**Figure 3**).

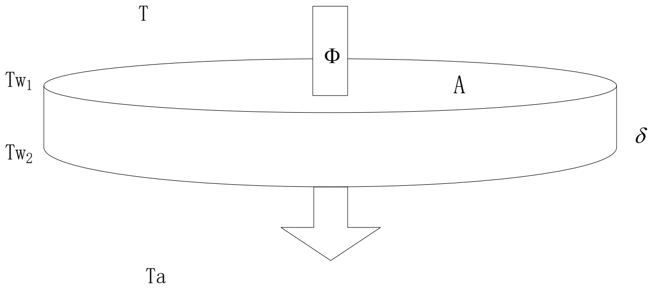

Figure 3. Schematic diagram of heat transfer process in the mud bottom

According to the law of steady-state heat flow [4], the boundary conditions can be expressed as follows:

$$r = Ha_0, kA\frac{dT}{dr} + \frac{T - T_a}{R_z + R_c + \frac{1}{\chi A}} = 0 \tag{30}$$

where k is the thermal conductivity of the pyrotechnic charge.

Introducing dimensionless parameters ρ_2 and θ, the above formula can be written as follows:

$$\rho_2 = 1, \frac{d\theta}{d\rho_2} + \frac{Ha_0}{Ak} \times \frac{1}{R_z + R_c + \frac{1}{\chi A}}\theta = 0 \tag{31}$$

The equivalent *Biot* number at the boundary of the upper and lower surfaces is defined:

$$Bi_{z1}^{'} = Bi_{z2}^{'} = \frac{Ha_0}{kA} \times \frac{1}{R_z + R_c + \frac{1}{\chi A}} \tag{32}$$

The corresponding boundary conditions are then transformed into:

$$\rho_2 = -1, \frac{d\theta}{d\rho_2} + Bi_{z2}^{'}\theta = 0 \tag{33}$$

5. Calculation of the thermal safety of fireworks products

Panda Fireworks' spray product "Blooming Butterfly Love" is taken as an example. The product is cylindrical, with a height of 188 mm, an inner diameter of 94 mm, a wall thickness of 2 mm, a thickness of

30 mm at the upper and lower end surfaces, an overall charge of 56 g, a charge height of 46 mm, and a charge density of 400 kg/m^3. The setting effect is as follows: white chrysanthemums, red and green beads, as well as 10 red, green, and yellow fried beads. This product uses black gunpowder as the injection gunpowder, and the effective gunpowder includes white gunpowder, red gunpowder, green gunpowder, and yellow gunpowder. Compared with the exothermic effect of black powder, the exothermic effect of the agent can be neglected [5]. The basic parameters of the spray are provided by Beijing Fireworks and Firecrackers Quality Supervision and Inspection Station.

5.1. Calculation parameters

Fountain tube height, l =188 mm; charge height, h = 46 mm; inner radius, $r_1 = a_0$ = 94/2 = 47 mm; outer-half diameter, r_2 = 98/2 = 49 mm; thickness of the upper and lower end surfaces, $\delta_1 = \delta_2 = 30$ mm; length-to-diameter ratio, $H = h/2a0 = 0.489$. The area of the end face, $A = \pi a_0^2$; the contact area between the pyrotechnic agent and the paper shell, $A_1 = 2\pi r_1 h$; the contact surface area between the shell of the fireworks and the air, $A_2 = 2\pi r_2 l$; the natural convection surface heat transfer coefficient χ of the air is 1–10 W/m^{2}K, where the heat transfer coefficient $\chi = 5$; the thermal conductivity of black powder, $k = 0.0847$ W/m^{2}K; the thermal conductivity of paper shell, $\lambda_r = 0.0357$ W/m^{2}K; and the thermal conductivity coefficients of the sealing powder and mud bottom, $\lambda_{z1} \approx \lambda_{z2} = 0.95$ W/m^{2}K. Other physical and chemical parameters of the spray pyrotechnic agent are shown in **Table 1**.

Table 1. Calculation results of steady-state critical parameters of the spray "Blooming Butterfly Love"

Substances	**Activation energy (J/kg)**	**Pre-exponential factor (s^{-1})**	**Heat of reaction (J/kg)**	**Thermal conductivity (W/m^{2}K)**
Black powder	1.9100×10^5	1.40×10^{16}	1.5263×10^6	0.0847
Upper surface material (sealing powder)	–	–	–	0.9500
Lower surface material (mud bottom)	–	–	–	0.9500
Side surface material (paper shell)	–	–	–	0.0357

From the basic theory of heat transfer, the thermal conduction resistance formulas of single-layer flat wall and single-layer cylindrical wall are $R_z = \dfrac{\delta}{\lambda_z A}$ and $R_r = \dfrac{\ln(r_2/r_1)}{2\pi\lambda_r l}$ [4], respectively, which can be calculated as follows: upper and lower end surface thermal resistance, $R_{z1} = R_{z2} = 4.55$KW, and side surface thermal resistance, R_r = 0.99KW. Substituting the above requirements into the formula (28), $Bi_r^{'} = \dfrac{a_0}{kA_1} \times \dfrac{1}{R_r + R_c + \dfrac{1}{\chi A_2}}$, and formula (32), $Bi_z^{'} = \dfrac{Ha_0}{kA} \times \dfrac{1}{R_z + R_c + \dfrac{1}{\chi A}}$, the side surface can be obtained. The equivalent *Biot* numbers of the upper and lower surfaces are $Bi_r^{'} = 2.7031$ and $Bi_{z1}^{'} = Bi_{z2}^{'} = 1.1726$.

5.2. Calculation results

When the spray product "Blooming Butterfly Love" is stored in a single cylinder, the corresponding critical parameter critical explosion criterion δ_{cr} is calculated to be 1.796424 by MATLAB programming [6,7], and the critical ambient temperature Ta,cr can be expressed by $\delta = a_0^2 Q E \sigma A \exp\left(-E/\mathrm{R}T_a\right)/k\mathrm{R}T_a^2$

(Frank-Kamenetskii expression), obtaining $T_{a,cr}$ = 450.5K. When the “Blossoming Butterfly Love” is stored in a single tube, the critical ambient temperature is 450.5K.

The critical ambient temperature of the product is higher than the extreme high temperature of civil aviation transportation (347.15K), indicating that the thermal safety of the product is stable in air transportation and theoretically there will be no risk of spontaneous combustion.

6. Conclusion

The range of civil aviation ground ambient temperature and civil aviation flight cabin temperature in civil aviation storage, ground operation, and other operations was found to be 178.15–347.15 K. Based on the theory of spontaneous combustion, we established a spontaneous combustion mathematical model for cylindrical fireworks products. The spray product “Blossoming Butterfly Love” was selected. The critical temperature of its spontaneous combustion was calculated to be 450.5 K, which is higher than the extreme high temperature in civil aviation transportation. This shows that the spray is thermally stable during air transportation and theoretically there will be no danger of spontaneous combustion. However, considering that fireworks products will be stacked during transportation, due to the mutual interference of thermal fields among multiple single-tube fireworks and poor heat dissipation conditions, the critical ambient temperature will be reduced, thereby reducing the thermal safety as well. The thermal safety of stacked fireworks products in civil aviation transportation requires further research.

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Disclosure statement

The author declares no conflict of interest.

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