

# Application of Hidden Markov Models in Stock Forecasting

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**Abstract:** In this paper, we tested our methodology on the stocks of four representative companies: Apple, Comcast Corporation (CMCST), Google, and Qualcomm. We compared their performance to several stocks using the hidden Markov model (HMM) and forecasts using mean absolute percentage error (MAPE). For simplicity, we considered four main features in these stocks: open, close, high, and low prices. When using the HMM for forecasting, the HMM has the best prediction for the daily low stock price and daily high stock price of Apple and CMCST, respectively. By calculating the MAPE for the four data sets of Google, the close price has the largest prediction error, while the open price has the smallest prediction error. The HMM has the largest prediction error and the smallest prediction error for Qualcomm's daily low stock price and daily high stock price, respectively.

**Keywords:** Hidden Markov model; Mean absolute error, Stock market

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## 1. Introduction

The stock market is unpredictable, but its trend can be predicted. Although the stock market cannot be predicted with 100% accuracy <sup>[1]</sup>, it can be operated probabilistically. For example, by combining historical data with other information, it is possible to determine the current position of the stock market and the probability of the next stock market rise or fall. The magnitude of this probability will affect the next operation strategy development. This is the importance of forecasting. Since the stock market is running on a cyclical pattern, this cyclical law is the only law that we humans can grasp from the stock market <sup>[2]</sup>. Therefore, although the stock market is said to be unpredictable, it is in fact predictable. Prediction is a prerequisite for operation. Therefore, improving the accuracy of prediction is an important basis for stockholders to make money with a high probability. Economists have established various non-linear equation models to study various movements in the economic and financial markets, such as stock market indices, exchange rate changes, *etc.* <sup>[3]</sup>.

Stock market forecasting has been one of the more active research areas in the past due to the interest of many large companies <sup>[4]</sup>. Historically, various machine learning algorithms have also been applied to this area with varying degrees of success. There are many ways to use deep learning and deep machine learning to predict the stock market. For example, moving average, linear regression, K-nearest neighbors, automatic auto-regressive integrated moving average (ARIMA), Prophet, and long short-term memory network (LSTM). However, stock price forecasting is still limited by many factors due to its volatile, seasonal, and unpredictable nature. Forecasting based on previous stock price data alone is an even more challenging task as some marginal factors are not considered. Stock prices are affected by company news

and other factors, such as demonetization or mergers/spinoffs of companies <sup>[5]</sup>. There are also intangible factors that often cannot be predicted in advance.

In this study, we used four different stocks to evaluate this approach: Apple, Google, Qualcomm, and Comcast Corporation (CMCST). In our analysis, a single variable was controlled, and the data for each stock were observed and recorded separately for their corresponding change curve. This paper is arranged as follows: in section 2, we review the specific elaboration of the hidden Markov model (HMM) technique from multiple dimensions; section III provides the mathematical proof of our approach to the HMM; in section IV, we describe the dataset and provide the experimental results; section V discusses the results and concludes the paper.

## 2. Literature review

Stock price prediction has become one of the hottest research fields in recent years as people have become increasingly enthusiastic about trading stocks based on the high returns they can bring. In this paper, HMM was used to predict the daily stock prices of four companies. HMM is the simplest structured dynamic Bayesian network that is capable of modeling hidden state transitions based on ordered observations <sup>[6]</sup>. HMM has been successfully applied in various fields, including speech recognition <sup>[7]</sup>, computational biology <sup>[8]</sup>, and signature verification <sup>[9]</sup>. Stock forecasts follow the same pattern, with stock prices depending on factors (implied variables) that are usually imperceptible to investors. Stock price changes are complex and volatile, and they are inextricably linked to corporate policies and decisions, financial conditions, and management decisions, all of which affect stock prices beyond the control of investors. Therefore, HMMs are naturally suitable for price prediction problems <sup>[10]</sup>.

HMM is now used in stock prediction analysis. In a study, Hassan and Nath <sup>[10]</sup> used HMM to forecast the share prices of four airlines, providing a new method for stock forecasting. The basic principle is to use HMM to screen the appropriate behavioral variables from previous data, and then combine the interpolation data near values, and use the mean absolute percentage error (MAPE) to predict the stock. Zhang *et al.* <sup>[11]</sup> have proposed the use of dynamic high-order HMM to forecast stock prices of CSI 300 index and S&P 500 index. High-order HMM transforms high-dimensional state vectors into single state vectors, allowing the simultaneous consideration of the stock market's short-term and long-term time dependency. The relationship between the hidden state and the predicted value can be obtained following the statistical analysis of historical daily returns. This suggests that dynamic high-order HMM has higher accuracy in predicting stock prices. In addition, Hassan <sup>[12]</sup> combined HMM with a fuzzy model to predict stock prices and found that the prediction accuracy could be improved. The key feature of this study is the division of the data space after the use of HMM for data training, the generation of a fuzzy model using the data space, and the use of the fuzzy model to predict the stock price. MAPE is then used to determine the prediction effect of the model. In a study conducted by Gupta and Dhingra <sup>[13]</sup>, MAPE was also used in the prediction of stocks using HMM; they took into account the fractional change of stock prices to train HMM, so as to improve the accuracy of predicting future stock prices as much as possible.

According to previous analysis of stock price prediction using HMM, HMM was also used in this study to train and model the open price, close price, daily high price, and daily low price of four companies, and these four hidden states were continuously observed. Following the stock price prediction, we calculated the MAPE to compare and analyze the accuracy of HMM in predicting the stock prices.

## 3. Methods

HMM was used to forecast the stock price of four companies. The formula of HMM is as follows:

$$\lambda = (\pi, A, B) \tag{1}$$

In this formula,

- (1)  $\pi$  refers to the initial probability, such as the probability when  $t = 1$ ;
- (2)  $A$  refers to the transition matrix, which represents the probability of state transition of an element;
- (3)  $B$  refers to the emission matrix; use  $b_j(\overrightarrow{O_t})$  to represent the probability of  $O$ , given state  $j$ .

Since all the observed variables in this experiment are random continuous variables, the emission distribution probability is assumed to be continuous, and the Gaussian mixture model (GMM) with parameter  $\mu$  is used.

$$b_j(\overrightarrow{O_t}) = \sum_{m=1}^M p(m)p(\overrightarrow{O_t} | m) = \sum_{m=1}^M c_{jm} N(\overrightarrow{O_t}, \overrightarrow{\mu_{jm}}, \Sigma_{jm}) \quad (2)$$

where,

- (1)  $c_{jm}$  is the weight of the  $m_{th}$  mixture component (Gaussian model) under state  $j$ ;
- (2)  $M$  is the number of components of the Gaussian mixture;
- (3)  $\overrightarrow{\mu_{jm}}$  is the mean vector for the  $m_{th}$  component in the  $j$  state;
- (4)  $p(\overrightarrow{O_t} | m) = N(\overrightarrow{O_t}, \overrightarrow{\mu_{jm}}, \Sigma_{jm})$  refers to the probability density function of the  $m_{th}$  Gaussian model.

In our experimental observations, four continuous random variables are used: the stock's opening price (open), closing price (close), daily high price (high), and daily low price (low). These data are stored in the form of four-dimensional vectors:

$$\begin{aligned} \overrightarrow{O_t} &= \left( \frac{close - open}{open}, \frac{high - open}{open}, \frac{open - low}{open} \right) \\ &= (fracChange, fracHigh, fracLow) \end{aligned} \quad (3)$$

#### 4. Analysis and results

##### (1) Datasets

The algorithm was tested on four different companies: Apple, CMCST, Google, and Qualcomm. For simplicity, we considered four main features in these stocks: open, close, high, and low prices. In the experiment, we collected information of the four groups of stocks of 2520 groups, and each group of information contains the information of four variables (open price, high price, low price, and close price). HMM was used to predict the next day's open price, close price, daily high price, and daily low price.

##### (2) Implementation

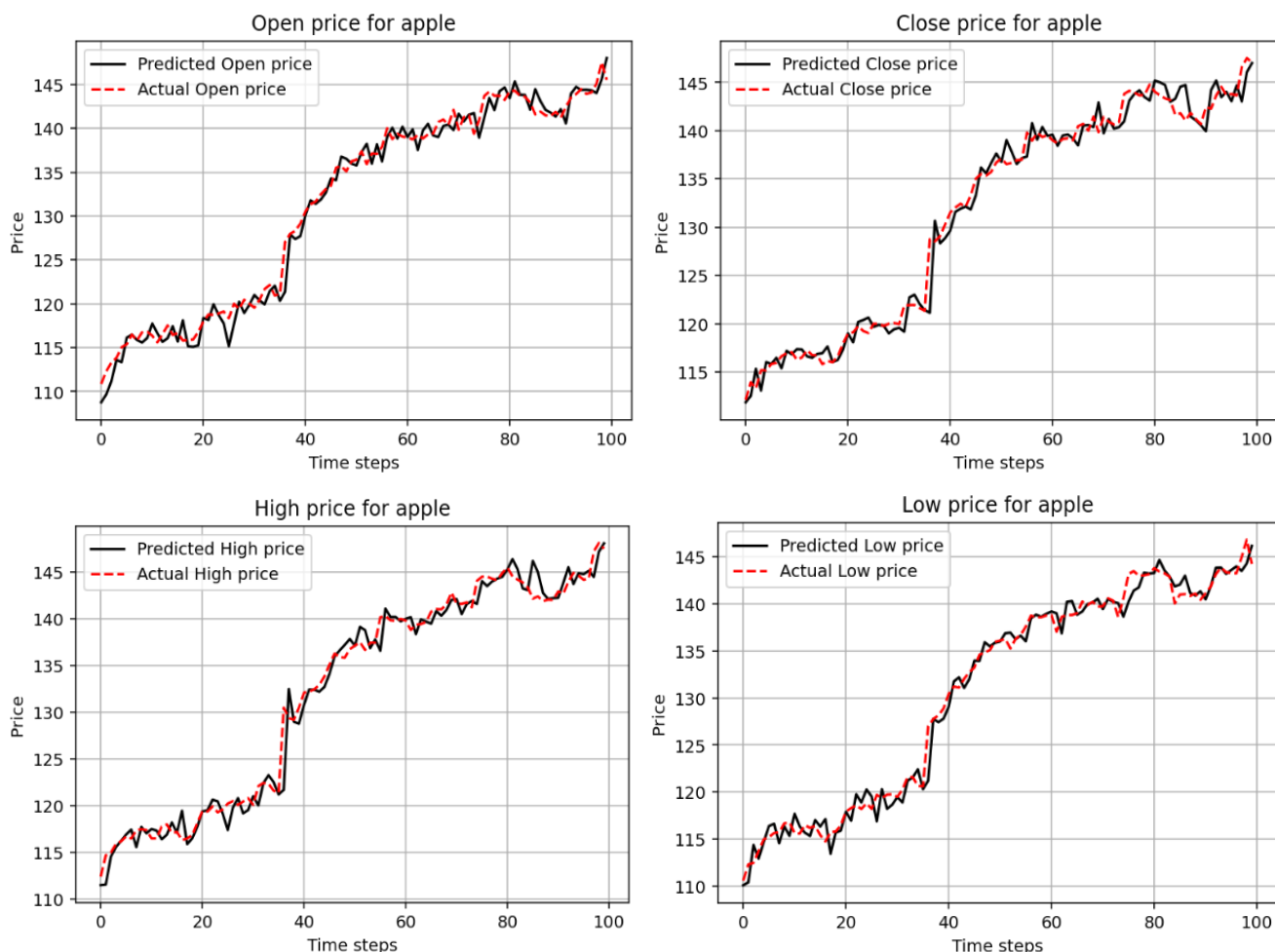
###### (a) HMM parameters

- (i) n\_components:  $N = 4$
- (ii) covariance\_type: full covariance matrix
- (iii) tol = 0.0001: stop threshold
- (iv) n\_iter = NUM\_ITERS: maximum number of iterations
- (v) init\_params: determine which parameters are initialized before iteration

###### (b) Result

In order to evaluate the prediction effect of HMM, the MAPE is calculated as the evaluation standard. Each MAPE in our experiment refers to the average absolute error between the actual data and the predicted data in percentage, such as actual open price and predicted open price.

According to the MAPE results calculated for Apple, it can be seen that the prediction error of its open price is the largest, followed by the prediction error of its close price, daily high stock price, and daily low stock price. Since the difference between the close price and the daily high stock price is 0.0001547, the prediction error is approximately the same, suggesting that the difference in the prediction effect is very small. According to the four panels in **Figure 1**, the fitting effect of the daily low stock price is relatively the best. Therefore, when using HMM for Apple's stock price prediction, HMM has the best prediction effect on the daily low stock price.



**Figure 1.** Actual and predicted values of the close, open, high, and low prices of Apple

**Table 1** shows that the prediction error of the open price of CMCST is the largest, while the prediction error of the daily high stock price of CMCST is the smallest. The difference between the close price and the daily low stock price is 0.00000622, which is approximately equal to zero, suggesting that the prediction effect of HMM is approximately the same for these two data sets. The four panels in **Figure 2** show that the deviation of the daily high stock price is the smallest, suggesting that the fitting effect is relatively the best. Therefore, when using HMM for CMCST's stock price prediction, HMM has the best prediction effect on the daily high stock price.

**Table 1.** Mean absolute percentage error of stock

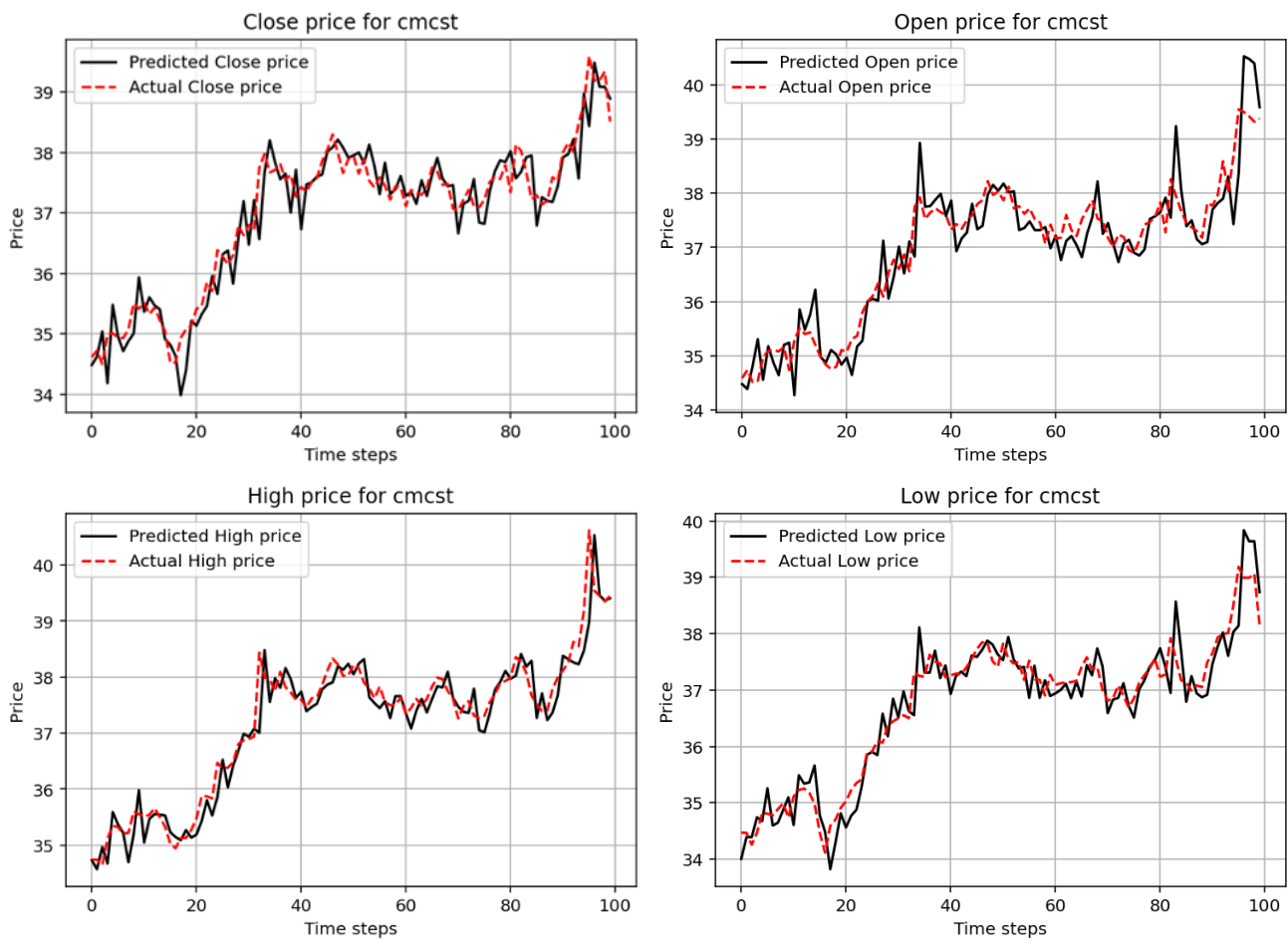
Stock name	MAPE for close price	MAPE for open price	MAPE for high price	MAPE for low price
Apple	0.00782133	0.00847303	0.00766663	0.0073681
CMCST	0.00807595	0.01008364	0.00657925	0.00806973
Google	0.00879857	0.01101522	0.01006594	0.00881552
Qualcomm	0.01475157	0.01488613	0.01332355	0.01612613

Abbreviation: MAPE, mean absolute percentage error.

According to the above data, the overall error of using HMM to predict stock prices is small. This shows that this model can be used in this kind of scenario.

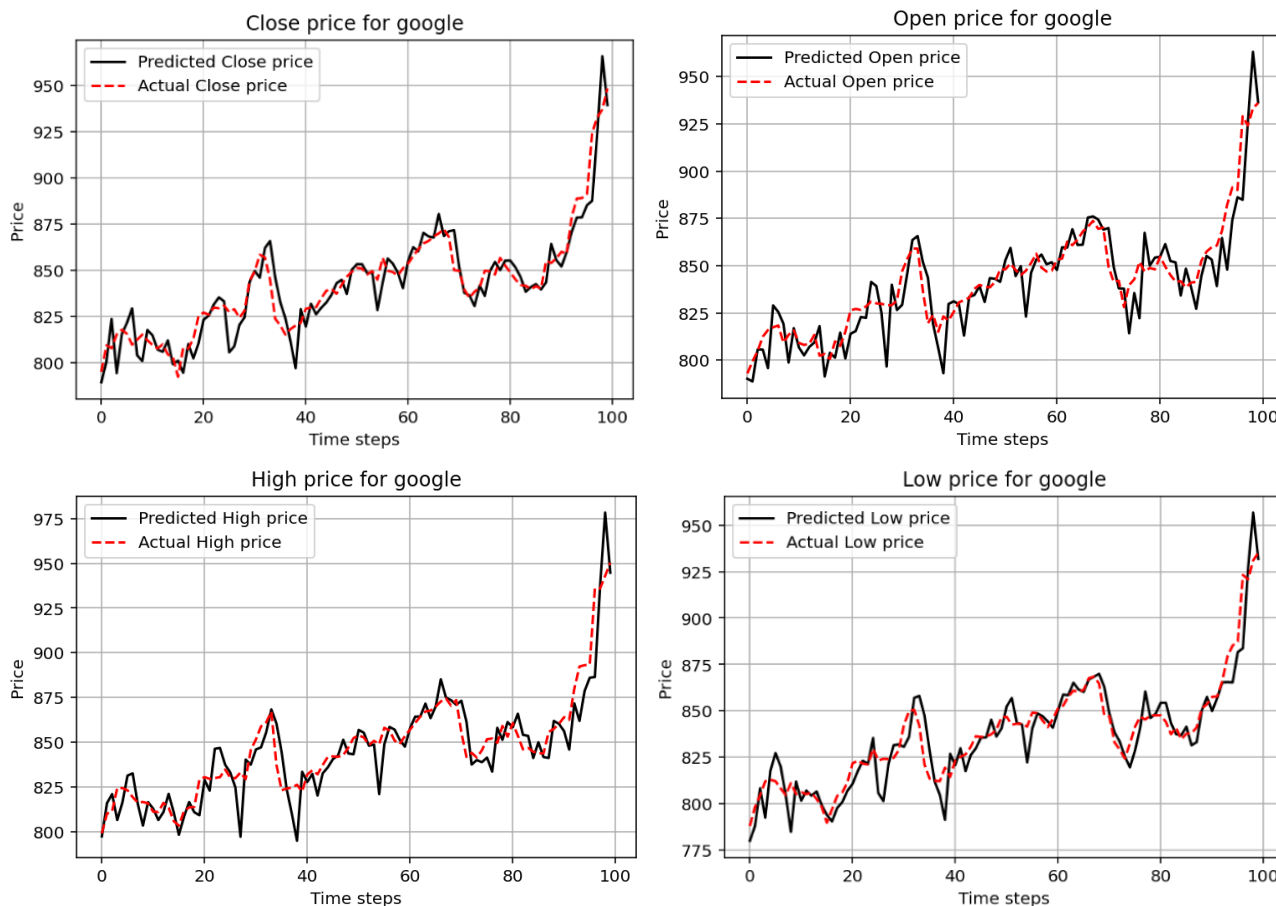
$$MAPE = \frac{1}{n} \sum_{i=1}^n \frac{|p_i - a_i|}{|a_i|} \times 100\% \quad (4)$$

where  $a_i$  is the actual stock value,  $p_i$  is the predicted stock value of  $i$ , and  $n$  is the number of days of test data.



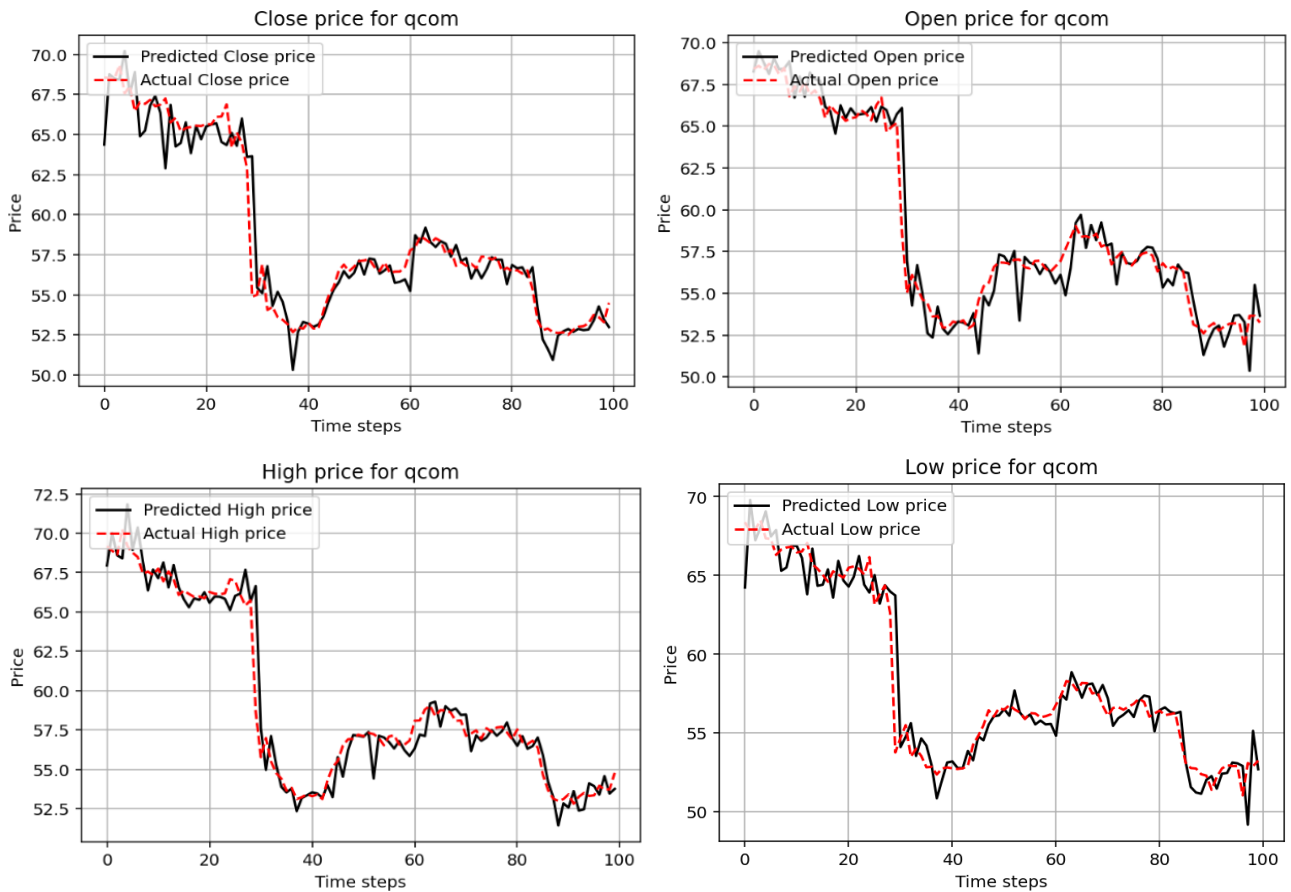
**Figure 2.** Actual and predicted values of the close, open, high, and low prices of CMCST

By calculating the MAPE for the four data sets of Google, the prediction error is the largest for the open price but the smallest for the close price. However, the difference between the close price and the daily low stock price is 0.00001695, while the difference between the open price and the daily high stock price is 0.00094928, both of which are approximately 0. HMM predicts that the open price and the daily high stock price are about the same, similar to the close price and the daily low stock price. According to the four panels in **Figure 3**, the fitting effect of the close price and the daily low stock price is the best, suggesting that the prediction effect is the best.



**Figure 3.** Actual and predicted values of the close, open, high, and low prices of Google

According to the data in **Table 1**, with regard to Qualcomm, it can be seen that HMM has the largest prediction error for its daily low stock price, and the smallest prediction error for its daily high stock price. The difference between the close price and open price is only 0.00013456. Therefore, HMM predicts approximately the same for these two sets of data. According to four panels in **Figure 4**, the fitting effect of the daily low stock price is the worst, while the fitting effect of the daily high price is the best. This suggests that HMM has the best prediction effect for the daily high stock price and the worst prediction effect for the daily low stock price.



**Figure 4.** Actual and predicted values of the close, open, high, and low prices of Qualcomm

## 5. Conclusion

We predicted the stock markets of Apple, CMCST, Google, and Qualcomm using HMM. By analyzing the values of the four continuous variables and MAPE listed in **Table 1**, the prediction results of Apple's stock price were found to be more accurate. Given that stock price fluctuations depend on a variety of factors, including environment and policy, errors in stock price prediction using HMM are inevitable. Therefore, the development of a new prediction model should be considered in the future to ensure more accurate prediction results.

## Disclosure statement

The author declares no conflict of interest.

## Author contributions

P.W. was responsible for collecting data and proposing ideas. T.W. was responsible for literature review and method selection. M.Y. was responsible for experimental analysis and conclusions.

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