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Exploring Apple's Stock Price Volatility Using Five GARCH Models

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Abstract: The financial market is the core of national economic development, and stocks play an important role in the financial market. Analyzing stock prices has become the focus of investors, analysts, and people in related fields. This paper evaluates the volatility of Apple Inc. (AAPL) returns using five generalized autoregressive conditional heteroskedasticity (GARCH) models: sGARCH with constant mean, GARCH with sstd, GJR-GARCH, AR(1) GJR-GARCH, and GJR-GARCH in mean. The distribution of AAPL's closing price and earnings data was analyzed, and skewed student t-distribution (sstd) and normal distribution (norm) were used to further compare the data distribution of the five models and capture the shape, skewness, and loglikelihood in Model 4 – AR(1) GJR-GARCH. Through further analysis, the results showed that Model 4, AR(1) GJR-GARCH, is the optimal model to describe the volatility of the return series of AAPL. The analysis of the research process is both, a process of exploration and reflection. By analyzing the stock price of AAPL, we reflect on the shortcomings of previous analysis methods, clarify the purpose of the experiment, and identify the optimal analysis model.

Keywords: Financial market; Stock price; Volatility; GARCH model

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1. Introduction

The stock market plays an important role in finance. In recent years, the rapid development of the stock market has concerned everyone related. It is imperative for investors and analysts to pay attention to the stock market because the fluctuation of stock prices directly affect income. In addition, the stock market also improves the flow of funds, helps solve financing problems, and provides more convenience for investors. Demirguc-Kunt and Levine emphasized that the international integrated stock market can disperse risks and promote economic growth [1]. The financial market of every country has a close relationship with the financial system of the United States. Taking Apple Inc., a large and successful modern company, as an example, the stock price changes of Apple Inc. reflect the development of the company and its partner companies. Investors and market participants will always pay attention to the stock fluctuations of Apple Inc., and the investment results are closely related to the stock market changes. Since the volatility of the stock is unclear, it is necessary to use relevant data analysis technology to calculate and analyze its model to present intuitive and understandable data to its investors and market participants. Therefore, it is necessary to apply statistics, modeling, and other technologies to it.

This paper comprises five sections. The first section is the introduction, which includes the background and reasons for the research. The second section is a literature review, where we discuss the contributions made by scientists in the past to our research field and the shortcomings in the analysis of stock price data in recent years. The third section describes our research methods. The fourth section is a detailed analysis and discussion of the experimental results. The last section concludes the paper. We were able to achieve the aim of the experiment and reach a deeper conclusion.

2. Literature review

Apple Inc. has been regarded as the most innovative technology company in the world over the past three decades [2]. In 2007, the stock price of Apple Inc. reached 200 dollars [3]. As of June 2015, Apple Inc. had been the largest listed company by market capitalization. In the past, many scholars have made many contributions to predicting stock prices. Mohan and Mullapudi studied deep learning models by gathering large sets of time series data and analyzing them to improve the accuracy of stock prices [4]. In order to obtain a better result on forecasting, Jeong took Apple's stack as a sample by using autoregressive integrated moving average (ARIMA) and seasonal autoregressive integrated moving average with exogenous regressors (SARIMAX) models [5]. Kim and Jun used a method that compared different time series models to determine the best time series model for predicting Apple's stock, and they concluded that the most appropriate one is IN-ARCH [6]. Silvennoinen and Teräsvirta also used a similar method; when surveying the MARCH, they made different generalized autoregressive conditional heteroskedasticity (GARCH) models that fit the same data and compared the result [7]. Ding and Zhang studied using Open Information Extraction (Open IE) techniques to withdraw the structured events from online data [8].

Recently, many scholars have taken an interest in GARCH comparison field study. Sharma *et al*. investigated five major emerging countries' volatility of financial markets by using univariate volatility models covering GARCH 1, 1, Exponential Generalized Autoregressive Conditional Heteroscedasticity (E-GARCH 1, 1) and Threshold Generalized Autoregressive Conditional Heteroscedasticity (T-GARCH-1, 1) models; they found that GARCH (1, 1) model is superior to nonlinear GARCH models on predicting volatility [9]. The study filled the gap on the choice of forecasting market volatility by linear versus nonlinear models. Ampountolas evaluated various traditional time series forecasting performance models for daily hotel demand at multiple horizons, achieved a stable forecast by comparing the different time series models, and eventually chose a suitable one to predict the daily hotel demand [10]. Fałdziński *et al*. took the forecasting performance of GARCH-type models and support vector regression (SVR) for futures contracts of selected energy commodities into comparison and found that SVR has a lower forecast error [11]. Several researchers modelled cryptocurrencies volatility by using GARCH models and carried out a comparison based on normal and student's t-error distribution [12]. The study not only identified the high volatility of cryptocurrency price volatility, but also obtained a better GARCH fitting model using an efficient measurement error distribution technique. Lee and Lee considered score vector and residual as the basis of cumulative sum (CUSUM) tests; they compared their performance and found that the standardized residual-based CUSUM is generally better than the other tests [13]. Their study is of great importance in internal risk modeling and regulatory oversight; it even strengthens the confidence in global precious metal investments. Even with the numerous achievements presented by previous researchers, there are still existing gaps. This study aimed to identify the most appropriate models among the time series models that can forecast volatility. Hence, we used the data of Apple Inc. for design comparison of GJRGARCH and sGARCH models.

3. Method

3.1. GJR-GARCH

Letting $\delta = 2$ yields the Glosten-Jagannathan-Runkle GARCH (GJRGARCH) model, which attempts to address volatility clustering in the innovation process.

When $\delta = 2$ and $0 \leq \gamma i < 1$,

$$\sigma_t^2 = \omega + \sum_{i=1}^{p} \alpha_i(|\varepsilon_{t-i}| - \gamma_i\varepsilon_{t-i})^2 + \sum_{j=1}^{q} \beta_j\sigma_{t-j}^2 \tag{1}$$

$$= \omega + \sum_{i=1}^{p} \alpha_i(|\varepsilon_{t-i}|^2 + {\gamma_i}^2{\varepsilon_{t-i}}^2 - 2\gamma_i|\varepsilon_{t-i}|\varepsilon_{t-i}) + \sum_{j=1}^{q} \beta_j\sigma_{t-j}^2 \tag{2}$$

$$\sigma_t^2 = \begin{cases} \omega + \sum_{i=1}^{p} \alpha_i^2(1+\gamma_i)^2\varepsilon_{t-i}^2 + \sum_{j=1}^{q} \beta_j\sigma_{t-j}^2, \varepsilon_{t-i} < 0 \\ \omega + \sum_{i=1}^{p} \alpha_i^2(1-\gamma_i)^2\varepsilon_{t-i}^2 + \sum_{j=1}^{q} \beta_j\sigma_{t-j}^2, \varepsilon_{t-i} > 0 \end{cases} \tag{3}$$

$$\sigma_t^2 = \omega + \textstyle\sum_{i=1}^{p} \alpha_i(1-\gamma_i)^2\varepsilon_{t-i}^2 + \sum_{i=1}^{p} \alpha_i\{(1+\gamma_i)^2 - (1-\gamma_i)^2\}S_i^-\varepsilon_{t-i}^2 + \displaystyle\sum_{j=1}^{q} \beta_j\sigma_{t-j}^2 \tag{4}$$

$$\sigma_t^2 = \omega + \sum_{i=1}^{p} \alpha_i(1-\gamma_i)^2\varepsilon_{t-i}^2 + \sum_{j=1}^{q} \beta_j\sigma_{t-j}^2 + \sum_{i=1}^{p} 4\alpha_i\gamma_iS_i^-\varepsilon_{t-i}^2 \tag{5}$$

where

$$S_i^- = \begin{cases} 1 \text{ if } \varepsilon_{t-i} < 0 \\ 0 \text{ if } \varepsilon_{t-i} \geq 0 \end{cases}$$

Also, define

$$\alpha_i^* = \alpha_i(1-\gamma_i)^2 \text{ and } \gamma_i^* = 4\alpha_i\gamma_i \tag{6}$$

then,

$$\alpha_t^2 = \omega + \sum_{i=1}^{p} \alpha_i(1-\gamma_i)^2\varepsilon_{t-i}^2 + \sum_{j=1}^{q} \beta_j\sigma_{t-j}^2 + \sum_{i=1}^{p} \gamma_i^*S_i^-\varepsilon_{t-i}^2 \tag{7}$$

This is the GJR-GARCH model [14].

However, when $-1 \leq \gamma_i < 0$, then recall equation (1).

$$\alpha_t^2 = \omega + \sum_{i=1}^{p} \alpha_i(|\varepsilon_{t-i}| - \gamma_i \varepsilon_{t-j})^2 + \sum_{j=1}^{q} \beta_j \sigma_{t-j}^2 \tag{8}$$

$$\alpha_t^2 = \begin{cases} \omega + \sum_{i=1}^{p} \alpha_i{}^2(1-\gamma_i)^2 \varepsilon_{t-i}^2 + \sum_{j=1}^{q} \beta_j \sigma_{t-j}^2, \varepsilon_{t-i} > 0 \\ \omega + \sum_{i=1}^{p} \alpha_i{}^2(1+\gamma_i)^2 \varepsilon_{t-i}^2 + \sum_{j=1}^{q} \beta_j \sigma_{t-j}^2, \varepsilon_{t-i} < 0 \end{cases} \tag{9}$$

$$\begin{aligned} \alpha_t^2 = \ & \omega + \sum_{i=1}^{p} \alpha_i(1+\gamma_i)^2 \varepsilon_{t-i}^2 + \sum_{j=1}^{q} \beta_j \sigma_{t-j}^2 + \sum_{i=1}^{p} \alpha_i \{(1+\gamma_i)^2 - (1-\gamma_i)^2\} S_i^+ \varepsilon_{t-i}^2 \\ & + \sum_{i=1}^{p} \alpha_i \{1 + \gamma_i^2 - 2\gamma_i - 1 - \gamma_i^2 - 2\gamma_i\} S_i^+ \varepsilon_{t-i}^2 \end{aligned} \tag{10}$$

where

$$S_i^+ = \begin{cases} 1 \text{ if } \varepsilon_{t-i} > 0 \\ 0 \text{ if } \varepsilon_{t-i} \leq 0 \end{cases}$$

Also, define

$$\alpha_i^* = \alpha_i(1+\gamma_i)^2 \text{ and } \gamma_i^* = -4\alpha_i\gamma_i \tag{11}$$

then,

$$\alpha_t^2 = \omega + \sum_{i=1}^{p} \alpha_i^* \varepsilon_{t-i}^2 + \sum_{j=1}^{q} \beta_j \sigma_{t-j}^2 + \sum_{i=1}^{p} \gamma_i^* S_i^+ \varepsilon_{t-i}^2 \tag{12}$$

This allows positive shocks to have a stronger effect on volatility than negative shocks [15]. However, when p = q = 1, the GJGARCH(1,1) model will be written as follows:

$$\alpha_t^2 = \omega + \alpha\varepsilon_t^2 + \gamma S_i \varepsilon_{t-1}^2 + \beta\sigma_{t-1}^2 \tag{13}$$

3.2. sGARCH

Standard Generalized AutoRegressive Conditional Heteroskedasticity (sGARCH) has a moving average (MA) part and an autoregressive (AR) part, which are mergers by GARCH, an extension of the ARCH model.

Define the GARCH (p,q) model as follows:

$$x_{t=\sigma_t \epsilon_t} \tag{14}$$

$$\sigma_t^2 = \omega + \sum_{i=1}^{p} \alpha_i x_{t-1}^2 + \sum_{j=1}^{q} \beta_j \sigma_{t-j}^2 \tag{15}$$

where $\omega > 0, \alpha_{i>0}, \beta_j > 0, \sum_{i=1}^{p} \alpha_i + \sum_{j=1}^{q} \beta_j < 1. \epsilon_t$ is a separate but identical sequence. For ease of processing, the order of all GARCH models used will be limited to one.

The standard GARCH model [16], represented as sGARCH(1,1), is given as follows:

$$\sigma_t^2 = \omega + \alpha_i \epsilon_{t-1}^2 + \beta_1 \sigma_{t-1}^2 \tag{16}$$

4. Empirical analysis

Figure 1 shows the closing price of Apple Inc. (AAPL) from January 2, 2008, to April 11, 2022. The fluctuations of the closing price and the turnover volume are clearly indicated in **Figure 1**. The closing price of AAPL showed an upward trend and tended to 165.75, fluctuating up and down. From 2008 to 2018, the closing price showed a relatively low value and a steady upward trend. In 2012, due to some factors, the closing price fell following a rise. From 2014 to the beginning of 2016, it still showed an upward trend, but after that, it began to fluctuate. There was an overall upward trend until 2018. In 2018, it began to rise rapidly, reaching nearly 170, and finally tending to 165.75. From 2020 to 2021, the closing price fluctuated significantly due to the impact of the COVID-19 pandemic, but it still showed an overall upward trend.

The trading volume of Apple Inc. is shown at the bottom of the figure. The trading volume in 2008 was high, but since 2016, the trading volume has been very low, from the initial 3,000 to several hundred(s). With the change in the stock's closing price, the trading volume also changes. Therefore, the trading volume showed a downward trend.

Figure 1. Closing price of Apple Inc. (AAPL)

Figure 2A shows AAPL's histogram of return, while **Figure 2B** shows AAPL.Close of Returns from January 2, 2008, to April 11, 2022. As can be seen from **Figure 2A**, the frequency decreases from 0.00 to positive and negative, respectively; the maximum positive return frequency and maximum negative return frequency can reach up to 1,500 and 1,000, respectively. The positive returns were greater than the negative returns, indicating that the company is profitable and ideal.

Figure 2B shows the curve of distribution density, the changing trend of the data set, and the normal distribution curve. As can be seen from **Figure 2B**, the highest values of the blue bar chart and the green curve appear on the right side of the 0.00 returns, which again suggest that AAPL is profitable. The red curve is a normal distribution curve. Compared with the red curve, the green curve is more consistent with the density of the data set distribution and is also approximately symmetrical.

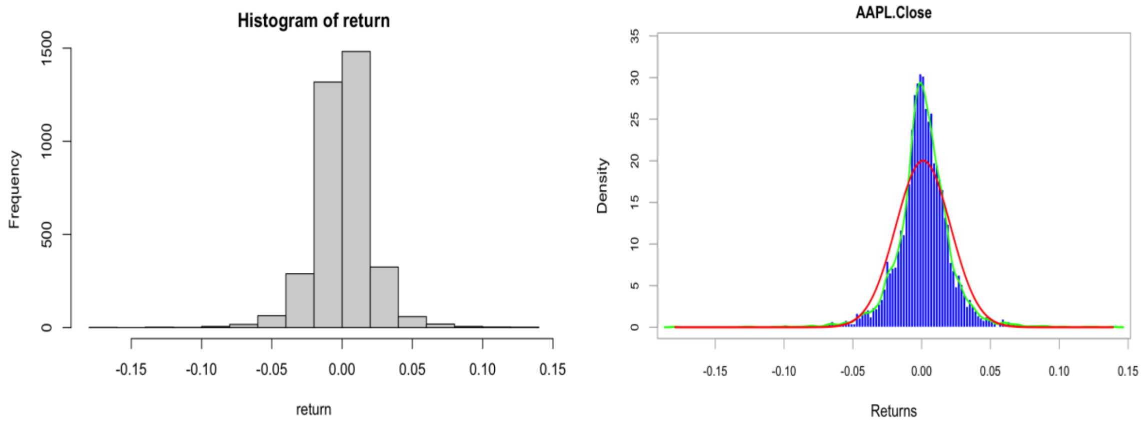

Figure 2. Histogram of return and AAPL close

Figure 3 shows the frequency range of return fluctuation around 0.00 from January 2, 2008, to April 11, 2022. Based on **Figure 3**, there was a maximum fluctuation in 2008, with a fluctuation range of nearly 0.3. The reason for that was the financial crisis, which brought a huge impact not only on the world economy, but also on Apple Inc. After that, the income fluctuation saw a gradual decrease, in which the fluctuation range was around 0.15. Until the middle of 2012, due to the influence of some factors, the fluctuation range slightly increased to 0.2, then gradually decreased and fluctuated in a small range. In 2020, the frequency of income fluctuation increased again. Although the range of fluctuation was small compared with that of 2008, it was the largest one in recent years. Affected by COVID-19, the world economy has been in turmoil and recession, ensuing a great fluctuation in the company's return frequency. Until 2022, Apple Inc.'s earnings fluctuated slightly, but the last was -0.0255.

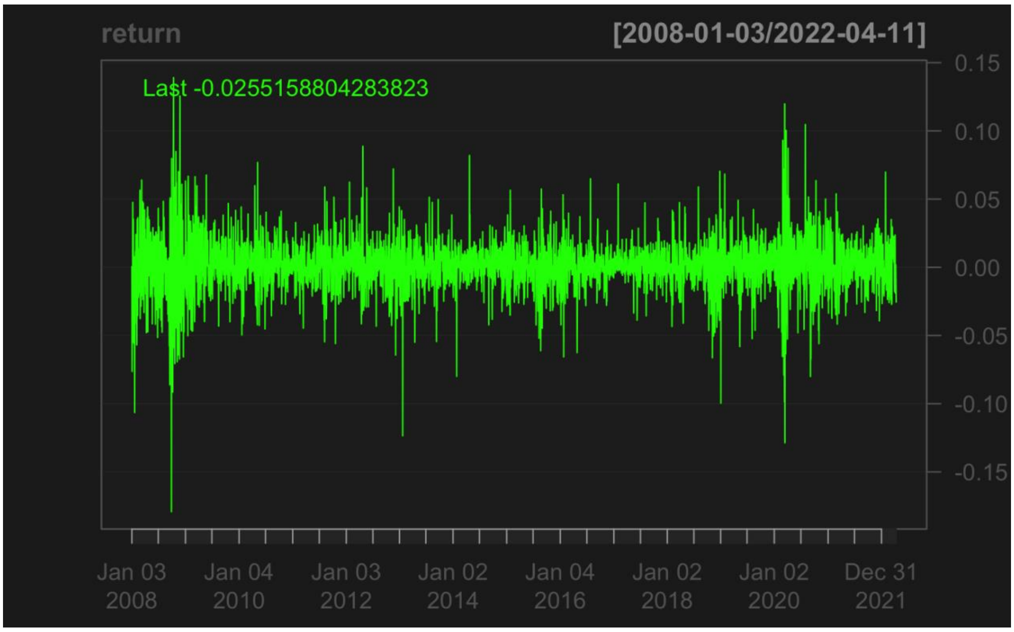

Figure 3. Returns of AAPL

Figure 4 shows Apple Inc.'s yearly rolling volatility from January 2, 2008, to April 11, 2022. It reflects the average fluctuation degree of AAPL price within one year. As can be seen in **Figure 4**, the rolling volatility showed a downward trend since 2008; thereafter, there was a sharp drop below 0.3 in 2009. These were the result of the financial crisis in 2008, in which the global economy was affected, and international trade declined. From 2010 to 2013, there was little volatility, and there was an overall upward trend, rising to above 0.3. However, at the beginning of 2014, the rolling volatility dropped again, and the decline was smaller than that in 2018, maintaining between 0.2 and 0.3. Until January 2017, the volatility dropped to 0.2, but it subsequently rose to more than 0.2 at the beginning of 2018, reaching more than 0.3 in 2019. However, the outbreak of the novel coronavirus has brought another great impact on the world economy. Therefore, as shown in the figure, we can see the significant fluctuations at the end of 2020, ranging from about 0.45 to 0.25.

Figure 4. Apple Inc.'s yearly rolling volatility

Table 1 shows the results of the maximum likelihood estimate (MLE) of sGARCH with constant mean, GARCH with sstd, GJR-GARCH, AR(1) GJR-GARCH, and GJR-GARCH in mean models for Apple Inc.'s returns. Model 1 uses normal distribution (norm), whereas Models 2, 3, 4, and 5 use skewed student t-distribution (sstd). From **Table 1**, the loglikelihood value (9605.649) is the maximum for Model 4 – AR(1) GJR-GARCH. When the skew is greater than 0, it indicates that the shape of the model distribution is right biased; when the skew is equal to 0, it indicates a normal distribution. When the shape is greater than 3, it indicates a spike, but when the shape is equal to 3, it indicates a normal distribution. The shape (5.448357) is the maximum and the skew (1.002632) is relatively low for Model 4 – AR(1) GJR-GARCH. These results indicate that Model 4, AR(1) GJR-GARCH, is the optimal model to describe the volatility of the return series of Apple Inc.

Table 1. Results of the maximum likelihood estimate (MLE) of sGARCH with constant mean, GARCH with sstd, GJR-GARCH, AR(1) GJR-GARCH, and GJR-GARCH in mean

Model	sGARCH with constant mean	GARCH with sstd	GJR-GARCH	AR(1) GJR-GARCH	GJR-GARCH in mean
ω	0.000015**	0.000008	0.000011***	0.000011***	0.000011***
α	0.111926***	0.101071***	0.034580***	0.033869***	0.034613***
β	0.848943***	0.882488***	0.863740***	0.863450***	0.863350***
γ	--	--	0.156976***	0.158980***	0.157047***
Skew	--	1.008920	1.002508	1.002632	1.002682
Shape	--	5.018493	5.423806	5.448357	5.425795
Loglikelihood	9425.7	9575.31	9605.39	9605.649	9605.395

Note: *The value of PR is less than 0.1; **the value of PR is less than 0.05; ***the value of PR is less than 0.01

5. Conclusion

We studied the volatility of AAPL's returns from January 2, 2018 to April 11, 2022. The results of the statistical properties revealed that the return of AAPL is leptokurtic and rightward. Five different GARCH type models (sGARCH with constant mean, GARCH with sstd, GJR-GARCH, AR(1) GJR-GARCH, and GJR-GARCH in mean) were compared, in which the AR(1) GJR-GARCH model was identified to be the most appropriate model for estimating the time-varying volatility in AAPL's returns. To account for the skewness and shape in AAPL's returns for the years under study, normal distribution (norm) and skewed student t-distribution (sstd) were used to capture the loglikelihood in the five GARCH models. The skewed student t-distribution (sstd) performed better in capturing the shape and skewness in the return series distribution. Hence, the AR(1) GJR-GARCH model is considered the optimal model for modeling and estimating the volatility in AAPL's returns. The results of this study are useful for investors and market players in investment decision-making and analysis of stock price fluctuations.

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Disclosure statement

The authors declare no conflict of interest.

Author contributions

Z.T. conceived the idea of the study and supervised the writing of the paper; S.F. analyzed the data and summarized the paper; K.H. collected the data and wrote the method; and J.L. wrote the introduction.

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