Real Estate Exotic Options based on Black-Scholes Model (BSM)

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Abstract: This paper analyzed the issue of high housing prices in China in view of exotic options using the traditional BSM and improved it while applying it to the current situation in the real estate market. A certain set time frame in the purchase of the options with real estate prices was designed in the implementation of exotic option pricings to ease the speculative pressures caused by high housing prices.

Keywords: Real estate; BSM option pricing model; Exotic option pricing

1. Principles of Real Estate Exotic Option Pricing

Based on the exotic option design, the traditional BSM was improved by redefining the underlying asset yield, K and option value, f in addition to defining the true value, C of the present real estate exotic option. Most of the basic assumptions were followed and the basic parameters were set as following:

C_i: rental rate per period; T_i: exotic option expiration date; S_0: price of the underlying asset on the first day of exotic option issuance; S_t: asset price of the exotic option maturity date; X: underlying asset price of exotic option due date; R_f: risk-free interest rate; σ: real estate volatility; K: present value of rent; V: present value benefit of opportunity cost; R_i: expected return per period of the underlying asset; f: value of the owner's rights to cession the property for sale; C: true value of the real estate exotic option; M: actual cash flow paid by expired option buyers upon expiration; $N(d_1), N(d_2)$: probability of cumulative normal distribution.

The sum of the present value of rent in each period of existing real estate exotic option is:

$$K = \sum_{i=1}^{T} \frac{C_i \times S_0 \times (1 + R_i)^{(i-1)}}{\left(1 + R_f\right)^i}$$

The present value gained by the sales of the property upon the sales of the option contract and the investments of the sales proceeds over the life of the option is:

$$V = \frac{S_0 \times \left(1 + R_f\right)^T - S_0}{\left(1 + R_f\right)^T} = S_0 - \frac{S_0}{\left(1 + R_f\right)^T}$$

Then, calculate d_1 and d_2:

$$
\begin{aligned}
d_1 &= \frac{\ln\left(\frac{S_0}{X}\right) + \left(R_f + \frac{\sigma^2}{2}\right) \times T}{\sigma \times \sqrt{T}} \\[2mm]
&= \frac{\ln\left(\frac{S_0}{S_t}\right) + \left(R_f + \frac{\sigma^2}{2}\right) \times T}{\sigma \times \sqrt{T}} \\[2mm]
&= \frac{\ln\left(\frac{S_0}{S_0 \times (1 + R_i)^T}\right) + \left(R_f + \frac{\sigma^2}{2}\right) \times T}{\sigma \times \sqrt{T}} \\[2mm]
&= \frac{-T \ln(1 + R_i) + \left(R_f + \frac{\sigma^2}{2}\right) \times T}{\sigma \times \sqrt{T}} \\[2mm]
d_2 &= d_1 - \sigma \times \sqrt{T}
\end{aligned}
$$

The value of the property owner's rights to cession the property for sales:

$$
\begin{aligned}
f &= S_0 \times N(d_1) - X \times e^{(-R_f \times T)} \times N(d_2) \\
&= S_0 \times N(d_1) - S_t \times e^{(-R_f \times T)} \times N(d_2) \\
&= S_0 \times N(d_1) - S_0 \times (1 + R_i)^T \times e^{(R_f \times T)} \times \\
&\quad N(d_2)
\end{aligned}
$$

Where, $N(x)$ is:

$$
N(x) = \int_{\infty}^{x} \frac{1}{\sqrt{2 \times \pi}} \times e^{\left(-\frac{x^2}{2}\right)} dx
$$

In summary, the true value of the real estate exotic option should be:

$$
C = max(K, V) + f
$$

If the option purchaser exercised their rights at the option expiration date, the cash flow M, should be:

$$
M = S_i - C = S_0 \times (1 + R_i)^T - max(K, V) - f
$$

2. Simulation Analysis of Real Estate Option

2.1. Parameter assignment

According to the actual situation of option, the parameters were assigned accordingly.

According to the statistics in 2016, the average house price in China's provincial capital city was 10000 yuan/m^2 and assuming that the standard housing was 100 square meters with monthly rental of 4000 yuan/month, the rental rate, C_i = (4000 x 12 / 10000 x 100) x 100% = 4.8%. The underlying asset price on the first day of exotic options issuance, S_0 would be 10000 x 100 = 1000000 yuan. The risk-free interest rate, R_f = treasury yield which was issued at 3.7% in March 2016. According to the statistics in June 2016, the average annual housing price growth rate in provincial capital cities, R_i was 8.6% while according to

the calculation results of statistical data, the average fluctuation rate of housing prices in China in 2016, σ was 25%. The option duration, T was set to 1-10 years.

2.2. Simulate results

In regard to the above parameters, the results were calculated in R language as shown in Table 1. The symbolic meanings are as follows:

Time: T, the option duration; Presentvalue: K, the present value of rent (the yield of the underlying asset); Optcost: V, the present value of opportunity cost; K_to_V is the maximum value of K and V; Optionvalue: f, the value of the owners' rights to sell their properties; Sum: C, the true value of real estate exotic option.

Table 1. Calculation results for different option durations

Time	Presentvalue	Optcost	K_to_V	Optionvalue	Sum
1	46287.37	35679.85	46287.37	80187.97	126475.3
2	94761.89	70086.64	94761.89	104152.9	198914.8
3	145526.9	103265.8	145526.9	119719.1	265246
4	198690.7	135261.1	198690.7	131192.1	329882.8
5	254366.5	166114.9	254366.5	140184.9	394551.4
6	312673.1	195867.8	312673.1	147500	460173.1
7	373734.8	224559.1	373734.8	153600.6	527335.4
8	437681.8	252226.7	437681.8	158780.5	596462.3
9	504650.4	278907.1	504650.4	163238.4	667888.8
10	574783.3	304635.6	574783.3	167115.3	741898.6

2.3. Graphical display

The horizontal coordinates of all three graphs in Figure 2 represent T in which the duration of real estate exotic option was 1-10 years. The vertical coordinates of Figure 2(a) are sums of the present value of all rents during each option period while the vertical coordinates of Figure 2(b) are the values of the owners' rights to sell their properties in each period, and the vertical coordinates of Figure 2(c) are the true value of the real estate exotic option in each period.
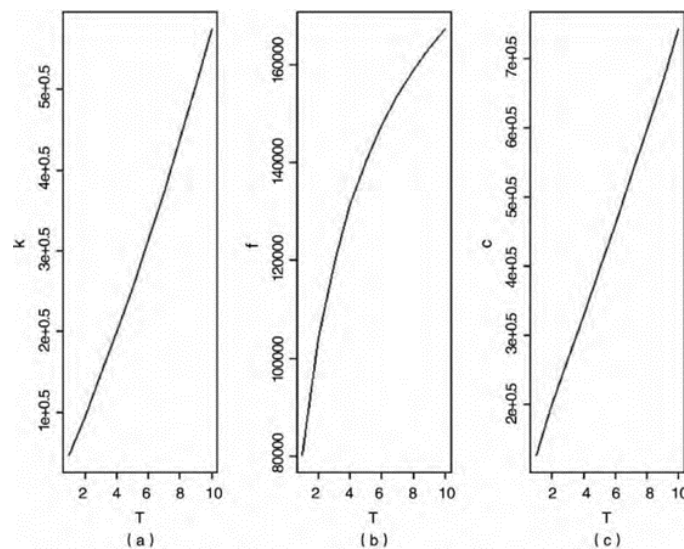

Figure 2.

2.4. Analysis of results

Based on the above analysis, a series of real estate exotic option structural components is exhibited such as the sum of the present values of rents for different terms from 1 to 10 years, the present values of opportunity cost, the values of the owners' rights to sell their properties, and the true values of real estate exotic option. In order to demonstrate the product structure in a clearer and more intuitive manner, the results of the data in Table 1 are applied. If an option buyer (future home buyer) purchases a 3-year real estate exotic option (the third row of Table 1), he or she will pay the option seller (property owner) the sum of the present value of the rent for 3 years (K) and the sum of the value of 3-years rights to sell the property ceded by the property owner on the contract date which is 145,526.90 yuan and 265,246 yuan respectively. The present value of opportunity cost (V) of risk-free investments for three years is 103,265.80 yuan in which K > V, thereby taking K = 145,526.90 yuan. The value of the owner's rights to sell the property for three years (f) is 119,719.10 yuan, so the real value of the real estate exotic option (C) is 265,246 yuan. If the option purchaser exercises their rights on the option expiration date, the property would be purchased with the cash flow (M) of the differences between the property's market price on the expiration date and the true value of the real estate exotic option. However, if the option purchaser abandons their rights on the option expiration date, the maximum loss in regard to the value of the owner's rights to sell the property for 3 years is 119,719.10 yuan.

Therefore, either under stable, declining or evenly rising home prices, the cost paid by the purchaser of a real estate exotic option would be less than the total cost of a bank loan to buy a home. The flexibility offered by the real estate exotic option to option buyers and the return of initial cash flow to option sellers, increases the liquidity of the real estate market, thus rendering this real estate derivative product as relevant and valuable.

3. Conclusion

The property exotic option described in this paper is important for stabilizing the real estate market as well as curbing the rapid increase of house prices. At the same time, it encourages more flexible options to be introduced to the property transaction market. From the perspective of supply and demand in the property market and the characteristics of exotic options, when the property market is in great demand and house prices rise significantly, purchasers may give up exercising their option at the time of the expiration due to the rapid rise of house prices which exceeds their expected payments. In the case whereby they are able to rent out the underlying property for the second time, they would only lose the option premium at most, but the property market may be dampened in view of the excessive numbers of purchasers that abandoned the exercise of their options. This would eventually suppress the rapid rise of property prices when the property market is in shock which eventually leads to a fall in property prices. Then, having paid the option premium, option purchasers may lose money during the expiration date of their options.

In order to minimize the loss in option fees, most option buyers would execute the option to purchase properties due to the expiration to prevent the fall of property prices. Thus, the precipitous fall of the property market could be prevented, and its stability would be regulated.

Other than hedging the risk, it may change the concept of ordinary home buyers, curb panic buying behaviors, reduce irrational speculations in the real estate industry, and promote the development of the real estate industry in a healthier direction.

Disclosure statement

The author declares no conflict of interest.

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