

Research on the Pricing of Convertible bonds in China--- Migration risk based on credit rating

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Abstract: At present, further research and exploration on credit risks are being carried out in the global field, and increasingly profound modern credit risks are exposed to the bond market. This requires that we cannot ignore the impact of credit rating migration risk on bond pricing, so as to adapt to the sustainable and healthy development of the bond market under the new normal of China's economy. The innovation point of this paper is to try to analyze the pricing of Convertible bonds in China from the perspective of credit rating migration risk. Tsiveriotis and Fernandes(1998) model is selected, and the credit risk in the model is assumed to be caused by the credit rating migration risk, and the credit spread is used to measure the credit rating migration risk. The research conclusion of this paper is as follows: First, it is valid to consider the risk of credit rating migration in the TF(1998) model. The market price of convertible bonds is on average 1.22% higher than the theoretical value of the model. In general, the theoretical value obtained from the model has little deviation from the market price, and has a good fitting degree. Second, from the Angle of credit rating, the selection of 32 samples of convertible bonds only empirical research shows that the credit rating of AA - convertible bonds average deviation rate is negative, suggest that the credit rating of AA - the phenomenon of convertible bonds value is underestimated, and AAA credit rating to AA, AA +, the average deviation rate of convertible bonds is positive, that credit rating AA (containing AA) more convertible bond value is overrated phenomenon, and the higher the credit rating of the average deviation rate of convertible bond, the greater the overvalued levels. It has certain guiding significance for participants in the convertible bond

market.

Key words: Convertible bond pricing; TF (98); Risk of credit rating transfer

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1 Review of domestic and foreign research

Outstanding achievements in the research of Convertible bonds in China focus on the empirical research and pricing theory, but there are few in-depth discussions on terms design, variety innovation, issuance barriers, and even separation transactions. However, the reality of China's convertible bond market is that the investigated issue size is small and the sample size is too small, which may easily lead to the lack of sufficient data for scientific research, which needs to enrich the scientific nature and interpretation of the empirical results. As the aftermath of the financial crisis impact on China's capital market, China's stock market still exists the phenomenon of instability, to reverse this situation, the urgent idle domestic scholars constantly in-depth study of Chinese convertible bond theory and application, in order to improve China's capital market, activate and expand the energy of the convertible bond market in China^{[1][2][3]}. In the face of the new round of convertible bonds issuance, it is necessary to conduct pricing research on Chinese convertible bonds based on the summary of previous research on convertible bonds and according to the development status of Chinese credit rating under the new situation and the uncertainty brought to investors

by the risk of credit rating migration.

2 Convertible bond pricing model theory

2.1 Analysis of TF(98) pricing model

The values of convertible bonds are as follows:

$$\frac{\partial \mu}{\partial t} + rS \frac{\partial \mu}{\partial S} + \frac{1}{2} \sigma^2 S^2 \frac{\partial^2 \mu}{\partial S^2} - r(u - v) - (r + r_c)v = 0 \quad (1)$$

For the value of the bond, there are:

$$\frac{\partial v}{\partial t} + rS \frac{\partial v}{\partial S} + \frac{1}{2} \sigma^2 S^2 \frac{\partial^2 v}{\partial S^2} - (r + r_c)v = 0 \quad (2)$$

The terminal condition of this model is $f(S, T) = \text{Max}(kS, f)$,

The boundary condition of this model is $B(S, T) =$

$$\begin{cases} F, & F > kS \\ 0, & F \leq kS \end{cases}$$

2.2 Description of credit spread caused by credit grade migration risk

Through the adjustment of default risk, considering the bonds with the credit rating of I, the risk-neutral interest rate process is as follows:

$$(t)^+ \quad (t) \quad (t) = (t)^+ \quad (t)^+ \quad (3)$$

At time T, the price of the corporate bond with credit rating I is

$$D_i(t, T) = \sum_{j=1}^K m_{ij}^* E_t^* \left[\exp \left(\int_t^T \mu_j^*(u) du \right) \right] = \sum_{j=1}^K m_{ij}^* \exp \left[\alpha_j (T - t) + \beta_j (T - t) X_j(t) \right] \quad (4)$$

If we take this into account in the risk-neutral environment as a whole, we can get the price at time T for the non-default risk of paying 1

$$D_0(t, T) = E_t^* \left(e^{-\int_t^T r^*(u) du} \right) \quad (5)$$

Finally, the credit risk spread ^[6] of corporate bonds can be expressed as:

$$s_i(t, T) = - \frac{\log D_i(t, T) - \log D_0(t, T)}{T - t} \quad (6)$$

3 Model determination

Because this paper focuses on applying the binary tree method to calculate the value of convertible bonds, it deals with the risk-free discount method for the equity part and the risk discount rate for the bond part. The value of partial differential equation can only be calculated according to the corresponding boundary conditions to determine the purpose of the boundary conditions of the convertible bond pricing model. Therefore, TF(98) pricing model is applied.

3.1 Estimation of volatility

The standard deviation of daily volatility can be expressed as:

$$\sigma_1 = \sqrt{\frac{1}{n-1} \sum_{i=1}^n (u_i - \bar{u})^2} \quad (7)$$

We selected the closing price of the underlying stock price 100 days from the most recent time to examine the samples as the historical data of the estimated volatility, and at the same time selected the appropriate correction of the historical volatility: the issue of convertible bonds itself would lead to an average price volatility of 11.52%, which would be used to correct the historical volatility.

The volatility of the underlying stock price of 32 convertible bonds is calculated as follows.

Table 1. Estimated price volatility of the underlying Convertible bond

| code | The convertible bond | The underlying stocks | During the period of the sample | Stock price volatility | Adjusted volatility | credit rating |
|-----------|----------------------|--------------------------------|---------------------------------|------------------------|---------------------|---------------|
| 128036.SZ | JINNONG BOND | JINXINNONG | 2018/7/19-2018/12/13 | 0.3840 | 0.2688 | AA- |
| 113504.SH | AIHUABOND | AIHUAGROUP | 2018/7/19-2018/12/13 | 0.3762 | 0.2610 | AA |
| 113019.SH | LINGLONGBOND | LINGLONGGROUP | 2018/7/19-2018/12/13 | 0.3094 | 0.1943 | AA+ |
| 128035.SZ | DAZUBOND | But the laser | 2018/7/19-2018/12/13 | 0.4487 | 0.3335 | AA+ |
| 123008.SZ | TAIKANGBOND | TAIKANGBIOLOGICAL | 2018/7/19-2018/12/13 | 0.6982 | 0.5830 | AA- |
| 128034.SZ | JIANGYINBOND | JINGYINBANK | 2018/7/19-2018/12/13 | 0.3846 | 0.2694 | AA+ |
| 113018.SH | CHANGSHUBOND | CHANGSHUBANK | 2018/7/19-2018/12/13 | 0.3457 | 0.2305 | AA+ |
| 123007.SZ | DAOSHIBOND | DAOSHITECHNOLOGY | 2018/7/19-2018/12/13 | 0.5258 | 0.4106 | AA- |
| 128033.SZ | DILONGBOND | XUEDILONG | 2018/7/19-2018/12/13 | 0.1259 | 0.0107 | AA |
| 110042.SH | HANGTIANBOND | CATIC ELECTRON | 2018/7/19-2018/12/13 | 0.4206 | 0.3054 | AAA |
| 110041.SH | NENGDIANBOD | INNER MONGOLIAHUADIAN | 2018/7/19-2018/12/13 | 0.2444 | 0.1292 | AAA |
| 128027.SZ | CHONGDABOND | CHONGDATECHNOLOGY | 2018/7/19-2018/12/13 | 0.4637 | 0.3485 | AA |
| 128026.SZ | ZHONGXINGBOND | ZHONGXING FLORA | 2018/7/19-2018/12/13 | 0.3037 | 0.1885 | AA- |
| 123003.SZ | LANSIBOND | LANSITECHNOLOGY | 2018/7/19-2018/12/13 | 0.4599 | 0.3447 | AA+ |
| 128025.SZ | TEYIBOND | TEYIPHARMACY | 2018/7/19-2018/12/13 | 0.2688 | 0.1536 | AA- |
| 128024.SZ | NINGHANGBOND | NINGBOBANK | 2018/7/19-2018/12/13 | 0.2946 | 0.1794 | AAA |
| 123002.SZ | GUOZHENBOND | GUOZHENENVIRONMENTALPROTECTION | 2018/7/19-2018/12/13 | 0.4232 | 0.3080 | AA |
| 113502.SH | JIAAOBOND | JIAAOENVIRONMENTALPROTECTION | 2018/7/19-2018/12/13 | 0.4023 | 0.2871 | AA- |
| 128019.SZ | JIULIBOND | JIULIHITTECH | 2018/7/19-2018/12/13 | 0.3143 | 0.1991 | AA |
| 113015.SH | LONGJIBOND | JONGJISHARES | 2018/7/19-2018/12/13 | 0.5030 | 0.3878 | AA+ |
| 128017.SZ | JINHEBOND | JHJ INDUSTRIAL | 2018/7/19-2018/12/13 | 0.1198 | 0.0046 | AA |
| 113013.SH | GUOJUNBOND | GUOTAIJUNAN | 2018/7/19-2018/12/13 | 0.3095 | 0.1943 | AAA |
| 128015.SZ | JIUQIBOND | JOIN-CHEER | 2018/7/19-2018/12/13 | 0.1159 | 0.0007 | AA |
| 128014.SZ | YONGDONGBOND | YONGDONGSHARES | 2018/7/19-2018/12/13 | 0.2688 | 0.1536 | AA- |
| 127003.SZ | HAIYINBOND | HAIYINSHARES | 2018/7/19-2018/12/13 | 0.1937 | 0.0785 | AA- |
| 113010.SH | JIANGNANBOND | JIANGNANWATERAFFAIRS | 2018/7/19-2018/12/13 | 0.2508 | 0.1356 | AA |
| 113011.SH | GUANGDABOND | GUANGDABANK | 2018/7/19-2018/12/13 | 0.2242 | 0.1091 | AAA |
| 113009.SH | GUANGQIBOND | GUANGQIGROUP | 2018/7/19-2018/12/13 | 0.2465 | 0.1314 | AAA |
| 110034.SH | JIUZHOUBOND | JIUZHOUTONG | 2018/7/19-2018/12/13 | 0.2380 | 0.1228 | AA+ |
| 110033.SH | GUOMAObOND | XIAMENINTERNATIONALTRADE | 2018/7/19-2018/12/13 | 0.0937 | -0.0215 | AA+ |
| 110031.SH | HANGXINBOND | AEROSPACEINFORMATION | 2018/7/19-2018/12/13 | 0.5111 | 0.3959 | AAA |
| 113008.SH | DIANQIBOND | SHANGHAI ELECTRIC | 2018/7/19-2018/12/13 | 0.3029 | 0.1878 | AAA |

Data source: Wind database

3.2 Estimation of the term structure of interest rates

Since the convertible bond itself is issued in the presence of bonds, if the convertible bond does not exercise the relevant rights in the conversion period but chooses to hold to maturity, then the convertible bond becomes a general sense of corporate bonds^[7]. Therefore, it is very important to determine the interest rate limit structure in the pricing process of convertible bonds. For the estimate of the term structure of the risk-free interest rate, according to the term structure of the interest rate, the yield of China's five-year zero-coupon Treasury bond^{[4][5]} is calculated to be 3.90, which is the risk-free interest rate of the five-year convertible bond in the pricing model.

3.3 Estimation of credit rating migration risk

The price of a corporate bond with an I credit rating and a risk of default is

$$D_i(t, T) = \sum_{j=1}^K m_{ij}^* E_t^* \left[\exp \left(\int_t^T u_j^*(u) du \right) \right] \tag{8}$$

$$= \sum_{j=1}^K m_{ij}^* \exp [a_j(T-t) + \beta_i(T-t)X_j(t)]$$

Assuming independence from each other, the mean recovery process of the jump: $X_j(t), j=1,2,3,4$,

The method of parameter estimation in this paper is: select parameter $\theta_j, j=2,3,4, \gamma, l$, so that

$$\sum_{i,k} \left(1 - \frac{S_{i,k}}{S_{i,k}} \right)^2 \text{ the smallest.}$$

This can be obtained by - obtaining the credit spread^[6], that is, the credit spread caused by the credit rating migration risk of 5-year ordinary corporate

bonds is 0.97%, which can be taken as the credit spread of convertible bonds caused by the credit rating migration risk.

Due to the characteristics of real-time changes in interest rates and the increasingly market-oriented interest rates in China, the interest rate changes have a relatively weak impact on the pricing of convertible bonds, so the choice is not considered.

4 Sample selection and processing

4.1 Selection of samples

Based on the above facts of China's convertible bond market, this paper selects convertible bonds with long maturity and convertible period, with sufficient trading days and redemption clause, sell-back clause and downward revision clause. However, except for those that have not entered the conversion period, have not set the conversion period, and have no right of downward correction, the 32 convertible bonds are issued by non-public companies, determined by the company and the lead underwriter according to

the market conditions and did not participate in the evaluation by the rating agencies, and the maturity is 6 years.

4.2 Processing of samples

The investigation period selected in this paper is the 100 consecutive trading days of each convertible bond from July 19, 2018 to December 13, 2018, solstice in total, so as to avoid possible abnormal changes in the market price of convertible bond at the initial stage of issuance.

5 Empirical analysis

In this section, based on the TF(98) model and with the help of Matlab software, the binary tree method is adopted to investigate the theoretical value calculated by the model of 32 convertible bonds in the sample for 100 consecutive days of trading, and the corresponding actual market price is compared.

A total of 3200 deviation rate intervals were analyzed for each trading day

Table 2. Investigate the deviation rate distribution interval of 3200 samples

| Distribution range | Sample Size | Distribution range | Sample Size | Distribution range | Sample Size |
|--------------------|-------------|--------------------|-------------|--------------------|-------------|
| Less than -15% | 21 | -6% to -3% | 353 | 6% to 9% | 661 |
| -15 % to -12% | 43 | -3% to 0 | 544 | 9% to 12% | 224 |
| -12% to-9% | 196 | 0 to 3% | 351 | 12% to 15% | 61 |
| -9% to -6% | 234 | 3% to 6% | 492 | More than 15% | 20 |

Data source: Wind database

Table 2 shows that most of the 3200 data in the 32 convertible bonds sample are in the range of [-12%,12%], while other ranges have a small distribution.

In this paper, the average deviation rate during the investigation period of 32 convertible bonds is obtained from the market price and theoretical value of the obtained convertible bonds, and the results are shown in Table 3 below.

The phenomenon of the undervaluation of convertible bonds is shown in 13 convertible bonds,

but the degree varies. The highest grade of Jinnong convertible bonds is AA-, and the lowest grade of Yongdong convertible bonds is -1.02%. There were only 19 cases of overvalued, with the highest value being 11.14% of Ningxing convertible bonds and the lowest value being 0.5% of Longji convertible bonds. In general, China's convertible bond market is still somewhat overvalued. Among them, variance can reflect the fitting degree of the model, and the variance data in the following table indicates that the model does not show too large deviation in pricing.

Table 3. Statistical table of the deviation rate of convertible bonds per trading day only

| Name | The sample points | Maximum deviation rate | Minimum deviation rate | Mean deviation rate | The variance |
|---------------|-------------------|------------------------|------------------------|---------------------|--------------|
| AIHUABOND | 100 | 0.1065 | -0.705 | 0.054 | 0.0007 |
| DILONGBOND | 100 | 0.0267 | -0.076 | -0.016 | 0.0005 |
| CHONGDABOND | 100 | 0.2051 | -0.0447 | 0.6909 | 0.0010 |
| GUOZHENBOND | 100 | 0.0816 | -0.1947 | -0.0150 | 0.0028 |
| JIANGNANBOND | 100 | 0.0762 | 0.0507 | 0.0606 | 0.00001 |
| JINHEBOND | 100 | 0.1567 | 0.0128 | 0.0541 | 0.0009 |
| JULIBOND | 100 | 0.0101 | -0.0601 | -0.0145 | 0.0001 |
| JIUQIBOND | 100 | 0.0134 | -0.0953 | -0.0448 | 0.0006 |
| DAOSHBOND | 100 | 0.1100 | -0.1112 | -0.0365 | 0.0019 |
| HAIYINBOND | 100 | -0.0412 | -0.1390 | -0.0759 | 0.0003 |
| AOJIABOND | 100 | -0.0521 | -0.1199 | -0.0825 | 0.0002 |
| JINNONGBOND | 100 | -0.0606 | -0.1490 | -0.1023 | -0.0004 |
| KANGTAIBOND | 100 | 0.5095 | -0.1584 | 0.0084 | 0.0018 |
| TEYIBOND | 100 | 0.0143 | -0.0802 | -0.0199 | 0.0002 |
| YONGDONGBOND | 100 | 0.1053 | -0.0607 | -0.0102 | 0.0008 |
| ZHONGXINGBOND | 100 | -0.0381 | -0.1302 | -0.0979 | 0.0005 |
| CHANGSHUBOND | 100 | 0.1046 | -0.0686 | 0.0156 | 0.0007 |
| DAZUBOND | 100 | 0.1453 | -0.0048 | 0.0530 | 0.0008 |
| GUOMAOBOND | 100 | 0.1207 | 0.0544 | 0.0789 | 0.0001 |
| JIANGYINBOND | 100 | 0.0415 | -0.0948 | -0.0219 | 0.0006 |
| JIUZHOBOND | 100 | 0.1115 | 0.0452 | 0.0734 | 0.0001 |
| LANSIBOND | 100 | -0.0110 | -0.0775 | -0.0461 | 0.0002 |
| LINGLONGBOND | 100 | 0.0662 | 0.0242 | 0.0415 | 0.00005 |
| LONGJIBOND | 100 | 0.0521 | -0.0507 | 0.0050 | 0.0007 |
| DIANQIBOND | 100 | 0.0478 | -0.1757 | -0.0182 | 0.0042 |
| GUANGDABOND | 100 | 0.1112 | 0.0467 | 0.0850 | 0.00004 |
| GUANGQIBOND | 100 | 0.0714 | 0.0389 | 0.0564 | 0.0006 |
| GUOJUNBOND | 100 | 0.0946 | 0.0417 | 0.0704 | 0.00008 |
| HANGDIANBOND | 100 | 0.1204 | 0.0153 | 0.0694 | 0.0002 |
| HANGXINBOND | 100 | 0.1077 | 0.0459 | 0.0790 | 0.0001 |
| MENGDIANBOND | 100 | 0.0342 | -0.0178 | 0.0086 | 0.0002 |
| NINGHANGBOND | 100 | 0.1527 | 0.0681 | 0.1114 | 0.00007 |
| MEAN VALUE | | ----- | ----- | 0.0122 | 0.0006 |

Data source: Wind database

5.1 Correlation analysis between the market price of convertible bonds and the underlying stock price

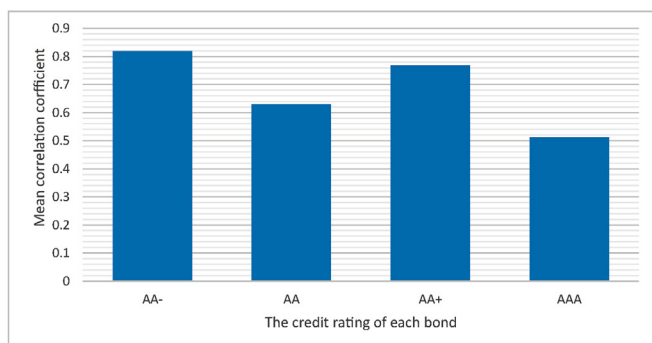


Figure 1. Variance of the bias rate of convertible bonds and mean of the population variance

5.1.1 General analysis

As shown in Figure 1, the correlation between the underlying stock price of 32 convertible bonds and its market price is 0.68, which means that the market price of convertible bonds is basically consistent with the trend of the underlying stock price. It can be seen that the underlying stock price of convertible bonds determines its market price to a large extent. The average correlation coefficient of 32 convertible bonds is 0.68 after calculation. The details are as follows: there are 21 convertible bonds with a correlation coefficient of more than 0.7, accounting for 65.6% of the total sample. The correlation coefficients of 12 convertible bonds of Dillon, Chongda, Jinhe, Dow, Kangtai, Teri, Yongdong, Changshu, Da Zang, Lanshi, Linglong and Longji all showed higher than 0.9. It can be further obtained that

the convertible bonds with a correlation coefficient of more than 0.9 were mainly distributed in the credit rating of AA- and AA+.

5.1.2 Analysis according to the credit rating of convertible bonds

Calculate the average correlation coefficient between the actual market price and the underlying stock price of the convertible bond with credit ratings of AA-, AA, AA+ and AAA respectively, as shown in Figure 1. Credit rating to AA - the market price of convertible bond and the underlying share price has the highest correlation, and the strongest positive correlation, the credit rating to AA - eight of the underlying stocks in the sample review poor market performance, and its lower credit ratings are closely related, and investors pay close attention to and consider the underlying share price movements, and fully consider the credit rating of bonds itself, make investment, manifests the market for credit rating to AA - the bond investment and financing of prudence. The correlation between the actual market price of eight convertible bonds with AAA credit rating and the underlying stock price is the weakest, and the electrical convertible bonds among them have a negative correlation between the actual market price and the underlying stock price. The reason is that the underlying stock price and the actual market price do not have the same trend within the sample survey range. But on the whole, the positive correlation between the underlying stock price and the actual market price of the convertible bond is strong.

5.2 Analysis on the credit rating and average deviation rate of convertible bonds

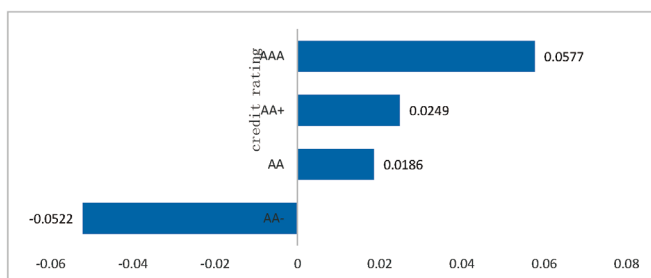


Figure 2. Relationship between the credit rating of convertible bonds and the average deviation rate

The deviation rate of each trading day of the convertible bond within each credit grade is counted, and the average deviation rate of each credit grade is calculated, as shown in Figure 2 above. According to the value deviation rate of convertible bond, it is the

percentage of the market price minus the theoretical value and the theoretical value.

Credit rating of convertible bonds is AA - the average deviation rate is negative, that is credit rating of AA - the phenomenon of convertible bonds value is underestimated, and AAA credit grade is AA, AA +, the average deviation rate for convertible bonds are, shows that credit ratings are AA (containing AA) more convertible bond value is overrated phenomenon, and the higher the credit rating of the average deviation rate of convertible bond, the greater the overvalued, the greater the degree of the law in accordance with the results before. In addition, this law also conforms to the psychology of investors in the convertible bond market. Investors are more willing to buy derivatives with higher credit ratings to reduce the probability of investment damage out of the psychology of risk aversion. The vast majority of investors buy convertible bonds with higher credit ratings at the same time, which is easy to cause credit ratings Higher convertible bonds in the market in short supply; From the perspective of the issuer, the issuer is bound to raise the price to maximize the financing effect by relying on its high credit rating and large audience. The combined forces of these two factors lead to the phenomenon that the actual market price of the convertible bond with high credit rating is often higher than the theoretical value of the convertible bond itself.

5.3 There is a certain deviation between the market price and the theoretical value calculated by the model.

The possible reasons are as follows:

First, the model itself is based on a set of idealized assumptions.

Second, given that the option pricing has not found an explicit solution so far, there is a relatively complex relationship between the option value, the put back value and the downward revised option value of the convertible bond, and the idealized assumption in the pricing process is that they are independent of each other and not affected.

Third, China's convertible bond market is not attractive to investors.

6 Countermeasures and Suggestions

Considering the current situation of the development of convertible bonds in China and

a series of problems in the pricing process of convertible bonds, the following countermeasures and Suggestions are put forward.

First, establish and improve China's credit rating system.

Second, we will relax market access mechanisms. Industries or enterprises with the highest financing demand for convertible bonds, such as those in capital-intensive industries and industries with high capital cost, can relatively relax the market mechanism for these industries to enter the convertible bond market, so as to give full play to the characteristics of low financing cost and effective prevention of investment risk of convertible bonds.

Thirdly, the terms of convertible bonds should be simplified and targeted. In view of the fact that the terms of convertible bonds in China are relatively complicated and the assimilation of terms of different convertible bonds is serious, the formulation of terms is not targeted to make clear expectations for investors, which increases the difficulty of pricing each convertible bond. According to the market demand theory, the issuers of convertible bonds regard convertible bonds as stock alternatives, and

fail to accurately locate the demand group in the investment market, resulting in pricing error.

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