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Employing Interval Grey Relational Analysis in ESG Investment Decision-Making Based on Ratings

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Abstract: ESG ratings have gradually become an important reference basis in investment decisions. Currently, different rating agencies often give significant differences in ESG scores to the same enterprise based on their own assessment criteria, data sources, and weight settings. The inconsistent scores caused by such multi-source heterogeneous data increase the cognitive uncertainty and decision-making complexity of investors when utilizing ESG information, affecting the accuracy and reliability of investment judgments. In this paper, by introducing the Interval Number Grey Relational Analysis (IGRA) method, an enterprise investment ranking model based on multi-source ESG scores is constructed. The scores from different rating agencies are integrated into the form of interval numbers, effectively reflecting the fluctuation range of the scores. And with the help of the grey system theory, the similarity degree between enterprises and ideal reference objects is measured. Realize the comprehensive processing and scientific ranking of multi-dimensional uncertain information. An empirical analysis was conducted based on the ESG rating data to verify the effectiveness of the method. This research provides methods for ESG investment practices and also offers theoretical references for dealing with uncertain investment issues.

Keywords: Interval number; Grey relational analysis; ESG rating; Uncertainty decision-making

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1. Introduction

As all sectors of society place increasing emphasis on sustainable development, regulatory policies continue to tighten requirements for corporate social responsibility, investors are paying more attention to companies' performance in non-financial areas, and public concern regarding environmental protection and employee rights is on the rise. ESG (Environmental, Social, and Governance) factors have thus begun to be integrated into investment decision-making frameworks and are regarded as critical indicators for assessing the long-term and stable development of companies [1,2].

However, major ESG rating agencies (such as MSCI, Sustainalytics, and Refinitiv) differ significantly in their rating models, indicator selection, data sources, and weighting methodologies. As a result, the ESG scores provided by different agencies are often inconsistent [2]. Importantly, there is no definitive basis to determine which score is "absolutely correct." This situation—characterized by "different sources, different numerical distributions, and a lack of certainty"—complicates investor evaluations of corporate sustainability performance and diminishes the practical usability and investment relevance of ESG information.

To address this issue, this paper introduces the Interval Grey Relational Analysis (IGRA) method, which converts ESG scores from multiple rating agencies into interval data. That is, for each ESG indicator, the diverse score values are represented as a numerical interval defined by upper and lower bounds. This approach avoids the shortcomings of traditional averaging methods, which tend to obscure rating differences and result in the loss of valuable information [3–5]. Based on these intervals, a unified model is constructed while preserving the heterogeneity across rating sources. By establishing grey relational degrees between a given company and an ideal reference object, a comprehensive ranking of corporate ESG performance can be achieved [5–8].

Compared to traditional methods, the interval-based approach retains valuable information under multi-source inconsistency and supports more accurate and robust decision-making in the presence of uncertainty [3,4]. This not only enhances the integrative capacity of ESG data but also improves its applicability in investment evaluation contexts. Based on empirical analysis, this paper seeks to verify the effectiveness of the IGRA method in multi-criteria decision-making under uncertainty and to explore its practical value in ESG-oriented investment strategies [8,9].

2. Basic terminology and theory

This section introduces the concepts and properties of interval numbers, grey relational analysis, etc.

2.1. Number of intervals

(1) Definition 1: A closed interval [a,b] is the set of real numbers satisfying $a \le x \le b$, that is,

$$[a,b]=\{x \in R|a\leq x\leq b\} \tag{1}$$

In this paper, the term interval refers to closed intervals [10].

(2) Definition 2: The left and right endpoints of an interval X are denoted by X^- and X^+, respectively, i.e.,

$$X=[X^-,X^+] \tag{2}$$

Two intervals are equal if and only if their corresponding endpoints are equal [10].

$$X=Y \Leftrightarrow X^-=Y^- \text{and} X^+=Y^+ \tag{3}$$

2.2. Grey relational degree

Let the reference sequence be

$$X_0=\{x_0(k)|k=1,2,...,n\} \tag{4}$$

The comparison sequence is

$$X_i=\{x_i(k)|k=1,2,...,n,i=1,2,...,m\} \tag{5}$$

The grey relational coefficient is defined as

$$\xi_i(k)=(\min\_(i,k)Δ(k)+\rho\max\_(i,k)Δ(k))/(Δ_i(k)+\rho\max\_(i,k)Δ(k)) \tag{6}$$

where

$$Δ_i(k)=|x_0(k)-x_i(k)| \tag{7}$$

$\rho \in (0,1)$ is the distinguishing coefficient, usually ρ=0.5.

The grey relational degree is

$$\gamma_i=\sum_{k=1}^{n} w_k \xi(k) \quad (8)$$

where the weights satisfy

$$\sum_{k=1}^{n} w_k=1$$

and equal weights are generally used [6–8]:

$$w_k=1/n \quad (9)$$

3. Steps of the IGRA method

This section details the steps of the interval grey relational analysis (IGRA) method combined with entropy weight calculation [3–5].

(1)Construct interval decision matrix

Collect evaluation data of each enterprises $i=1,2,\dots,m$ on each indicator $j=1,2,\dots,n$, expressed as intervals:

$$x_{ij} \in [x_{ij}^{-},x_{ij}^{+}]$$

(2) Calculate interval means and normalize

Calculate the mean of each interval:

$$\bar{x}_{ij}=(x_{ij}^{-}+x_{ij}^{+})/2 \quad (10)$$

Normalize the means for each indicator j to obtain the probability distribution:

$$p_{ij}=\bar{x}_{ij}/\left(\sum_{i=1}^{m} \bar{x}_{ij}\right) \quad (11)$$

(3) Calculate indicator entropy

Based on p_{ij}, calculate the entropy of the j-th indicator [11]:

$$e_j=-k\sum_{i=1}^{m} p_{ij} \ln(p_{ij}),\quad k=1/(\ln m) \quad (12)$$

where when $p_{ij}=0$ define $p_{ij} \ln p_{ij}=0$.

(4) Calculate indicator difference coefficient and weight

Define the difference coefficient of indicator j as:

$$d_j=1-e_j \quad (13)$$

Normalize the weights:

$$w_j=d_j/\left(\sum_{j=1}^{n} d_j\right) \quad (14)$$

(5) Construct ideal reference sequence

Select the maximum upper bound of the interval for each indicator among all enterprises to form the ideal reference sequence:

$$X_0=\{[x_{0j}^{-},x_{0j}^{+}] \mid x_{0j}^{+}=\max_i x_{ij}^{+}\}$$

(6) Calculate interval distances

Use Euclidean distance to calculate the distance between each enterprise's interval and the ideal reference interval:

$$\Delta_{ij}=(x_{ij}^{-}-x_{0j}^{-})^2+(x_{ij}^{+}-x_{0j}^{+})^2 \quad (15)$$

(7) Calculate grey relational coefficients

Find the minimum and maximum distances:

$$\Delta_{min}=\min_{(i,j)} \Delta_{ij},\quad \Delta_{max}=\max_{(i,j)} \Delta_{ij}$$

Set distinguishing coefficient $\rho = 0.5$, then calculate the grey relational coefficient:

$$\xi_{ij}=(\Delta_{min}+\rho\Delta_{max})/(\Delta_{ij}+\rho\Delta_{max}) \quad (16)$$

(8) Calculate comprehensive grey relational degree

Calculate the comprehensive grey relational degree for each enterprise by weighted summation:

$$\gamma_i=\sum_{(j=1)}^{n} w_j \ \xi_{ij} \tag{17}$$

(9) Ranking and analysis

Rank enterprises based on the comprehensive grey relational degree γi. A higher degree indicates that the enterprise's overall ESG performance is closer to the ideal enterprise.

4. Empirical results analysis

This paper selects listed commercial banks in China in 2023 as the research sample. Considering the core position of commercial banks in the financial system and the significant impact of their ESG performance on investment decisions, four representative banks were selected from state-owned banks, joint-stock banks, and city commercial banks, denoted as A, B, C, and D.

ESG indicator data is sourced from multiple rating agencies, including MSCI ESG Ratings, Sustainalytics, and the Wind ESG database. Due to differences in evaluation methods and indicator dimensions across rating agencies, this paper adopts interval numbers to model each indicator, representing the range of values across different data sources. This approach effectively addresses the uncertainty and heterogeneity of multi-source information [10].

4.1. Index system and interval modeling

The ESG indicator system consists of three dimensions: Environmental (E), Social (S), and Governance (G), with two specific indicators under each dimension. The directionality of each indicator is also specified—positive indicators mean the higher the value, the better, while negative indicators imply the opposite, as shown in **Table 1**.

Table 1. ESG indicator system construction

Dimension	Indicator ID	Indicator name	Direction
E	E1	Carbon emission intensity	Negative
E	E2	Proportion of green credit	Positive
S	S1	Employee satisfaction score	Positive
S	S2	Frequency of customer data leaks	Negative
G	G1	Board independence ratio	Positive
G	G2	Information disclosure transparency score	Positive

After normalizing the values from different data sources, the interval number matrix is obtained as shown in **Table 2**.

Table 2. ESG index interval number matrix (Normalized)

	E	S	G
A	[0.5, 0.61]	[0.5, 0.53]	[0.5, 0.52]
B	[0.5, 0.63]	[0.5, 0.55]	[0.5, 0.54]
C	[0.5, 0.60]	[0.5, 0.52]	[0.5, 0.51]
D	[0.2, 0.58]	[0.2, 0.5]	[0.2, 0.49]

The ideal reference sequence is constructed by taking the maximum upper bound of each indicator as the upper limit, and the maximum lower bound as the lower limit [6,8]. Next, the entropy weight method is used to calculate the weights of each indicator [3,4]. Combined with the interval grey relational analysis model, the grey relational degrees of each company under the three dimensions are obtained, as shown in **Table 3** [5].

Table 3. ESG comprehensive correlation results

Company	E Correlation	S Correlation	G Correlation	Comprehensive correlation result
A	0.9914	0.9914	0.9914	0.9914
B	1.0000	1.0000	1.0000	1.0000
C	0.9809	0.9809	0.9809	0.9809
D	0.3333	0.3333	0.3333	0.3333

The ranking based on the comprehensive grey relational degree from highest to lowest is:

4.2. Methods and results analysis

Table 3 shows the grey correlation degrees and comprehensive correlation results of four companies under three indicators: Environmental Performance (E), Social Responsibility (S), and Corporate Governance (G). The comprehensive correlation scores indicate that Company B ranks first with the highest score of 1.0000, indicating that its ESG performance is closest to the ideal reference sequence, achieving the best overall performance. Companies A and C scored 0.9914 and 0.9809, respectively, showing performances close to the ideal state, which means these two companies have relatively balanced and good ESG performance. Company D scored significantly lower at only 0.3333, reflecting that its performance on each indicator is noticeably weaker than that of other companies, with substantial room for improvement.

At the indicator level, the correlation degrees of the three indicators—Environmental Performance, Social Responsibility, and Corporate Governance—among the companies are relatively consistent, reflecting a relatively balanced development trend. The Social Responsibility and Corporate Governance indicators were given higher weights, emphasizing their importance in the ESG comprehensive evaluation. Since the correlation values are relatively close, the weighting makes the comprehensive score more accurately reflect the overall ESG performance of the companies.

For specific companies, Company D needs to strengthen environmental protection, social responsibility implementation, and governance mechanisms to improve its overall ESG level. Companies A and C, although performing well, should optimize their weak points. Company B should maintain its advantages, further promote sustainable development and governance innovation, and consolidate its leading position.

In conclusion, this study utilizes interval grey relational analysis combined with the entropy weight method to effectively capture differences in ESG performance among companies. The method demonstrates strong adaptability in handling heterogeneous data and uncertainty, providing a solid basis for enterprise ESG improvement and relevant policy formulation [3,5,6].

5. Conclusion

With the deepening of the concept of sustainable development, the concept of ESG has been valued and has become an important reference indicator for measuring the long-term value of enterprises. However, there are obvious differences in methods, data caliber, and evaluation standards among different rating agencies in the current market, which makes it impossible for investors to use evaluation information to make accurate market judgments, causing trouble for investors. In this context, this paper introduces the method of interval numbers and gray correlation analysis to integrate and quantitatively sort the results of multi-source ESG ratings, providing an auxiliary basis for investment decisions.

From the research results, the constructed model has certain advantages in integrating multi-institutional scores and reflecting the uncertainty of information intervals. The introduction of interval numbers not only retains the differences between different scores, but also reduces the risk of misjudgment caused by forcibly taking the average or a single data source to a certain extent. By calculating the correlation of the ideal reference sequence, the model can intuitively reflect the comprehensive performance of each investment object in the ESG dimension.

In terms of indicator weights, this paper uses the entropy weight method to objectively calculate the importance of each evaluation indicator, avoiding the bias of relying entirely on the subjective judgment of experts. This weighting method, based on the degree of variation of the data itself, helps to improve the interpretability and practical applicability of the model. Especially when the amount of information is uneven, or the data of some dimensions fluctuate significantly, the entropy weight method can more reasonably reflect the relative influence of indicators.

From an application perspective, the model has strong promotion potential. Whether in fund rating, enterprise screening, or in the construction of responsible investment portfolios, it can provide quantitative support and provide investment institutions with a transitional evaluation method that is both robust and flexible.

It is worth noting that although the method in this paper performs well in integrating uncertain information, it still has some limitations. For example, the construction of interval data is highly dependent on the coverage and quality of rating agency data. If there are extreme values or some types of data are missing, it may affect the rationality of the interval setting. At the same time, the ESG performance of enterprises is somewhat dynamic, and this paper mainly analyzes based on static data, which fails to reflect its trend over time.

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