The Prediction of Engel's Coefficient and Education Expenditure Based on the Linear Regression Model for Heilongjiang and Ontario

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Abstract: It is meaningful to study trends in food and education expenditure as proportions of total household expenditure. In this study, based on year 2006 to 2017 data from Heilongjiang province in China and Ontario province in Canada, a linear regression model is used to forecast the Engel's coefficients (proportion spent on food) and the education proportion from year 2018 to 2027 for those two regions. The results suggest that in both regions the Engel's coefficients show a decreasing trend, while the education expenditure proportions show an increasing trend. The ratios of education expenditure to food expenditure in both places show an increasing trend.

Keywords: Engel's coefficient; Education expenditure; Linear regression; Heilongjiang; Ontario

1 Introduction

In general, household expenditures include food, housing, education, recreation, and so many other expenditures. Among these various types of expenditures, the fundamental one is food. As an economy develops and its residents' incomes increase, the proportion of food expenditure (FE) in the total of household expenditure tends to decrease since people's basic demand for food is met. As early as the nineteenth century, German statistician Ernst Engel (1821-1896) proposed Engel's coefficient to refer to the proportion of residents' income spent on food[1]. This coefficient is usually used to measure a population's standard of living. However, some other types of expenditure may be increasing. The proportion of education expenditure (EE) is one of those showing the increasing trend.

According to Maslow's hierarchy of needs, proposed by American psychologist Abraham Maslow, people pursue esteem needs, such as education needs[2], in addition to the physiological needs, such as food needs. Dai and Zhou discussed the needs of happiness by the middle class in Guangdong province and concluded that education is one of the key contributing factors to happiness[3]. In a modern competitive society, education is a key prerequisite for winning the competition. So people are spending more and more money on education. Wang analyzed the advantages and disadvantages of measuring Shanghai citizens' living standard by using the Engel coefficient and concluded that the structure of food expenditure was gradually increasing with the improvement of people's living standard[4]. Gu's work, the Engel's coefficient in China was decreasing and the living standard of Chinese residents was improving[5]. Wang studied the influence of the price of commodities on Engel's coefficient and compared the similarities and differences of FE between urban and rural residents based on his econometric model[6]. Wang and Woo used Engel's coefficient to discuss the real level of the household income in China, from 2005 to 2008, the degree of corruption grew 91%. So this country must give institution reforms[7]. Tang used GM(1,1)[8] to predict the EE and school enrolment rates in the future. Also, to verify GM(1,1) can predict the same accurate number as NCES (National Center for Education Statistics). In the future, the GM(1,1) model can provide a short-term prediction on education[9]. It is

clear that research on Engel's coefficient is abundant, but research on education as a proportion of household expenditure is limited. It is the first attempt to compare the changing trend of the proportion of EE and FE in the total household expenditure in two regions of a developing country and a developed country.

In this study, Heilongjiang (HLJ) province in China and Ontario province in Canada were chosen as comparable research regions. The reason is because HLJ is representative of a province in a developing country while Ontario is a special province in a developed country. HLJ is a landlocked province with relatively sluggish economic development. Compared with China's overall Gross Domestic Product (GDP) per capita of 59,210 CNY (about 7,786 USD) in 2017, GDP per capita of HLJ was only 41,916 CNY (about 6,208 USD)[10]. In addition, in 2017, the GDP growth rate of HLJ province was 6.3%[11], much lower than the national annual GDP growth rate of 6.9%[12]. As the highest GDP province, Ontario's GDP reached 857,384M CAD (about 608,158M USD), contributing to 38.55% of Canadian total GDP (2,223,856M CAD, about 1,577,421M USD) in 2018[13].

In this study, a linear regression model is used to predict trends of both FE and EE proportion to the total household expenditure in HLJ and Ontario.

2 Methods

Regression analysis generates a 'best-fit' mathematical equation that can be used to predict the values of the dependent variable as a function of the independent variable. In this study, a simple linear regression model is adopted, which is an effective predicting method used in many fields, such as finance, medicine, and economy.

This method mainly studies the relationship between the dependent variable and the independent variable. The equation of the linear regression is written as follows:

$$y = a + bx + \varepsilon \qquad (1)$$

Where x is the independent variable, y is the dependent variable, a and b are regression parameters estimated from historical data, and ε is an error random variable that has expected value 0. The ordinary least square (OLS) is used to estimate the parameters a and b.

Intuitively, OLS is fitting a line through the sample points such that the sum of squared residuals is as small as possible[14].

The estimated regression line is written as:

$$y = \hat{a} + \hat{b}x \qquad (2)$$

In Equation (2),

$$\hat{a} = \frac{(\sum_{i=1}^{n} x_i^2)(\sum_{i=1}^{n} y_i) - (\sum_{i=1}^{n} x_i)(\sum_{i=1}^{n} y_i x_i)}{n \sum_{i=1}^{n} (x_i)^2 - (\sum_{i=1}^{n} x_i)^2} \qquad (3)$$

$$\hat{b} = \frac{n \sum_{i=1}^{n} x_i y_i - (\sum_{i=1}^{n} x_i)(\sum_{i=1}^{n} y_i)}{n \sum_{i=1}^{n} (x_i)^2 - (\sum_{i=1}^{n} x_i)^2} \qquad (4)$$

The significance test of the linear regression involves *F-test, t-test, R^2* and confidence interval.

The *t-test* can be used to test whether the mean of a random variable is equal to any particular number, even when the variance of the random variable is unknown. The *t-test* is written as follows:

$$|t| = \frac{|\hat{b}|}{\hat{\sigma}}\sqrt{S_{xx}} \sim t_{a/2}(n - 2) \qquad (5)$$

Where $S_{xx} = \sum_{i=1}^{n} (x_i - \underline{x})^2$,

and $\hat{\sigma} = \sqrt{\frac{\sum_{i=1}^{n} (y_i - \hat{y}_i)^2}{n-2}}$.

t distribution critical value with significance level α and *(n-2)* degrees of freedom, where n is the number of samples.

The *F-test* tests the significance of the equation and verifies the interpretation ability of x to y. The *F-test* is as follows:

$$F = \frac{SSR/(k-1)}{SSE/(n-k)} \sim F(k - 1, n - k) \qquad (6)$$

Where *SSE* is the sum of squared residuals, *SSR* is the sum of squares for regression and *F* statistic with *(k-1)* and *(n-k)* degrees of freedom.

R^2 is goodness of fit, or the volatility of y relative to its mean. The larger R^2 is, the better the regression fitting is. The expression of the R^2 is as follows:

$$R^2 = \frac{(\sum_{i=1}^{n} (x_i - \underline{x})(y_i - \underline{y}))^2}{\sum_{i=1}^{n} (x_i - \underline{x})^2 \sum_{i=1}^{n} (y_i - \underline{y})^2} \qquad (7)$$

The sample means are y and x.

A confidence interval is used to measure the degree of uncertainty for predicting. In this study, the confidence interval refers to the probability that the predicted parameter will fall between two set values with 95% confidence level. The form of the confidence interval is written as follows.

$$\hat{a} + \hat{b}x_0 \pm t_{a/2}(n - 2)\hat{\sigma}\sqrt{1 + \frac{1}{n} + \frac{(x_0 - \underline{x})^2}{S_{xx}}} \qquad (8)$$

3 Data

The linear regression model incorporated 2006 to 2017 data on Engel's coefficients, EE, and total household expenditure in HLJ and Ontario. The data for HLJ are obtained from the annual social development report[15-16], shown in Table 1. The data

for Ontario are obtained from Statistics Canada[17], shown in Table 2.

Table 1. HLJ's household expenditure from 2006 to 2017

Year	Total expenditure (CNY)	Food & alcohol (CNY)	Engel's coefficient (%)	Education & recreation (CNY)	Education in total expenditure (%)
2006	6655.4	2215.7	33.3	843.9	6.34
2007	7519.3	2633.2	35.0	938.2	6.30
2008	8623.0	3138.5	36.3	906.2	5.36
2009	9629.6	3397.4	35.3	956.9	5.13
2010	7632.0	2751.0	36.0	767.5	5.32
2011	8616.0	3182.0	36.9	882.0	5.22
2012	9268.0	3437.0	37.1	948.0	5.34
2013	10087.0	3453.0	34.2	1178.0	6.90
2014	11461.0	3360.0	29.3	1217.0	6.37
2015	12162.0	3521.0	27.9	1293.0	6.38
2016	13134.0	3746.0	28.5	1567.0*	7.16
2017	13468.0	3866.0	28.7	1599.0	7.12

*According to "The report read | Heilongjiang blue book: Heilongjiang social development report (2018) https://www.pishu.cn/zxzx/xwdt/516803.shtml", the resident income increased by 60% from 2010 to 2016. The per capita spending on medical care, transportation and education, and recreation expenditure increased by 70.1%, 68.4% and 65.3% over 2012, in the first three quarters of 2012, the per capita expenditure on education and recreation of residents is : 948, 948*1.653 = 1567 CNY.

Table 2. Ontario per household average expenditure from 2006 to 2017

Year	Total Expenditure (CAD)	Food & Alcoholic (CAD)	Food & Alcoholic in Total (%)	Education (CAD)	Education in Total (%)
2006	52664	8722	16.56	1402	2.66
2007	53938	8842	16.39	1220	2.26
2008	55002	8977	16.32	1614	2.93
2009	53572	8690	16.22	1551	2.89
2010	55995	8830	15.77	1446	2.59
2011	57301	8772	15.31	1552	2.70
2012	57963	8495	14.66	1876	3.24
2013	60572	8833	14.58	2033	3.36
2014	61660	9061	14.69	1897	3.07
2015	62719	9638	15.37	1962	3.13
2016	66220	9925	14.99	2258	3.41
2017	66855	9936	14.84	2223	3.34
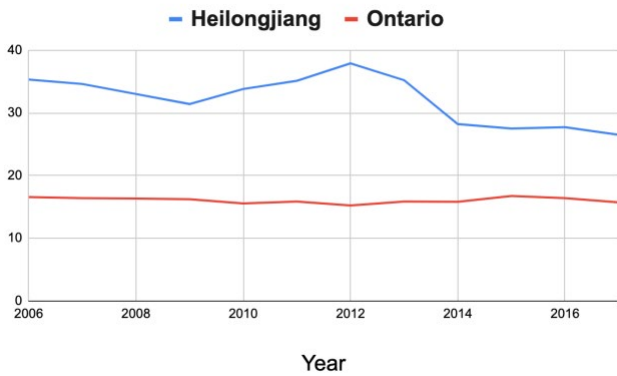
Figure 1. The FE proportion curve for HLJ and Ontario from 2006 to 2017.
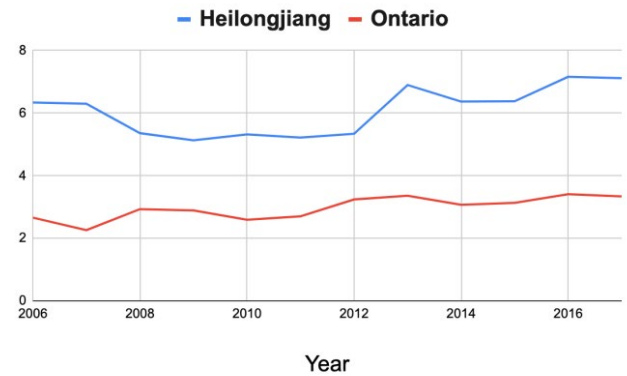
Figure 2. The EE proportion curve for HLJ and Ontario from 2006 to 2017

The FE proportion curve for HLJ and Ontario can be seen from Figure 1, the FE proportion curve for HLJ shows a declining trend with fluctuation while the FE proportion curve for Ontario shows a more stable trend.

Figure 2 shows that the EE in HLJ and Ontario have been overall increasing from 2006 to 2017. HLJ's EE proportion increased at an average of 0.08% per year. Ontario's EE proportion increased at an average of 0.04% per year. HLJ's EE proportion increased at a twice faster speed than Ontario's did.

4 Simulation and analysis

4.1 The Engel's coefficient in HLJ and Ontario

linear regression model determination

4.1.1 HLJ's Engel's coefficient regression model

From the data in Table 1, OLS can be used to calculate the linear regression model for HLJ's Engel's coefficient as follows:

$$y_1 = -0.7101x + 1461.6547$$

In this model, the slope is -0.7101, representing the decrease in HLJ's Engel's coefficient. The *y-intercept* is 1461.6547. The Engel's coefficient will gradually decrease with the increase of time *x*. The goodness of fit R^2 is 0.5127. HLJ's Engel's coefficient fitted value and original data are shown in Table 3 and Figure 4.


Figure 3. The parameter estimation of the Engel's coefficient linear regression for HLJ

In Figure 3, the coefficient of determination R^2 is 0.5127. *F-test* is 10.5224, *t-test* is 3.3192, -3.2438, so HLJ's regression model passes the *F-test* and *t-test*.

Table 3. HLJ's Engel's coefficient fitted value and original data from 2006 to 2017

Year	2006	2007	2008	2009	2010	2011
Fitted Value	37.114	36.404	35.694	34.984	34.274	35.563
Original Data	33.3	35.0	36.3	35.3	36.0	36.9
Residual	3.814	1.404	0.606	0.316	1.726	3.337
Year	2012	2013	2014	2015	2016	2017
Fitted Value	33.563	32.143	31.433	30.723	30.013	29.303
Original Data	37.1	34.2	29.3	27.9	28.5	28.7
Residual	4.247	2.057	2.133	2.823	1.513	0.603
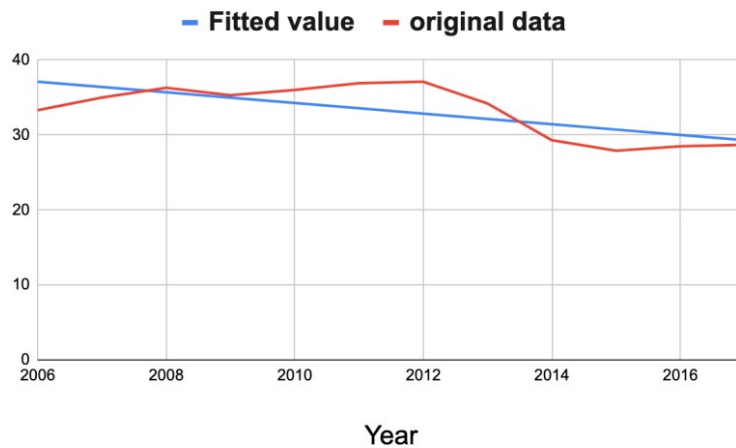
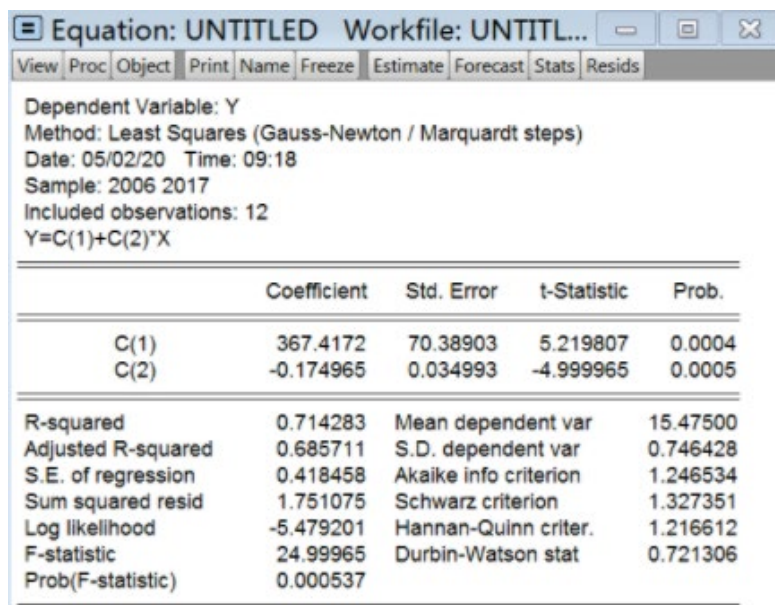**Figure 4.** HLJ's Engel's coefficient fitted value and original data from 2006 to 2017

4.1.2 Ontario's Engel's coefficient regression model

From the data in Table 2, OLS can be used to calculate the linear regression model for Ontario's Engel's coefficient as follows:

$y_2 = -0.1750x + 367.4172$

In this model, the slope is -0.1750, representing the decrease in Ontario's Engel's coefficient. The *y-intercept* is 367.4172. The Engel's coefficient will gradually decrease with the increase of time x. The goodness of fit R^2 is 0.7143. Ontario's Engel's coefficient fitted value and original data are shown in Table 4 and Figure 6.

Figure 5. The parameter estimation of the Engel's coefficient linear regression for Ontario

In Figure 5, the coefficient of determination R^2 equals 0.7143. *F-test* is 24.9997, *t-test* is 5.2198, -4.9999. Ontario's regression model can pass the *F-test* and *t-test*.

Table 4. Ontario's Engel's coefficient fitted value and original data from 2006 to 2017

Year	2006	2007	2008	2009	2010	2011
Fitted Value	16.437	16.262	16.087	15.912	15.737	15.387
Original Data	16.56	16.39	16.32	16.22	15.77	15.31
Residual	0.1227	0.1277	0.2326	0.3076	0.0326	0.2525
Year	2012	2013	2014	2015	2016	2017
Fitted Value	15.387	15.213	15.038	14.863	14.688	14.513
Original Data	14.66	14.58	14.69	15.37	14.99	14.84
Residual	0.7275	0.6326	0.3476	0.5074	0.3023	0.3273
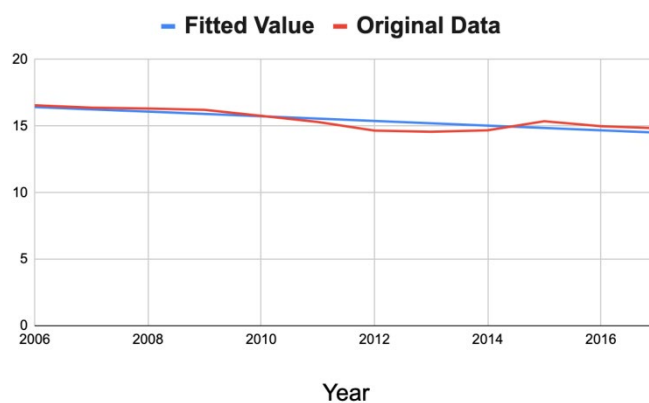
Figure 6. Ontario's Engel's coefficient fitted value and original data trend 2006 to 2017

4.2 The EE in HLJ and Ontario linear regression model determination

4.2.1 HLJ's EE regression model

From the data in Table 1, OLS can be used to calculate the linear regression model for HLJ's EE as follows：

$$z1 = 0.1207x - 236.7083$$

In this model, the slope is 0.1207, representing the increase of the EE proportion in HLJ. The *y-intercept* is -236.7083. The EE proportion will gradually increase with the increase of time *x*. The goodness of fit R^2 is 0.3207. The EE proportion in HLJ fitted value and original data are shown in Table 5 and Figure 8.
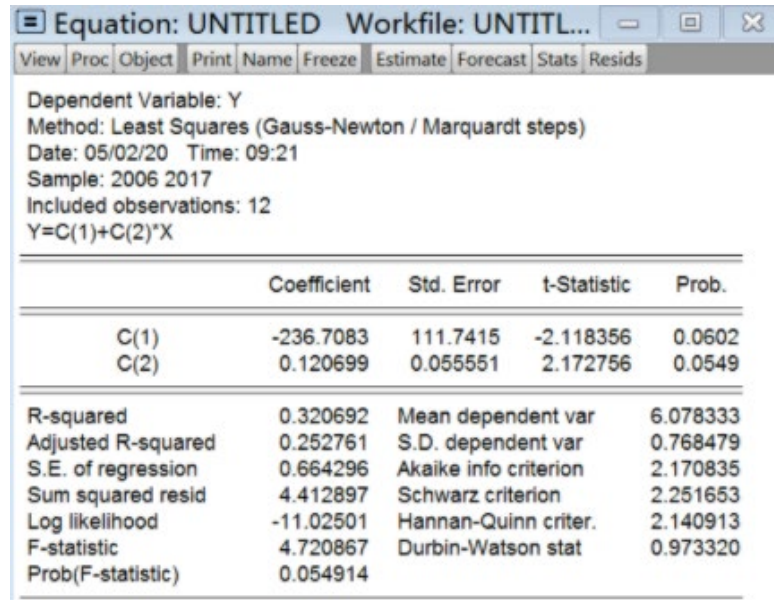

Figure 7. The parameter estimation of the EE proportion linear regression for HLJ.

In Figure 7, the coefficient of determination R^2 equals 0.3207. *F-test* is 4.7209, *t-test* is 2.1728, -2.1184. HLJ's regression model can pass the *F-test* and *t-test*.

Table 5. The fitted value and original data of the EE proportion in HLJ from 2006 to 2017

Year	2006	2007	2008	2009	2010	2011
Fitted Value	5.414	5.535	5.656	5.777	5.897	6.018
Original Data	6.34	6.30	5.36	5.13	5.32	5.22
Residual	0.9255	0.7648	0.295	0.6466	0.5773	0.8000
Year	2012	2013	2014	2015	2016	2017
Fitted Value	6.139	6.259	6.380	6.501	6.625	6.742
Original Data	5.34	6.90	6.37	6.38	7.16	7.12
Residual	5.347	0.6406	0.0101	0.1208	0.5385	0.3778
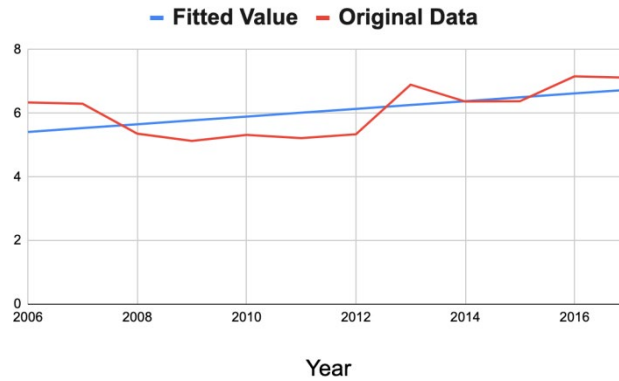
Figure 8. The fitted value and original data of the EE proportion in the HLJ from 2006 to 2017

4.2.2 Ontario's EE regression model

From the data in Table 2, OLS can be used to calculate the linear regression model for Ontario's EE as follows:

$z_2 = 0.0803x - 158.6583$

In this model, the slope is 0.0803, representing the increase of the EE proportion in Ontario. The *y-intercept* is -158.6583. The EE proportion will gradually increase with the increase of time x. The goodness of fit R^2 is 0.6519, the EE proportion in Ontario fitted value and original data are shown in Table 6 and Figure 10.

Figure 9. The parameter estimation of the EE proportion linear regression for Ontario.

In Figure 9, the coefficient of determination R^2 equals 0.6519. *F-test* is 18.7234, *t-test* is 4.3271, -4.2477. Ontario's regression model can pass the *F-test* and *t-test*.

Table 6. The fitted value and original data of the EE proportion in Ontario from 2006 to 2017

Year	2006	2007	2008	2009	2010	2011
Fitted Value	2.523	2.603	2.684	2.764	2.844	2.925
Original Data	2.66	2.26	2.93	2.89	2.59	2.70
Residual	0.1369	0.3434	0.246	0.1259	0.2545	0.2248
Year	2012	2013	2014	2015	2016	2017

Continued table 6

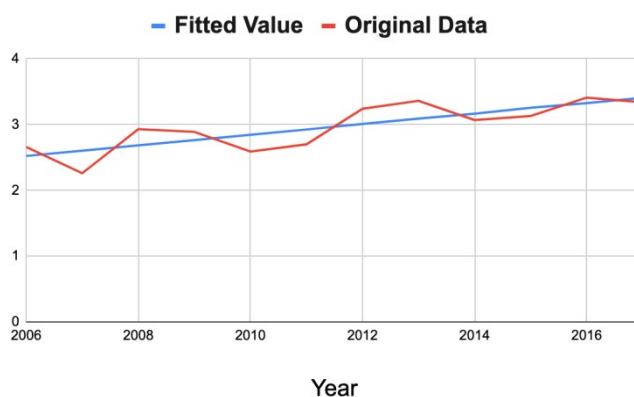| Year	2006	2007	2008	2009	2010	2011
Fitted Value	3.010	3.090	3.166	3.256	3.327	3.407
Original Data	3.24	3.36	3.07	3.13	3.41	3.34
Residual	0.2348	0.2745	0.9587	0.1162	0.8343	0.6692

Figure 10. The fitted value and original data of the EE proportion in Ontario from 2006 to 2017

4.3 The analysis of the ratio of EE to FE in HLJ and Ontario

Table 7. The ratio of EE to FE in HLJ and Ontario from 2006 to 2017

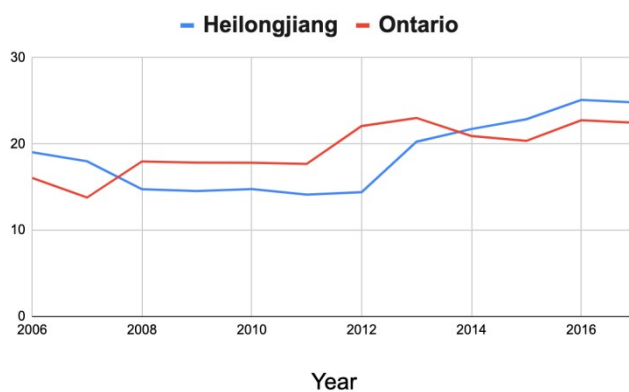| Year	2006	2007	2008	2009	2010	2011
HLJ	19.04	18.00	14.76	14.54	14.78	14.14
Ontario	16.07	13.80	17.98	17.85	17.82	17.69
Year	2012	2013	2014	2015	2016	2017
HLJ	14.42	20.27	21.74	22.87	25.11	24.82
Ontario	22.08	23.02	20.93	20.36	22.75	22.46

Figure 11. The ratio of EE to FE in HLJ and Ontario from 2006 to 2017

Figure 11 shows the ratio curves of EE to FE for HLJ and Ontario from 2006 to 2017. These two curves show an increasing trend with fluctuation. The EE to FE proportion for HLJ decreased from 19.04 to 14.42 from 2006 to 2012 and increased from 20.27 to 24.82 for the period of 2012-2017. From 2008 to 2012, EE as a fraction of total household expenditure in HLJ is higher than that in Ontario. The FE proportion decreasing and the EE proportion increasing caused this increasing trend. The EE proportion in HLJ is higher than Ontario, but because the FE proportion in total expenditure is much higher, from 2008 to 2012, the ratio of EE to FE in HLJ is lower than Ontario. With the diminishing of the FE proportion in total expenditure, from 2013 to 2017, the ratio of EE to FE in HLJ is gradually higher than Ontario.

4.4 Prediction and Comparison

The linear regression model is then used to forecast Engel's coefficient and EE from 2018 to 2027, as shown in Table 8 and 9 and Figure 8 and 9. According to the linear regression model, the future Engel's coefficient and EE from 2018 to 2027 are predicted. The predicted values of Engel's coefficient are shown in Table 8 and Figure 8. The predictions of EE are shown in Table 9 and Figure 9.

4.4.1 The prediction of Engel's coefficient and EE in HLJ and Ontario from 2018 to 2027

Table 8. Engel's coefficient in HLJ and Ontario from 2018 to 2027

| x (Year)	y_1 (HLJ)	Confidence interval (HLJ)	y_2 (Ontario)	Confidence interval (Ontario)
2018	27.96	(21.11, 34.81)	14.34	(13.25, 15.43)
2019	27.18	(20.09, 34.27)	14.16	(13.03, 15.29)
2020	26.39	(19.04, 33.74)	13.99	(12.82, 15.16)
2021	25.61	(17.97, 33.25)	13.81	(12.59, 15.03)
2022	24.82	(16.88, 32.76)	13.64	(12.37, 14.91)
2023	24.03	(15.76, 32.30)	13.46	(12.14, 14.78)
2024	23.25	(14.65, 31.85)	13.29	(11.91, 14.67)
2025	22.46	(13.50, 31.00)	13.11	(11.68, 14.54)
2026	21.68	(12.36, 31.00)	12.94	(11.45, 14.43)
2027	20.89	(11.20, 30.59)	12.76	(11.21, 14.31)
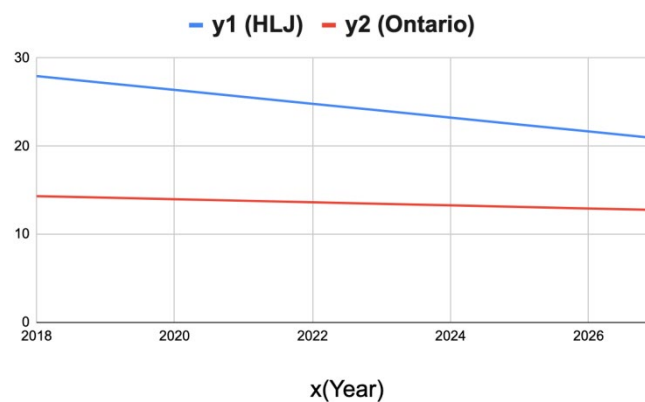
Figure 12. Engel's coefficient in HLJ and Ontario from 2018 to 2027

Figure 12 the trend of Engel's coefficient in HLJ and Ontario. Engel's coefficients in both provinces are forecasted to gradually decrease in the period of 2018 to 2027. Ontario's Engel's coefficient to 12.76%, while HLJ's Engel's coefficient to 20.89%. Engel's coefficient in HLJ is expected to decrease at a greater rate than in Ontario. As time goes on, the difference between the Engel's coefficient of the two provinces will likely become smaller and smaller.

Table 9. The EE proportion in HLJ and Ontario 2018 to 2027

x (Year)	z_1 (HLJ)	Confidence interval (HLJ)	z_2 (Ontario)	Confidence interval (Ontario)
2018	6.85	(5.11, 8.59)	3.49	(2.91, 4.07)
2019	6.97	(5.17, 8.77)	3.57	(2.97, 4.17)
2020	7.09	(5.22, 8.96)	3.65	(3.03, 4.27)
2021	7.21	(5.27, 9.15)	3.73	(3.08, 4.38)
2022	7.33	(5.31, 9.35)	3.81	(3.14, 4.48)
2023	7.45	(5.35, 9.55)	3.89	(3.19, 4.59)
2024	7.57	(5.39, 9.75)	3.97	(3.24, 4.70)
2025	7.69	(5.42, 9.96)	4.05	(3.29, 4.81)
2026	7.81	(5.45, 10.17)	4.13	(3.34, 4.92)
2027	7.93	(5.47, 10.39)	4.21	(3.39, 5.03)
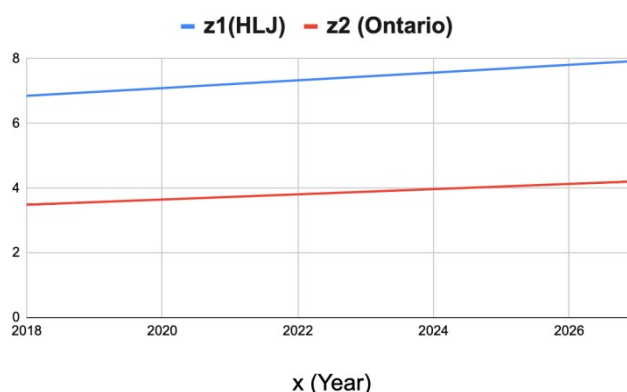
Figure 13. The EE proportion in HLJ and Ontario from 2018 to 2027

Figure 13 the trend of the EE proportion in HLJ and Ontario. In the next decade, the EE in the two provinces are gradually increasing. The EE in HLJ increased faster than in Ontario. As time goes on, the differences between the EE of the two provinces could become smaller and smaller. But in 2027, the EE proportion in HLJ is 7.93%, and the EE proportion in Ontario is 4.21%, so the EE in HLJ is still higher than Ontario's.

4.4.2 Historical data comparison: from 2006 to 2017

(1) Engel's coefficient in HLJ province decreased from 33.3% in 2006 to 28.7% in 2017 with an average annual decrease rate of 4.6%.

(2) It indicates that the standard of living in HLJ's residents is still improving.

(3) The Engel's coefficient in Ontario province decreased from 16.56% in 2006 to 14.84% in 2017 with an average annual decrease rate of 1.72%.

(4) The two provinces both show a decreasing trend in their Engel's coefficients. The decreasing rate in HLJ's is higher than Ontario's, but Ontario's Engel's coefficient is lower than HLJ's Engel's coefficient. The standard of living of Ontario's residents is still higher than HLJ's residents' living level.

(5) The EE proportion in HLJ has increased from 6.34% in 2006 to 7.12% in 2017, and Ontario's EE proportion increased from 2.66% in 2006 to 3.34% in 2017. The increasing trend of the two provinces indicates that residents spend more and more money on education. From 2006 to 2017, the EE proportion in HLJ increased by an average of 0.08% per year. The

EE proportion in Ontario increased by an average of 0.04% per year. Although both provinces have shown an increasing trend, HLJ's EE proportion is still higher than Ontario's.

5 Conclusion

Using data from 2006 to 2017, a linear regression model is used to analyze trends in Engel's coefficient and the EE proportion in HLJ and Ontario, as well as to forecast 2018 to 2027. Key insights from the forecast include:

(1)HLJ's Engel's coefficient shows a decreasing trend, with average annual decline rate of 0.38% during the period while the change of Ontario's Engel's coefficient during the research period is relatively steady, with an average annual decline rate of only 0.14%. Ontario's Engel's coefficient is lower than HLJ's Engel's coefficient in each year.

(2)HLJ's EE shows an increasing trend, with average annual increase rate of 0.08% during the period while the change of Ontario's EE during the research period is relatively steady, with an average annual increase rate of only 0.04%.

(3)The ratio of EE to FE in HLJ shows an increasing trend, with average annual increase rate of 0.53% during the period; the ratio of EE to FE in Ontario shows an increasing trend, with average annual increase rate of 0.58% during the period.

(4)The predicted results and comparison of Engel's coefficient and EE show that HLJ's standard of living is relatively low. In the future, HLJ's Engel's coefficient will reach 20.89% in 2027, while Ontario's Engel's coefficient will reach 12.76% in 2027. The living level of residents in the two provinces is rising, and the differences are getting smaller and smaller.

6 Acknowledgements

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