

# A Study on Calculation Theory and Method of Transverse Distribution Coefficient of the Corrugated Steel Web Box Girder Based on the "Modified Rigid Jointed Beam Method"

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## 1. The Mechanical Model of the Modified Rigid Jointed Beam Method

This paper selects the single-box five-chamber cross section corrugated steel web box girder bridge as the research object to deduce the "modified rigid jointed beam method" of the single-box multi-chamber corrugated steel box girder bridge. For the application of the rigid-connected beam method for the single-box multiple-chamber corrugated steel web box girder, the essential of the modification is to supplement the redundants at the cut to achieve the purpose of restoring the actual force condition of the structure<sup>[1]</sup>. The box girders are cut along the top and bottom boards of the web. Each chamber has two cuts. For the "traditional rigid jointed beam method", the single-box multi-chamber corrugated steel web box girder is equivalent to a flat plate beam model, and only the vertical shear at the cut and the transverse bending moment, other than the transverse force, are considered. In the single-box multi-chamber corrugated steel web box girder, the longitudinal shear force is

further study. method of the transverse load distribution of the steel box girder bridge need further study. Based on the "traditional rigid jointed process and the programming ideas of the transverse distribution coefficient of corrugated steel web composite box girder of the "modified rigid jointed beam method". beam method" and the existing research, this paper proposes the "modified rigid-jointed beam method" in combination with the specific internal force distribution of corrugated steel web composite box girder. The computational scheme and formula of mechanics, the calculation process and the programming ideas of the transverse distribution coefficient of corrugated steel web composite box girder of the "modified rigid jointed beam method".

also ignored<sup>[2]</sup>. The mechanical properties of the single-box multi-chamber corrugated steel web box girder indicate that the top and bottom boards and the webs of the box girder are subjected to shear flow under the eccentric load. After the box girder is cut, the transverse force of the

**Abstract:** The PC box girder Bridge with corrugated steel web, as a new kind of bridge structure, has different mechanical properties from that of the ordinary concrete box girder bridge. Due to the late development of the corrugated steel web pre-stressed box girder structure, the domestic experts have made little research on the transverse load distribution of PC box Girder Bridge with corrugated steel webs. Whether the method of calculating the transverse distribution coefficient in the classical box girder theory can be applied to the corrugated steel web composite box girder and how to further improve the calculation theory and method of the transverse load distribution of the steel box girder bridge need

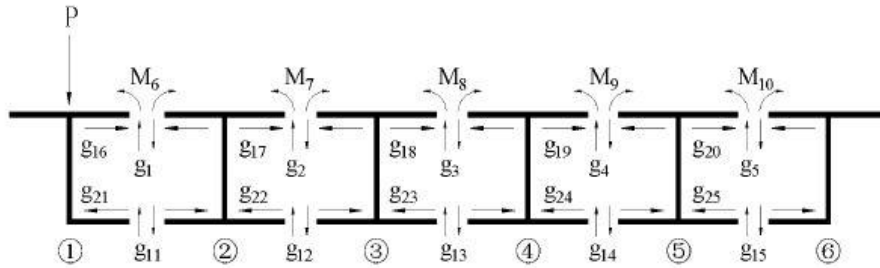
**Key words:** Corrugated steel web composite box girder; Transverse distribution coefficient; Traditional rigid jointed beam method; Modified rigid jointed beam method; Mechanical model

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redundants of the top and bottom boards cannot be ignored due to the existence of the shear flow. In addition, the load is generally applied to the top board of the box girder. Thus, for the redundants, the transverse bending moment is

on the top board, and that on the bottom board is relatively small and can be ignored. In summary, the redundants at the cut of the single-box multi-chamber corrugated steel web box girder bridge according to the "modified rigid jointed beam method", the top

board bears combination of the vertical shear, transverse bending moment and transverse force, the bottom board bears the vertical shear and transverse force, with the mechanical diagram shown in Figure 1.



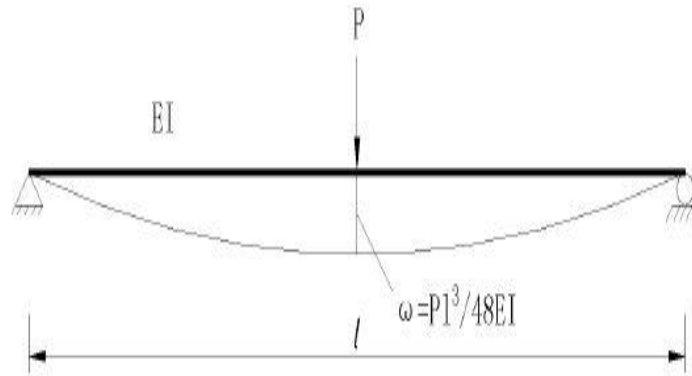
**Figure 1.** Modified Rigid Beam Method

## 2. Half-Wave Sinusoidal Load Equivalent

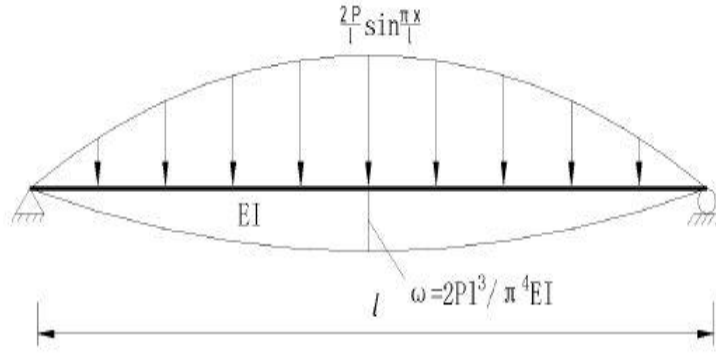
When the No.1 beam midspan is subjected to the concentrated force, the slit generates and distributes the

vertical shear force in the longitudinal direction and distributes the load on the whole bridge structure. In order to facilitate the analysis of the load

distributed on each beam, the concentrated force is equivalent to a continuous distributed load with a sinusoidal distribution<sup>[3-6]</sup>, as shown in Figure 2 and Figure 3.



**Figure 2.** Schematic Diagram of Simply Supported Beam under the Action of  $P=1$



**Figure 3.** Schematic Diagram of Simply Supported Beam under Sinusoidal Load

Setting the deflection expression as:

$$\omega(x) = \sum_{n=1}^{\infty} b_n \sin \frac{n\pi x}{l} \quad (1)$$

It satisfies the boundary condition of the endpoint

$$X=0, l, w=0$$

$$\frac{d^4 \omega(x)}{dx^4} = \sum_{n=1}^{\infty} b_n \left(\frac{n\pi}{l}\right)^4 \sin \frac{n\pi x}{l} \quad (2)$$

The equilibrium differential equation is:

$$EI \frac{d^4 \omega(x)}{dx^4} = p(x) \quad (3)$$

Substituting the formulas (1-8) and  $p(x) = \sin \frac{\pi x}{l}$  into (3-9):

$$EI \sum_{n=1}^{\infty} b_n \left(\frac{n\pi}{l}\right)^4 \sin \frac{n\pi x}{l} = p(x) = \sin \frac{\pi x}{l} \quad (4)$$

The above formula should be treated as below, both should be multiplied by  $\sin \frac{m\pi x}{l}$ , and interval integrals between 0 to l are:

$$EI b_1 \left(\frac{\pi}{l}\right)^4 \frac{l}{2} = \frac{l}{2}, \quad m = n = 1 \quad (5)$$

As a result:

$$b_1 = \left(\frac{l}{\pi}\right)^4 \frac{1}{EI}, \quad m = n = 1 \quad (6)$$

$$EI b_n \left(\frac{n\pi}{l}\right)^4 \frac{l}{2} = 0, \quad m = n \neq 1 \quad (7)$$

As a result:

$$b_n = 0, \quad m = n \neq 1 \quad (8)$$

$$\omega(x) = \frac{2P}{EI} \left(\frac{l}{\pi}\right)^4 \sin \frac{\pi x}{l} = \frac{2Pl^3}{\pi^4 EI} \quad (9)$$

The rationality of the sinusoidal load equivalent:

1) The midspan deflection is approximately equal:

$$\frac{Pl^3}{48EI} \approx \frac{2Pl^3}{\pi^4 EI} \quad (10)$$

2) The internal forces  $M(x)$  and  $Q(x)$  generated by the sinusoidal load satisfy the simply supported beam boundary condition.

### 2.1 Derivation of the Transverse Load Distribution Influence Line

Since the load is sinusoidal distributed, the shear and bending moments of the beams are sinusoidal distributed. Thus, the transverse distribution of the unit sinusoidal load can be simply expressed by the values of the load and redundants in the center of the bridge span, that is, the peak value of the sinusoidal load:

No. 1 beam transverse load distribution influence line vertical coordinate:

$$\left. \begin{aligned} \eta_{11} &= 1 - g_1 - g_{11} \\ \eta_{21} &= g_1 + g_{11} - g_2 - g_{12} \\ \eta_{31} &= g_2 + g_{12} - g_3 - g_{13} \\ \eta_{41} &= g_3 + g_{13} - g_4 - g_{14} \\ \eta_{51} &= g_4 + g_{14} - g_5 - g_{15} \\ \eta_{61} &= g_5 + g_{15} \end{aligned} \right\} \quad (11)$$

$\eta_{11} - \eta_{61}$  is the vertical coordinate of the transverse load

distribution influence line of No. 1 beam under the 1-6 beam axis, and  $\eta_{ij}$  is the vertical coordinate of the

load horizontal distribution influence line of No. 1 beam, so

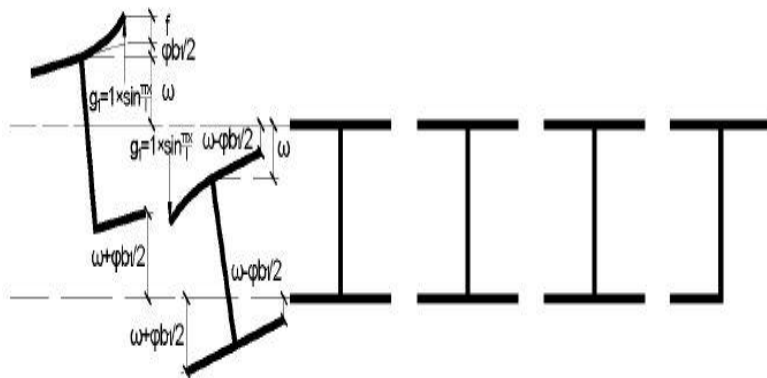
according to the reciprocal theorem of work:  $\eta_{ij} = \eta_{ji}$ . 25 canonical equations can be listed as follows according to the basic structure of Figure 2 above:

$$\left. \begin{aligned} \delta_{1,1}g_1 + \delta_{1,2}g_2 + \dots + \delta_{1,24}g_{24} + \delta_{1,25}g_{25} + \delta_{1,p} &= 0 \\ \delta_{2,1}g_1 + \delta_{2,2}g_2 + \dots + \delta_{2,24}g_{24} + \delta_{2,25}g_{25} + \delta_{2,p} &= 0 \\ \dots & \\ \delta_{24,1}g_1 + \delta_{24,2}g_2 + \dots + \delta_{24,24}g_{24} + \delta_{24,25}g_{25} + \delta_{24,p} &= 0 \\ \delta_{25,1}g_1 + \delta_{25,2}g_2 + \dots + \delta_{25,24}g_{24} + \delta_{25,25}g_{25} + \delta_{25,p} &= 0 \end{aligned} \right\} (12)$$

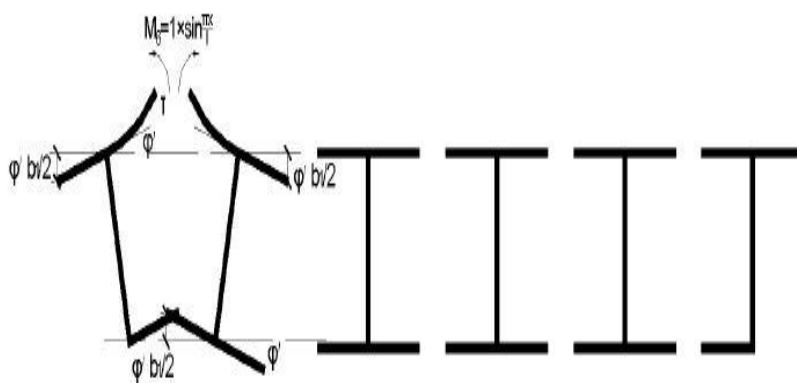
In the above formula:

$\delta_{(i, k)}$ -vertical relative displacement at the cut  $i$  caused by the effect unit sinusoidal load at the cut  $k$

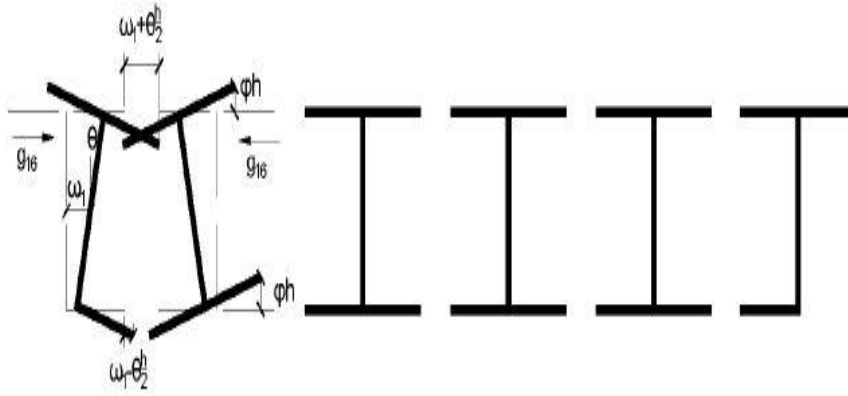
$\delta_{(i, p)}$ -vertical displacement at the cut  $i$  caused by the external load  $P$ .



a)



b)



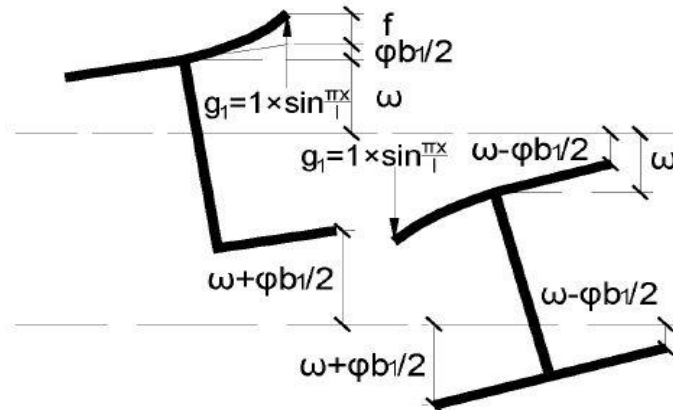
c)

**Figure 4.** Modified Rigid Jointed Beam Method for the Cut Point Displacement

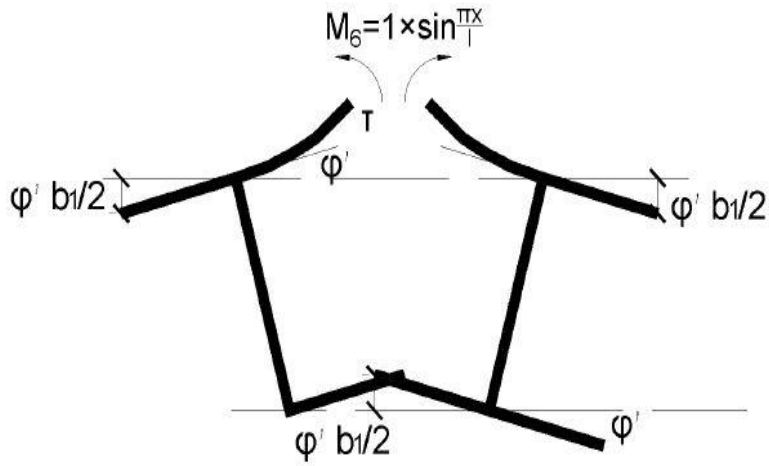
In Figure 4, a) represents the bridge span central displacement along the direction; Figure b) represents the displacement in the direction of  $M_6$  ; and figure c) shows the

displacement along the direction of  $g_{16}$  ; the displacement along each direction generated by the redundants is similar to that being shown in the figure above, only the

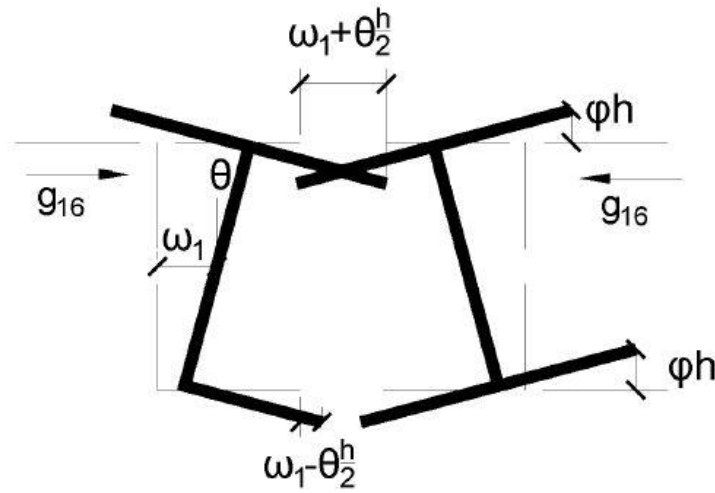
subscript changes. Hereby the figure is not necessary; Figure 5, Figure 6 and Figure 7 are the enlarged view of partial displacements generated along  $g_1$ ,  $M_6$  and  $g_{16}$  respectively.



**Figure 5.** Large Displacement  $g_1$  Partial Discharge along the Redundants



**Figure 6.** Large Displacement  $M_6$  Partial Discharge along the Redundants



**Figure 7.** Large Displacement  $g_{16}$  Partial Discharge along the Redundants

The coefficient involving only the vertical redundants is subject to the following rules:

$$\delta_{1,1} = \delta_{2,2} = \delta_{3,3} = \delta_{4,4} = \delta_{5,5} = \delta_{11,11} = \delta_{12,12} \\ = \delta_{13,13} = \delta_{14,14} = \delta_{15,15} = 2 \left( \omega + \varphi \frac{b}{2} + f \right) \quad (13)$$

$$\delta_{1,2} = \delta_{2,3} = \delta_{3,4} = \delta_{4,5} = \delta_{5,5} = \delta_{11,12} = \delta_{12,13} \\ = \delta_{13,14} = \delta_{14,15} = \delta_{15,15} = - \left( \omega - \varphi \frac{b}{2} \right) \quad (14)$$

$$\delta_{6,2} = \delta_{7,3} = \delta_{8,4} = \delta_{4,5} = \delta_{9,5} = \delta_{10,4} = \delta_{6,12} \\ = \delta_{7,13} = \delta_{8,14} = \delta_{9,15} = \delta_{10,14} = -\varphi \quad (15)$$

Where  $\omega$  is the vertical deflection of the main beam,  $\frac{\varphi b_1}{2}$  is the

displacement caused by the twist angle  $\varphi$ ,  $f$  is the deflection of the flange plate, and the negative sign indicates the opposite direction of  $\delta$

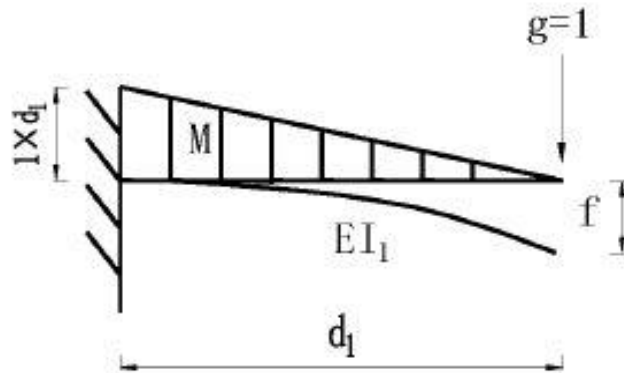
and  $g$ ; he girder span central deflection  $\omega$  and the twist angle generated by the unit sine eccentric load  $\varphi$  are as follows:

$$\varphi = \frac{b_1 l^2}{2\pi^2 G I_T} \quad \left. \begin{aligned} \omega &= \frac{l^4}{\pi^4 E I} \\ (16) \end{aligned} \right\} \quad f = \frac{d_1^3}{3E I_1}$$

Where E and G are the modulus of elasticity and shear of the material respectively; I-beam cross section vertical bending resistance inertia moment;  $I_1$  -The transverse bending resistance inertia moment of the unit long flange plate;  $I_2$  -beam cross section transverse

bending resistance inertia moment;  $I, b_1, d_1$  -the torsion inertia moment of the beam;  $I_T$  -are the bridge span length, the widths of the beam cross section and the suspension plate. The deflection at the cantilever end of the main beam flange plate under the sinusoidal load is also the sinusoidal curve

$f(x) = f * \sin(\frac{\pi x}{l})$  midspan peak value, which equals to the deflection of the upper flange plate with unit width under the action unit load. The deflection of the upper flange plate is calculated by the bending moment from the plate edge to the flange as shown in Figure 8:



**Figure 8.** The Deformation of Beams under the Action of Unit Load

The coefficient involving only the excess bending moment is subject to the following rules:

$$\left. \begin{aligned} \delta_{6,6} = \delta_{7,7} = \delta_{8,8} = \delta_{9,9} = \delta_{10,10} &= 2 (\varphi' + \tau) \\ \delta_{1,7} = \delta_{2,8} = \delta_{3,9} = \delta_{4,10} &= \varphi' \frac{b_1}{2} \\ \delta_{6,7} = \delta_{7,8} = \delta_{8,9} = \delta_{9,10} &= \varphi' \end{aligned} \right\} (17)$$

The coefficient involving only the transverse redundants is subject to the following rules:

$$\left. \begin{aligned} \delta_{16,16} = \delta_{17,17} = \delta_{18,18} = \delta_{19,19} = \delta_{20,20} = \delta_{21,21} \\ = \delta_{22,22} = \delta_{23,23} = \delta_{24,24} = \delta_{25,25} &= 2 \left( \omega_1 + \theta \frac{h}{2} \right) \end{aligned} \right\} (18)$$

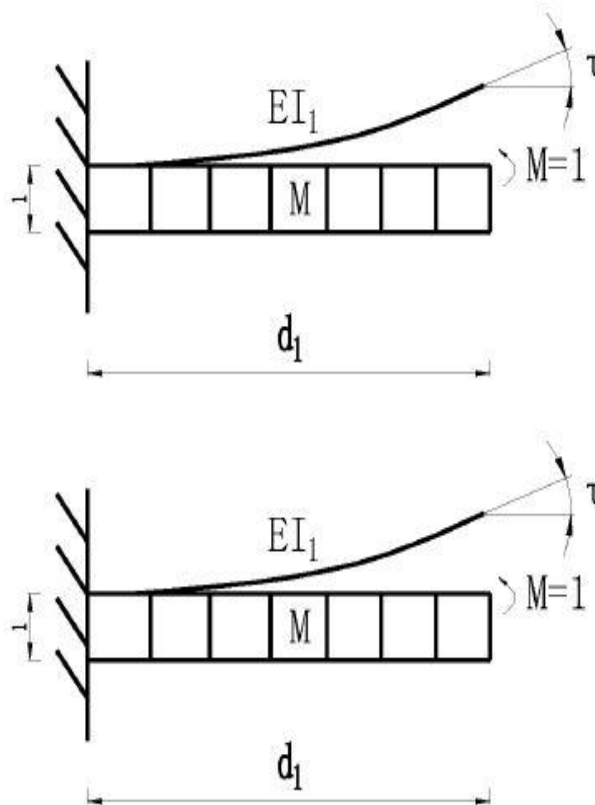
$$\left. \begin{aligned} \delta_{1,17} = \delta_{2,18} = \delta_{3,19} = \delta_{4,20} = \delta_{1,22} = \delta_{2,23} = \delta_{3,24} \\ = \delta_{11,17} = \delta_{12,18} = \delta_{13,19} + \delta_{14,20} + \dots &= -\theta \frac{b_1}{2} \end{aligned} \right\} (19)$$

$$\begin{aligned}
 &\delta_{16,17} = \delta_{17,18} = \delta_{18,19} = \delta_{19,20} = \delta_{16,22} = \delta_{17,23} \\
 &= \delta_{18,24} = \delta_{19,25} = \delta_{25,19} = \delta_{21,17} = \delta_{22,18} = \delta_{23,19} \\
 &= \delta_{24,20} = \delta_{21,22} = \delta_{22,23} = \delta_{23,24} = \delta_{24,25} \\
 &= \delta_{25,24} = -\left(\omega_1 + \theta \frac{h}{2}\right)
 \end{aligned}
 \tag{20}$$

Being caused by the unit transverse sine bending moment, the midspan twist angle of the beam is  $\varphi'$ , and

the deflection of the suspension plate is  $\tau$ . Under the action of the transverse unit sinusoidal load, the midspan transverse deflection is

$\omega_1$  and the twist angle is  $\theta$ . Wherein  $\tau$  of the I-beam with the flange cantilever  $d_1$  is calculated as shown in Figure 9.



**Figure 9.** Deflection of the Beam under the Bending Moment of 1 Units

It can be obtained according to the material and the structural mechanic formulas:

$$\begin{aligned}
 \tau &= \frac{1 \cdot d_1}{EI_1} \\
 \varphi' &= \varphi \frac{1}{b_1/2} = \frac{2\varphi}{b_1} \\
 \theta &= \frac{hl^2}{2\pi^2GI_T} \\
 \omega_1 &= \frac{l^4}{\pi^4EI_2}
 \end{aligned}
 \tag{21}$$



When the unit sinusoidal load acts on the No. 1 beam, the load factor can be obtained (when acting on other beams, it can be treated similarly):

$$\left. \begin{aligned} \delta_{1,p} &= \delta_{11,p} - \omega \\ \delta_{2,p} &= \delta_{3,p} = \dots = \delta_{10,p} = 0 \\ \delta_{12,p} &= \delta_{13,p} = \dots = \delta_{25,p} = 0 \end{aligned} \right\} \quad (22)$$

In addition, it can write:

$$\begin{aligned} \Delta_{1,6} &= \delta_{2,7} = \delta_{3,8} = \delta_{4,9} = \delta_{5,10} = \delta_{1,16} = \delta_{2,17} = \delta_{3,18} = \delta_{4,19} = \\ \delta_{5,20} & \\ &= \delta_{1,21} = \delta_{2,22} = \delta_{3,23} = \delta_{4,24} = \delta_{5,25} = \delta_{1,3} = \delta_{1,4} = \delta_{1,5} = \delta_{1,8} = \\ \delta_{1,9} & \\ &= \delta_{1,10} = \delta_{1,13} = \delta_{1,14} = \delta_{1,15} = \delta_{1,18} = \delta_{1,19} = \delta_{1,20} = \delta_{1,23} = \\ \delta_{1,24} & \\ &= \delta_{1,25} = \delta_{2,4} = \delta_{2,5} = \delta_{2,7} = \delta_{2,9} = \delta_{2,10} = \delta_{2,14} = \delta_{2,15} = \delta_{2,17} = \\ \delta_{2,19} & \\ &= \delta_{2,20} = \delta_{2,22} = \delta_{2,24} = \delta_{2,25} = \delta_{3,8} = \delta_{3,10} = \delta_{3,11} = \delta_{3,13} = \delta_{3,15} = \\ \delta_{3,16} & \\ &= \delta_{3,18} = \delta_{3,20} = \delta_{3,21} = \delta_{3,23} = \delta_{3,25} = \delta_{4,6} = \delta_{4,7} = \delta_{4,9} = \delta_{4,11} = \\ \delta_{4,12} & \\ &= \delta_{4,14} = \delta_{4,16} = \delta_{4,17} = \delta_{4,19} = \delta_{4,21} = \delta_{4,22} = \delta_{4,24} = \delta_{5,6} = \delta_{5,7} = \\ \delta_{5,8} &= \delta_{5,10} \\ &= \delta_{5,11} = \delta_{5,12} = \delta_{5,13} = \delta_{5,15} = \delta_{5,17} = \delta_{5,18} = \delta_{5,20} = \delta_{5,21} = \delta_{5,22} = \\ \delta_{5,25} & \\ &= \delta_{6,8} = \delta_{6,9} = \delta_{6,10} = \delta_{6,13} = \delta_{6,15} = \delta_{6,18} = \delta_{6,19} = \delta_{6,20} = \delta_{6,23} = \\ \delta_{6,24} & \\ &= \delta_{6,25} = \delta_{7,9} = \delta_{7,10} = \delta_{7,14} = \delta_{7,15} = \delta_{7,19} = \delta_{7,20} = \delta_{7,24} = \delta_{7,25} = \Delta_{8,10} = \delta_{8,11} \\ &= \delta_{8,15} = \delta_{8,16} = \delta_{8,20} = \delta_{8,21} = \delta_{8,25} = \delta_{9,11} = \delta_{9,12} = \delta_{9,16} = \delta_{9,17} = \\ \delta_{9,21} & \\ &= \delta_{9,22} = \delta_{10,11} = \delta_{10,12} = \delta_{10,13} = \delta_{10,16} = \delta_{10,17} = \delta_{10,18} = \delta_{10,21} = \\ \delta_{10,22} & \\ &= \delta_{10,23} = \delta_{11,13} = \delta_{11,14} = \delta_{11,15} = \delta_{11,18} = \delta_{11,19} = \delta_{11,20} = \delta_{11,21} = \\ \delta_{11,23} & \\ &= \delta_{11,24} = \delta_{11,25} = \delta_{12,14} = \delta_{12,25} = \delta_{13,15} = \delta_{13,16} = \delta_{13,20} \\ & \\ &= \delta_{13,21} = \delta_{13,23} = \delta_{13,25} = \delta_{14,16} = \delta_{14,17} = \delta_{14,21} = \delta_{14,22} = \delta_{14,24} = \delta_{15,16} \end{aligned}$$

$$= \delta_{15,18} = \delta_{15,21} = \delta_{15,22} = \delta_{15,23} = \delta_{15,25} = \delta_{16,18} = \delta_{16,19} = \delta_{16,20} = \delta_{16,23}$$

$$= \delta_{16,24} = \delta_{16,25} = \delta_{17,19} = \delta_{17,20} = \delta_{17,24} = \delta_{17,25} = \delta_{18,20} = \delta_{18,21} = \delta_{18,25}$$

$$= \delta_{19,21} = \delta_{19,22} = \delta_{20,21} = \delta_{20,22} = \delta_{20,23} = \delta_{21,23} = \delta_{21,24} = \delta_{21,25} = \delta_{22,24} = \delta_{22,25} = \delta_{23,25} = 0$$

(23)

The remaining coefficients can be obtained according to the reciprocity theorem of the work. There is no more detailed description.

Hereby, the coefficient  $\delta_{ij}$  has been fully calculated. All the redundants are solved after substituting the coefficient into the canonical equation and the solving polynomial matrix equation. The transverse load distribution influence line is calculated through

the formula. To solve the rigid jointed beam method canonical equation,  $\delta_{i,p}$  can be moved to the right of the equation, and both sides of the equation are divided by the displacement  $\omega$ .

### 3. Modified Rigid Jointed Beam Method Calculation Procedures and Essentials

#### 3.1 Classification of Longitudinal Beam Elements of the Corrugated Steel Web Box Girder

The treatment of corrugated steel webs applied by the "modified rigid jointed beam method" is the same with that of the traditional "rigid jointed beam method". That is, with the conversion conditions that the total shear force and the shear strain are the same, the cross section of the corrugated steel web is converted to that of the equivalent concrete with corresponding thickness as shown in Figure 10.

$$t_e = \frac{G_e t}{G_c} = n_0 t \quad (24)$$

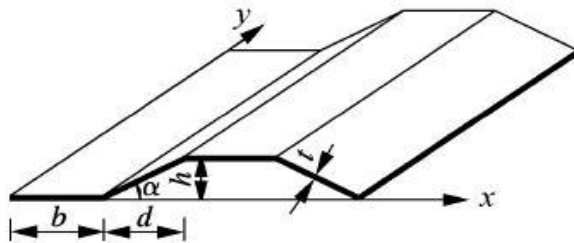


Figure 10. Structural Shape of Corrugated Steel Webs

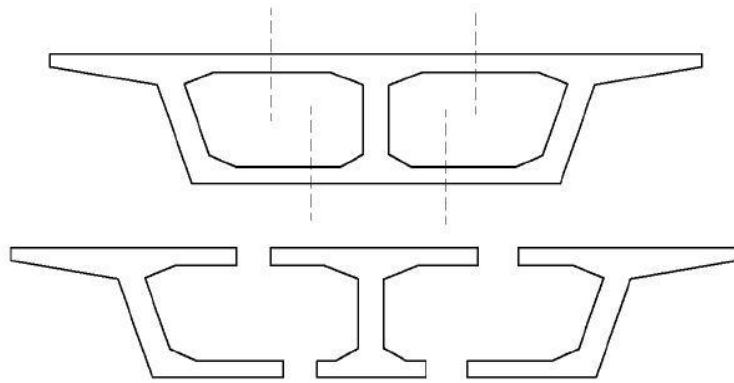
Among them:

$n_0 = \frac{G_e}{G_c}$  is the ratio between the corrugated steel web equivalent shear modulus and the concrete shear modulus;

$G_e = \frac{b+d}{b+d \sec \alpha} G_0$ ,  $G_0$  is the steel shear modulus, and the meanings of  $b$ ,  $d$ ,  $\alpha$  are shown in Figure 10.

After the equivalent treatment of the corrugated steel web box girder bridge as described above, the original corrugated steel box girder bridge is equivalent to the corresponding concrete web box girder bridge. The single box girder is taken as an example to illustrate. As shown in Figure 11 (equivalent conversion of the box girder), the equivalently converted

box girder of the single-box double-chamber cross section is cut into the structure composed by the I-beam. The centroids of each I-beam are on the main axis of the superstructure. In this case, one-third of the top board and one-third of the bottom board are reckoned in for each beam. The inertia moment of each beam is one-third of the total inertia moment of the upper structure<sup>[7]</sup>.

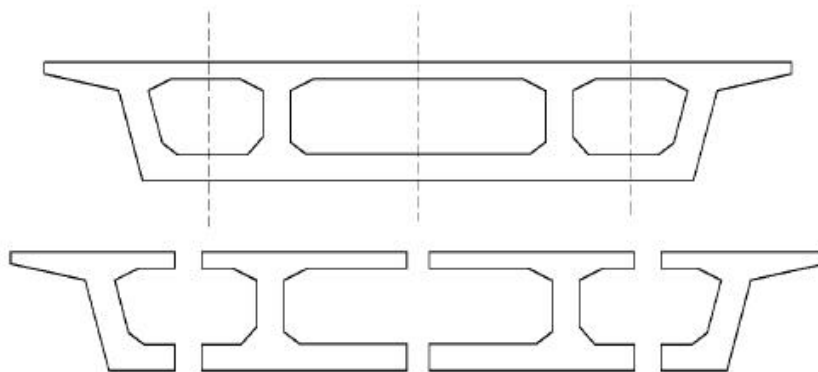


**Figure 11.** Single box and double chamber section

For the superstructure of the single-box multi-chamber cross section, it is impossible to section in accordance with Figure 10 generally. Thus, the box girder can only be cut along the top and bottom boards in the middle of the

two webs as shown in Figure 12. Thus, the centroids of the center beam coincide with the main axis of the upper structure, and the centroids of both sides of the I-beam are on the horizontal line near the top board, but their inertia

moments are calculated according to the overall structure of the spindle. Thus, the vertical I-beam of the "modified rigid jointed beam method" proposed by the paper is classified according to the method indicated by Figure 11.



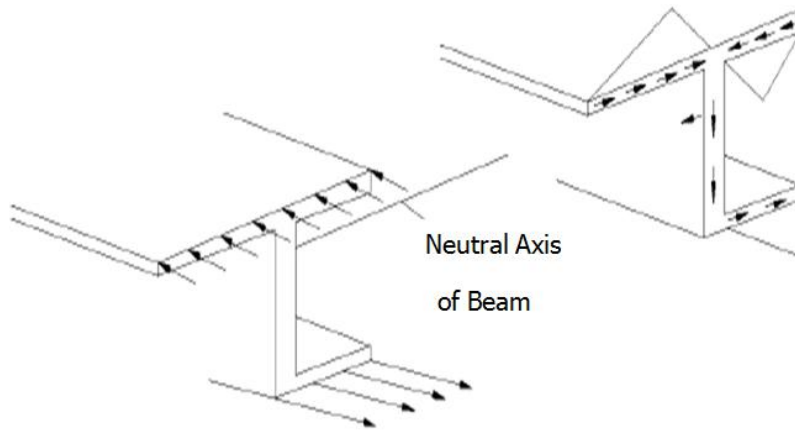
**Figure 12.** Multi-box Section

### 3.2 Determination of Cross Section Characteristics

#### 3.2.1 Bending rigidity

As shown in Figure 12, the equivalent box girder structure along the top and bottom boards are cut into the structure composed

by the I-beam. The bending stress distribution is shown in Figure 13, which is similar to the result of the simply supported beam theory.



**Figure 13.** Normal Stress and Shear Stress Generated by the Bending of the Side Beam Around the Self-neutral Axis

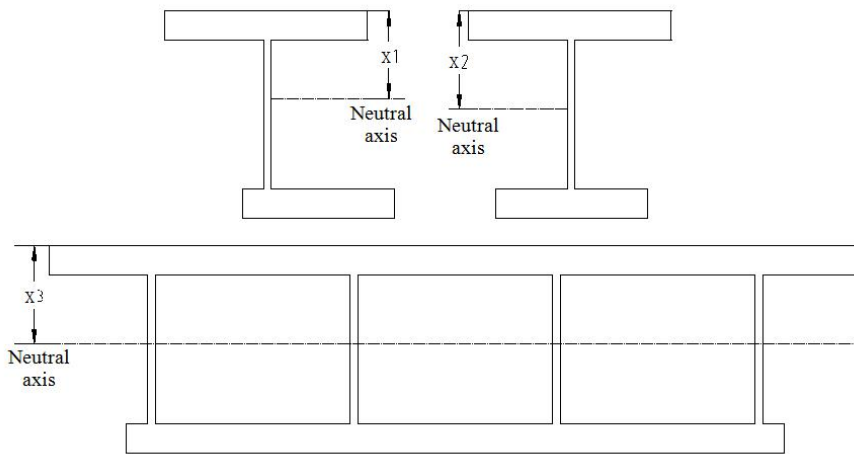
$$\frac{\delta}{z} = \frac{M}{I} = \frac{E}{R} \quad \left. \vphantom{\frac{\delta}{z}} \right\} \quad (25)$$

$$\tau = \frac{Q_M A z}{I}$$

Vertical Shear Stress Produced by Bending of I - beam:

$$Q_M = \frac{dM}{dx} \quad (26)$$

$Q_M$  is part of the web shearing force, and the web has the shear component due to the torsion  $Q_T$ .



**Figure 14.** Schematic Diagram of the Position of the Neutral Axis of the Beam

In Figure 14, the neutral axis of the I-beam will not be on the same horizontal line, and  $x_1$ ,  $x_2$  and  $x_3$  are not equal, which is inconsistent with the situation of the actual structure. The actual engineering

structure, the corrugated steel web box girder is bent under the action of the load and should be bent around the same neutral axis. Thus, the cross section characteristic of each I-beam represented by the stringer element should be

calculated according to the overall superstructure. Though the suspension plate may be larger, and the thickness of the edge and the middle webs is different, it is proper to cut the superstructure along the longitudinal direction.

When the suspension plate is large or the flange is wide, consider the effective width of the cross section.

In summary, the bending stiffness of each beam cross section is:  $EI$

$E$ -the elastic modulus.

$I$ -the inertia moment applied by the cross section of each beam on the neutral axis of the box girder.

### 3.2.2 Torsional rigidity

In this chapter, the effect of the cross section distortion is not taken into account for the value choosing for the box girder torsional rigidity. When the box girder is twisted as a whole, the shear stress flows in the cross section of the structure as being concluded based on the

previous analysis result. As shown in Figure 15 and 16, most of the shear stress flows around the top and bottom boards and webs on both sides, and a small amount flows through the middle web. The shear stress flow of the inner web of the multi-chamber is relatively small and can be ignored as being compared with that of webs on both sides. The inner web stress flow from both sides to the middle of the web is gradually reduced. The middle inner web stress flow can be regarded as zero<sup>[8]</sup>. It can be concluded that it is not necessary to consider the inner web for the torsional stiffness of the corrugated steel web box girder under the action of the torsional load. After cutting the box girder between the

webs, the total torque in the box girder is composed of the longitudinal beam element torque and its shear force  $Q_r$ , where the longitudinal beam element torque represents the torque formed in the upper structure due to the opposite shear stress in the top and bottom boards, while the shear force  $Q_T$  represents the stress flow in the web,<sup>[9-12]</sup> thus, the torsional rigidity of the longitudinal beam element is:

$$GJ_x$$

$G$ -the shear modulus

$J_x$  -(the inertia moment of the top and bottom boards flanges represented by the longitudinal beam element on the central axis)

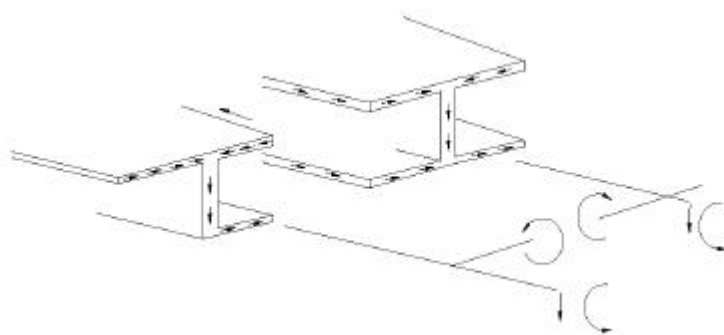
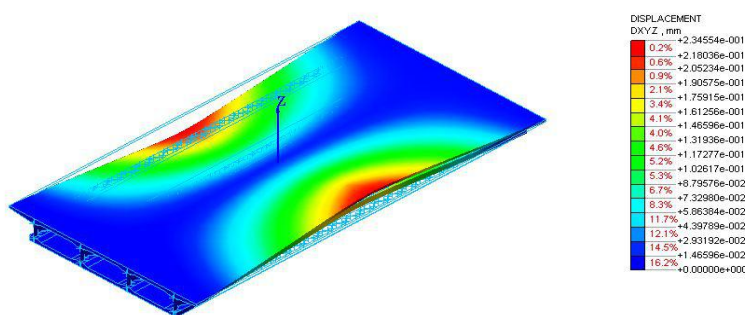
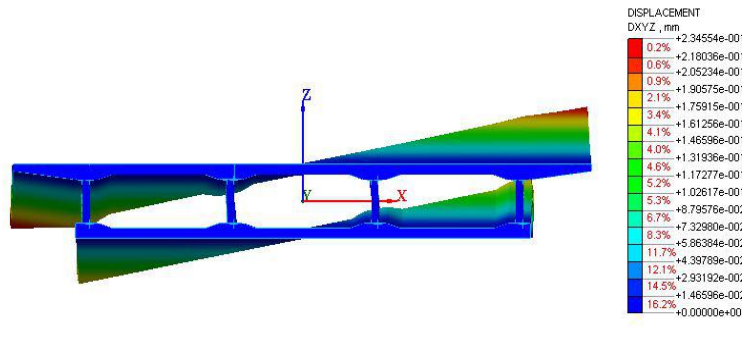


Figure 15. Shear Distribution of Cross Section under Torsion



(a) axis mapping



(b) positive view

Figure 16. Vertical displacement cloud after torsional loading

### 3.3 Calculation Procedures of the "Modified Rigid Jointed Beam Method" Transverse Distribution Influence Lines

The treatment of corrugated steel webs applied by the "modified rigid jointed beam method" is the same with that being indicated above. That is, the corrugated steel web is equivalent to the concrete slab with corresponding thickness. Then it is calculated according to the method described in the paper. [13-15]

In order to reflect calculation procedures and rules of the "modified rigid jointed beam method" visually, this section takes the single-box three-chamber box girder for example to specifically describe the transverse distribution coefficient calculation method of the single-box multi-chamber corrugated steel web box girder of the "modified rigid jointed beam method".

For the transverse load distribution of the box girder calculated by the "modified rigid jointed beam method", the calculated span is 20m and the bridge width is 16m.

The calculation of the transverse distribution influence line of the "modified rigid jointed beam method":

- (1) The thickness of the equivalent concrete web:

$$t_e = \frac{G_e t}{G_c} = n_0 t = 15 \text{ (mm)} \quad (27)$$

After equivalent conversion, the concrete web box girder with the corresponding thickness is obtained

with equivalent stiffness, as shown in Figure 17.

- (2) According to key points of the longitudinal beam element

classification, the single-box three-chamber cross section box girder is cut along the web into the structure composed by the I-beam, as shown in Figure 18.

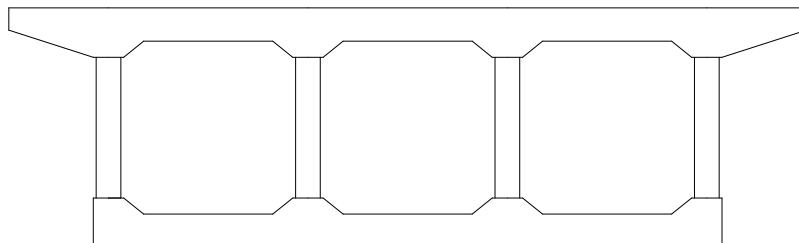
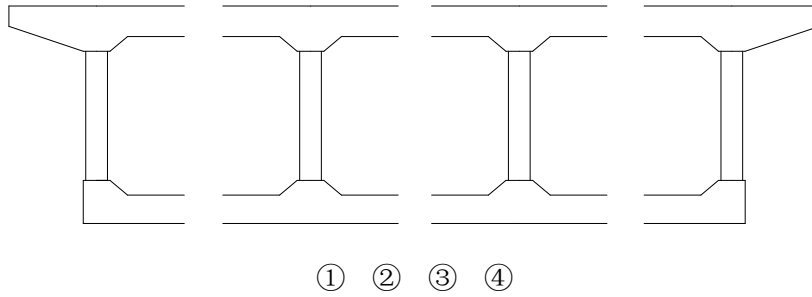
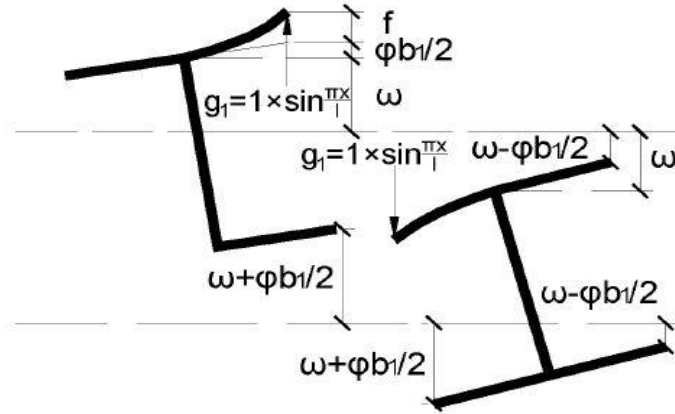


Figure 17. Transverse Section of Box Girder



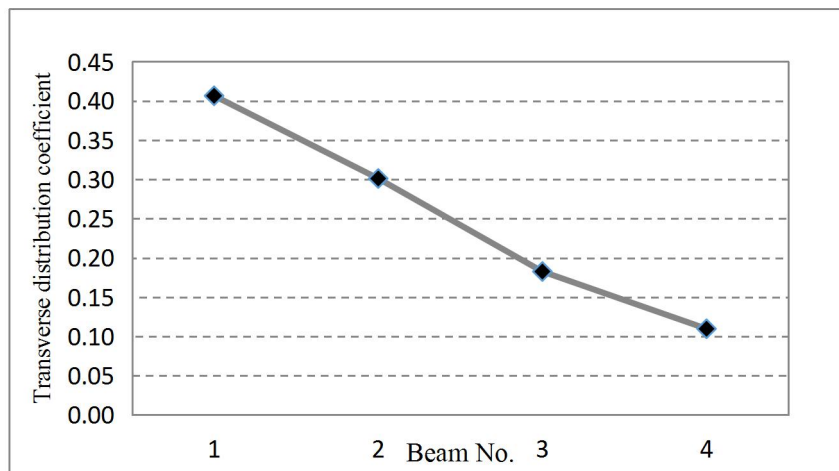
**Figure 18.** Single Box and Three Section Box Girder Section Cut



**Figure 19.** Large displacement  $g_1$  partial discharge along the redundants

When  $q(x) = \frac{2P}{l} \sin \frac{\pi x}{l}$  is on top of the No. ① beam shown in Figure

19, the transverse distribution coefficient is calculated according to the formula (11):



**Figure 20.** Influence Line of Transverse Distribution of No. ① Beam Load

No. ① beam:



$$\left. \begin{aligned}
 \eta_{11} &= 1 - g_1 - g_{11} = 0.41 \\
 \eta_{21} &= g_1 + g_{11} - g_2 - g_{12} = 0.30 \\
 \eta_{31} &= g_2 + g_{12} - g_3 - g_{13} = 0.18 \\
 \eta_{41} &= g_3 + g_{13} = 0.11
 \end{aligned} \right\} \quad (28)$$

Similarly, transverse distribution coefficient of the load acted on tops on beams ②, ③ and ④ can be calculated and there is no need to be repeated hereby. The above are the calculation procedures using the "modified rigid jointed beam method" to obtain the transverse distribution coefficient of the corrugated steel web box girder.

#### 4. The "Modified Rigid Jointed Beam Method" Calculation Programming

After the chambers of the multi-chamber corrugated steel web box girder are added, the number of  $\delta_{i,j}$  coefficients in the canonical equation increases orderly. If it is still calculated manually, the cumbersome calculation process will greatly increase the probability of workload and error. In order to improve the situation, the paper compiles the solver of the redundants of the "modified rigid jointed beam method" through the simulation software Matlab.<sup>[16-19]</sup>

Matlab is a powerful and computationally efficient mathematical tool developed by the American company MathWorks in C language, which is called Matrix Laboratory. At the beginning, Matlab can only carry on the matrix computation. Due to the enhancement of computer and programming technologies, Matlab's function has also been perfected constantly, with the addition of functions including the numerical computation, the symbol operation as well as the graphic processing, etc. Matlab is widely

used, covering almost all areas of scientific and engineering computing, such as automatic control, vibration theory, optimization design, timing analysis and modeling, neural network control, digital signal processing, chemical statistics, dynamic simulation system, special functions, etc. Matlab, in addition to its powerful computing functions, is an advanced computer language, which can be used to do programming. Matlab computer language is called "The Language of the Mathematic Version". Compared with advanced computer languages, including Fortran and C language, etc., Matlab, with simpler grammar rules and being closer to the general way of thinking, liberates programmers, especially non-professional origin programmers from writing cumbersome codes. Using Matlab to write programs is as intuitional as deriving a formula on the paper. Thus, it is also known as "Draft Paper" scientific engineering algorithm language. It follows that Matlab language is simple, natural and easy to learn and use.

Being a plain text file and called as M file with the expanded name .m, Matlab program file can be divided into scripting and functional types, which can be used to edit with any word processing software. Matlab itself is an interpretation system. The implementation of the function program is in a way explanation. The program can run directly without being compiled, and report and analyze the cause of the error occurred in time. Thus, the

program debugging is easy with high programming efficiency.

In addition, Matlab has developed a powerful set of modules or toolboxes for many specialized fields and users can use these toolboxes directly without having to write their own code. At present, Matlab has expanded the scope of the toolboxes to many fields of the scientific research and engineering applications. In order to make full use of various resources, Matlab has developed interfaces with other practical programs, which makes it easy to dock with Fortran, C and other languages to achieve mixed programming.<sup>[20-22]</sup>

The solver of the "rigid jointed beam method".

The basic solution unit of Matlab is a matrix whose instruction expressions are very close to that of mathematics and engineering. For example, the matrix  $BX=A$ , expressed as  $B*X=A$  in the Matlab software.  $X$  is solved by the expression  $X=B/A$ . There is no need to program the matrix multiplication and the inversion. Thus, it is simpler to use Matlab to solve the matrix than using languages including C and Fortran, etc. In the process of solving the redundants, moving to the right side of the equation to solve the following matrix  $\delta_{i,p}[\delta_{i,p}][X_i] = [\delta_{i,p}]$ .<sup>[23]</sup>

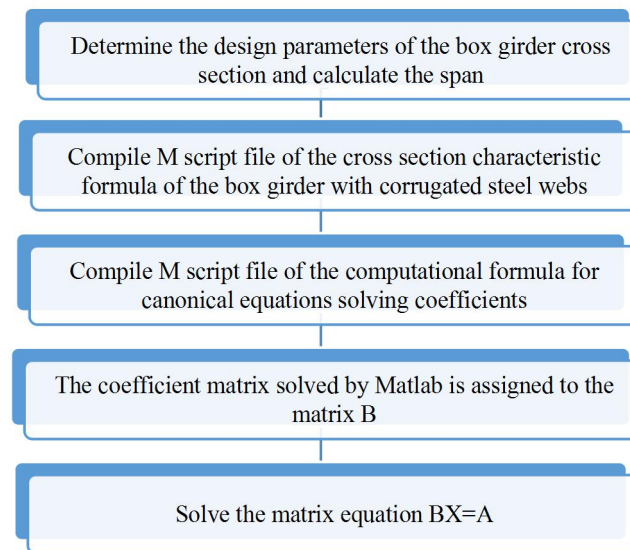
For the solution of the matrix,  $\delta_{i,p}$  will be moved to the right of the equation and then the equation is divided by the displacement  $\omega$  of both sides. In this way, all the coefficients are only related to the



cross section parameters and the calculation span of the box girder

during the solving process, with no relations to elastic model E. The

following figure is a flow chart that uses the solver to solve the matrix.



**Figure 21.** "The Modified Rigid Beam Method" for Solving the Redundants Program Flow Chart

For the Figure 21 of solving the redundants program flow chart, Matlab will automatically calculate by M script file as long as the corrugated steel web box girder cross section parameters are input during the second and the third steps; according to the "modified rigid jointed beam method" derivation process, it can be concluded that: In the case of each additional chamber of multi-chamber, the number of redundants is increased, and the coefficients  $\delta_{i,p}$  in the canonical equations are increased by the square of the number of redundants. The change rule is obtained. The coefficients solved by the third step are given the matrix  $[\delta_{i,p}]$ . Then all the redundants at the cut of the corrugated steel web box girder are solved by the solver.

## 5. Conclusions

In this paper, the "modified rigid jointed beam method" is introduced in detail and the formula is given based on the calculation principle

of the "traditional rigid jointed beam" of the corrugated steel web composite box girder. The conclusions are as follows:

1) Based on the "traditional rigid jointed beam method" algorithm of the corrugated steel web composite box girder transverse distribution coefficient and combining with the existing research results of the stress distribution of the roof plate and the web of the corrugated steel web composite box girder, the conclusion is that the key to the modification of the "rigid jointed beam method" is to supplement the redundants at the cut of the top and bottom boards, that is, the redundants at the incision of the top board is the transverse bending moment, and of the bottom board is the lateral shear force and the transverse force.<sup>[24]</sup>

2) Through the mechanical analysis, this paper abstracts the basic mechanics calculation model of the "modified rigid jointed beam method" and obtains the formula of

"modified rigid jointed beam method" of corrugated steel web composite box girder by solving the corresponding force equation.

3) The process of "modified rigid jointed beam method" to calculate the transverse distribution coefficient of corrugated steel web composite box girder is: The steel web of the corrugated steel web composite box girder is equivalent to the corresponding concrete web through the equivalent stiffness principle.<sup>[25]</sup> Then, the transverse distribution coefficient of a cross section is solved by the force method equation established according to the computational scheme of the corresponding box chamber.

4) After the corrugated steel web composite box girder is equivalent to the concrete web box girder, the top and bottom boards along the web are cut into I-beams, the stress of which should be the same with that of the original box girder. Thus, the neutral beam of the original box

girder is used as the neutral axis of each I-beam to do relevant calculations of the corrugated steel web composite box girder.

5) The number of parameters of the basic force method equation of the "modified rigid jointed beam method" of the corrugated steel web composite box girder increases with that of the box chamber of the corrugated steel web composite box girder. That is, the number of parameters will increase by 5 when 1 chamber is added. [26] In order to simplify the calculation, this paper gives the programming ideas of "modified rigid jointed beam method", which can be used as a reference for relative scientific and research personnel to solve the transverse distribution coefficient of corrugated steel web composite box girder.

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