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Nonconvex Constrained Consensus of Discrete-Time Heterogeneous Multi-Agent Systems with Arbitrarily Switching Topologies

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Abstract: This paper mainly focuses on the velocity-constrained consensus problem of discrete-time heterogeneous multi-agent systems with nonconvex constraints and arbitrarily switching topologies, where each agent has first-order or second-order dynamics. To solve this problem, a distributed algorithm is proposed based on a contraction operator. By employing the properties of the stochastic matrix, it is shown that all agents' position states could converge to a common point and second-order agents' velocity states could remain in corresponding nonconvex constraint sets and converge to zero as long as the joint communication topology has one directed spanning tree. Finally, the numerical simulation results are provided to verify the effectiveness of the proposed algorithms.

Keywords: Heterogeneous; Multi-agent systems; Nonconvex constraint; Consensus

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1. Introduction

In the past decades, the consensus control problem of multiagent systems has always been a hot issue in systems and control communities. Many related researchers transferred their research directions to this field and many interesting works have been reported, such as the results in [1–7]. Most of the published works rarely concerned the constraints on the velocity states of agents and assumed that all agents work in an ideal setting. In reality, the velocities of agents cannot be too large and often remain in a particular range, for example, the dead zone of the velocity of physical vehicles [8]. Recently, the constrained consensus problem has received more and more attention and some distributed algorithms were designed to solve such problems. For example, based on the projection operator, some gradient and sub-gradient distributed algorithms were proposed for convex-constrained consensus problems [9,10]. Noticing that the constrained sets might be nonconvex in practical engineering, such as the velocity of the quadrotor, some scholars have begun to investigate the nonconvex constrained consensus problems. By introducing a contraction operator and the multiple model transformation method, a distributed algorithm was proposed and it is proven that the nonconvex constrained consensus can be achieved provided the joint topology has a directed spanning tree [11]. Later, this result was extended to the nonconvex input-constrained consensus problem and distributed optimization problem over discrete time or continuous-time multi-agent networks [12–14].

Most of the existing works on nonconvex-constrained consensus problems assumed that all agents have the same dynamics, i.e., the dynamics of all agents are the same order integrators. In practical engineering, the dynamics of the agents might be different due to the restriction of the environment. This kind of multi-agent system is called heterogeneous multi-agent systems. The consensus problems of heterogeneous multi-agent systems without any constraints have been extensively studied [15]. However, few works have been concerned with the nonconvex-constrained consensus problem of heterogeneous multi-agent systems. In, the mean-square nonconvex constrained consensus problem was solved for heterogeneous multi-agent systems with Markovian switching topologies, but through calculating its mathematical expectation, the average topology at any moment essentially has a directed spanning tree. Hence, the nonconvex-constrained consensus problem remains an unsolved issue when the communication topology arbitrarily switches.

This paper performs further research on nonconvex velocity-constrained consensus problem for heterogeneous multi-agent system with arbitrary switching topologies. A distributed algorithm is first designed by introducing a contraction operator. Then, the closed-loop system is changed into an equivalent one by a model transformation and it is proven that the nonconvex velocity-constrained consensus can be achieved as long as the joint topology has a directed spanning tree. In contrast, the Markovian topologies are considered and average communication topology has a directed spanning tree in essence, this paper considers the case that the communication topology switches arbitrarily which is more general, and the convergence analysis methods of this paper are totally different from due to different switching mechanism. Notations: In this paper, R^r represents Euclidean space with dimensionr. ‖x‖ represents the Euclidean normal of a vector x. For matrix A, A^T represents its transpose. I^r represents identicalmatrix with dimension r. ⊗ represents kronecker product. 1 represent a vector with all of its entries being one.

2. Preliminaries

Let $\mathcal{G} = (\mathcal{V}_\mathcal{G}, \mathcal{E}_\mathcal{G})$ be a directed graph, where $V_\mathcal{G} = \{1, \cdots, n\}$ is the set of nodes and $\mathcal{E}_\mathcal{G}$ is the set of edges. $a_{ij} > 0$ if and only if $(j,i) \in \varepsilon_\mathcal{G}$. Let $N_i = \{j \in V{:}(j,i) \in \varepsilon\}$ be the neighbor set of node i. The Laplacian L is defined as $[L]_{ii} = \sum_{j=1}^{n} a_{ij}$ and $[L]_{ji} = -a_{ij}$ for all i ≠ j. The union of some graphs is a new graph whose node set and edge set are the unions of the node sets and edge sets of the corresponding graphs. A series of edges $(v_{ik-1},v_{ik})(v_{ik-2},v_{ik-1})\ldots,(v_{i1},v_{i2})$ is called a directed path from node v_{ik} to node v_{i1}. A directed graph is said to have a directed spanning tree if there is one node, which is called as root, such that there is a directed path from the root to any other node.

Lemma 1: Let A be a stochastic matrix and $\mathcal{G}(A)$ be its corresponding graph, i.e., A is the adjacent matrix of $\mathcal{G}(A)$. Then $\mathcal{G}(A)$ has a spanning tree if and only if the algebraic multiplicity of eigenvalue 1 of A is one.

3. Main results

Consider a discrete-time heterogeneous multi-agent system with second-order agents and first-order agents. Suppose the th second-order agent has the following dynamics:

$$x_i(k+1) = x_i(k) + v_i(k)T \qquad (1)$$
$$v_i(k+1) = u_i(k), i \in J_s$$

where x_i ;v_i ; $u_i \in R^r$ are the position state, velocity state and control input of the i th agent. Assume that the velocity $v_i(k)$ of the the i th second-order agent is constrained to stay in a nonempty and nonconvex set $V_i \subseteq R^r$ which is only accessed by the i th agent, $\mathcal{J}_s = \{1,...,m\}$. Suppose the i th first-order agent has the following dynamics:

$$x_i(k+1) = u_i(k), i \in \mathrm{I}_f, \quad (2)$$

where x_i ; $u_i \in R^r$ are the position state and control input of the i th agent, $\mathcal{J}_f = \{m+1,...,m+n\}$.

Assumption 1 [11]: Let $V_i \subseteq \mathbf{R}^r$, $i \in \mathcal{J}_s$ be nonempty bounded closed sets, which satisfied that $\sup\|S_{Vi}(x)\| = \bar{\rho}_i > 0$, $\inf_{x \notin V_i} \|S_{V_i}(x)\| = \underline{\rho}_i > 0$ and $0 \in V_i$ for all $i \in \{1,2,...,m\}$, where $S_{Vi}(\cdot)$ is a constraint operator with $S_{Vi}(x) = \frac{x}{\|x\|} \sup_{0 \le \beta \le \|x\|} \left\{\beta: \frac{\alpha\beta x}{\|x\|} \in V_i, 0 < \alpha < \beta\right\}$ when $x \neq 0$ and $S_{Vi}(0)=0$.

The aim of this paper is to design an algorithm for each agent based on local information to assure all agents reach an agreement on position state while the velocity of the second-order agent remains in its corresponding nonconvex constraint set. To complete this task, we propose the following algorithm:

$$u_i(k) = \begin{cases} S_{V_i}[v_i(k) - p_i(k)v_i(k)T + \beta_i(k)], i \in \mathrm{I}_s, \\ x_i(k) + \beta_i(k), \qquad i \in \mathrm{I}_f, \end{cases} \quad (3)$$

where $p_i(k)>0$ is the time-varying gain, $\beta_i(k) = \sum_{j \in Ni(k)} a_{ij}(k)[x_j(k) - x_i(k)]\, T$.

Denote $a_i(k) = \frac{\|S_{V_i}[v_i(k) - p_i(k)v_i(k)T + \beta_i(k)]\|}{\|v_i(k) - p_i(k)v_i(k)T + \beta_i(k)\|}$, $i=1,...m$ if $v_i(k) - p_i(k) \cdot v_i(k)T + \beta_i(k) \neq 0$, otherwise, $a_i(k)=1$. It is clear that $0 < a_i(k) \le 1$. Define $b_i(k) = \frac{1 - \alpha_i(k)(1 - p_i(k)T)}{T}$. It follows from the definition of $a_i(k)$ that $S_{V_i}[v_i(k) - p_i(k)v_i(k)T + \beta_i(k)] = v_i(k) - v_i(k)b_i(k)T + \alpha_i(k)\beta_i(k)$. Let $y_i(k) = x_i(k) + \frac{2}{b_i(k)} v_i(k), i \in \mathcal{J}_s$ We have:

$$x_i(k+1) = \left(1 - \frac{b_i(k)}{2}T\right)x_i(k) + \frac{b_i(k)}{2}Ty_i(k) \quad (4)$$

$$y_i(k+1) = \left(1 - c_i(k)\right)x_i(k) + c_i(k)y_i(k) + \frac{2}{b_i(k+1)}\alpha_i(k)\beta_i(k)$$

where $c_i(k) = \frac{b_i(k)}{2}T + \frac{b_i(k)}{b_i(k+1)} - \frac{b_i(k)^2}{b_i(k+1)}T$. Define $Z(k) = \left[x_s^T(k), y_s^T(k), x_f^T(k)\right]^T$, where $x_s(k) = [x_1(k)^T,...,x_m(k)]^T$, $y_s(k) = [y_1(k)^T, ..., y_m(k)]^T$, $x_f(k) = [x_{m+1}(k)^T, ..., x_{m+n}(k)]^T$, $E(k) = \mathrm{diag}\{a_1(k), ..., a_m(k)\}$, $A(k) = \begin{bmatrix} A_1 & A_2 & 0 \\ A_3 & A_4 & 0 \\ 0 & 0 & I_n \end{bmatrix}$, where $A_1 = \mathrm{diag}\{1 - \frac{b_1(k)}{2}, \cdots, 1 - \frac{b_m(k)}{2}\}$, $A_2 = \mathrm{diag}\left\{\frac{b_1(k)}{2}, \cdots, \frac{b_m(k)}{2}\right\}$, $A_3 = \mathrm{diag}\{1 - c_1(k), \cdots, 1 - c_m(k)\}$, $A_4 = \mathrm{diag}\{c_1(k), \cdots, c_m(k)\}$, $A_4 = \mathrm{diag}\{c_1(k), \cdots, c_m(k)\}$. Let $L_s(k)$ and $L_f(k)$ be the corresponding. Laplacian of the second-order and first-order agents networks. Thus, the Laplacian matrix of $\mathcal{G}(k)$ has the following form:

$$L(k) = \begin{bmatrix} L_s(k) + D_{sf}(k) & -A_{sf}(k) \\ -A_{fs}(k) & L_f(k) + D_{fs}(k) \end{bmatrix} \quad (5)$$

where $A_{sf}(k)$ and $A_{fs}(k)$ are the corresponding sub-matrix of $L(k)$, $N_{i,f}$ and $N_{i,s}$ are the i th agent's neighbor sets of first-order agents and second-order agents respectively,

$$D_{sf}(k)=diag\left\{\sum_{j\in N_{1,f}} a_{1j}(k),\cdots,\sum_{j\in N_{m,f}} a_{mj}(k)\right\} \tag{6}$$

$$D_{fs}(k)=\mathrm{diag}\left\{\sum_{j\in N_{m+1,s}} a_{m+1,j}(k),\cdots,\sum_{j\in N_{m+n,s}} a_{nj}(k)\right\}. \tag{7}$$

Let $B(k)=\mathrm{diag}\left\{\frac{2T\alpha_1(k)}{b_1(k+1)},\cdots,\frac{2T\alpha_m(k)}{b_m(k+1)}\right\}$, and $L_0(k)=\mathrm{diag}\{L_{0s}(k),L_{0f}(k)\}=\mathrm{diag}\{L(k)\}$ be a diagonal matrix and the diagonal entries are the corresponding ones of $L(k)$. Then the closed-loop system can be summarized as:

$$Z(k+1)=([A(k)-E_0(k)]\otimes I_r)Z(k), \tag{8}$$

where

$$E_0(k)=\begin{bmatrix} 0 & 0 & 0 \\ B(k)\left(L_s(k)+D_{sf}(k)\right) & 0 & -B(k)A_{sf}(k) \\ -A_{fs}(k) & 0 & L_f(k)+D_{fs}(k). \end{bmatrix} \tag{9}$$

Furthermore, let $\Psi(k)=A(k)-E_0(k)$, then system (5) can be changed into:

$$Z(k+1)=[\Psi(k)\otimes I_r]Z(k). \tag{10}$$

To continue, we need the following assumptions [11].

Assumption 2: There is a positive number $\mu_c>0$ such that for any $i,j\in\{1,2,\cdots,n+m\}$, $a_{ij}(k)\geq\mu_c$ when $a_{ij}>0$.

Assumption 3: For each $i\in\mathcal{J}_s$ and $k\geq 0$, $0<b_i(k)\leq p_i(k+1)<\frac{1}{T}$, $[L(k)]_{ii}\leq d_i<\frac{1}{4}p_i(k)^2$ where $d_i>0$, and for each $i\in\mathcal{J}_f$, $[L(k)]_{ii}<1$.

Assumption 4: Suppose the communication topology switches or not at any k and there exist a sequence $\{k_l,l\geq 0\}$ and a constant $\eta>0$ such that $\lim_{l\to\infty}k_l=\infty, k_0=0$, and $k_{l+1}-k_l\leq\eta, l\geq 0$, and the joint graph of graphs $\mathcal{G}(k_l),\mathcal{G}(k_l+1),\cdots,\mathcal{G}(k_{l+1}-1)$ has at least one directed spanning tree.

Lemma 2: The matrix $\Psi(k)$ is a stochastic matrix for any $k\geq 0$.

Proof: From $0<a_i\leq 1$ and $p_i(k)\leq b_i(k)\leq p_i(k+1)\leq b_i(k+1)<\frac{1}{T}$, it follows that $b_i(k)^2\geq p_i(k)^2>4d_i$ and $\frac{b_i(k)^2}{4\alpha_i(k)}\geq\frac{b_i(k)^2}{4}>d_i$. Hence, $\frac{b_i(k)^2T}{2b_i(k+1)}=\frac{2T\alpha_i(k)}{b_i(k+1)}\cdot\frac{b_i(k)^2}{4\alpha_i(k)}>\frac{2T\alpha_i(k)}{b_i(k+1)}d_i\geq\frac{2T\alpha_i(k)[L(k)]_{ii}}{b_i(k+1)}$, which leads to $1-c_i(k)=1-\frac{b_i(k)}{2}T-\frac{b_i(k)}{b_i(k+1)}+\frac{b_i(k)^2}{b_i(k+1)}T=\left(1-\frac{b_i(k)}{b_i(k+1)}\right)\left(1-\frac{b_i(k)T}{2}\right)+\frac{b_i(k)^2T}{2b_i(k+1)}>\frac{2T\alpha_i(k)[L(k)]_{ii}}{b_i(k+1)}$, i.e., $1-c_i(k)-\frac{2T\alpha_i(k)[L(k)]_{ii}}{b_i(k+1)}>0$, $i\in\mathcal{J}_s$. Since $[L(k)]_{ii}<1$, $i\in\mathcal{J}_f$, we have that $A_3-B(k)L_s(k)$ is a nonnegative matrix, which suggests that $\Psi(k)$ is a nonnegative matrix. It follows from the definition of $E_0(k)$ that $-E_0(k)\mathbf{1}=0$. Besides, It is easy to see that $A(k)\mathbf{1}=1$ and $\Psi(k)\mathbf{1}=\mathbf{1}$. Therefore, $\Psi(k)$ is a stochastic matrix for all $k\geq 0$.

Next, we will prove that $\alpha_i(k)$ has strictly positive lower boundary.

Lemma 3: Under Assumptions 1 - 4. For all $i\in\mathcal{J}_s$, $\alpha_i(k)\geq\frac{\rho_i}{\left(\frac{1}{T}+2Td_i\right)\max_j\{\|Z_j(0)\|,\gamma\}}$, where $\gamma>0$ is a constant.

Proof: It follows from (6) that $Z(k)=[\Psi(k-1)\otimes I_r]Z(k-1)=\cdots=[\Gamma(k-1,0)\otimes I_r]Z(0)$, where $\Gamma(k-1,0)=\Psi(k-1)\Psi(k-2)\cdots\Psi(0)$. From Lemma 2, it is clear that $\Gamma(k-1,0)$ is a stochastic matrix. Hence, $\alpha_i(k)$ is the convex combination of $Z_j(0), j\in\{1,2,\cdots,2m+n\}$ for all $i\in\{1,2,\cdots,2m+n\}$, which suggests that $\| Z_i(k\|\leq\max_{\to}\{\|Z_j(0)\|\}$. Hence, $\|\beta_i(k)\|=\|\sum_{j\in N_i(k)}a_{ij}(k)[x_j(k)-x_i(k)]T\|\leq T\left[\sum_{j\in N_i(k)}a_{ij}(k)\|x_j(k)\|+\right.$

$[L(k)]_{ii}\|x_i(k)\|] \leq 2T[L(k)]_{ii}\max\{\|Z_i(0)\|\}$. Besides, for $i \in \{1,2,\cdots,m\}$, $\| v_i(k) - p_i(k)v_i(k)T \| = \| (1 - p_i(k)T)v_i(k) \| = (1 - p_i(k)T) \| v_i(k) \| \leq \| v_i(k) \| =) \|\frac{b_i(k)}{2}(y_i(k) - x_i(k))\| \leq \frac{b_i(k)}{2}\|Z_{m+i}(k) - Z_i(k)\| \leq b_i(k)\max\{\|Z_j(0)\|\} \leq \frac{1}{T}\max\{\|Z_j(0)\|\}$. Noticing that $[L(k)]_{ii} \leq d_i, i \in \mathcal{I}_s$, we get $\|v_i(k) - p_i(k)v_i(k)T + \beta_i(k)\| \leq \|v_i(k) - p_i(k)v_i(k)T\| + \|\beta_i(k)\| \leq \left(\frac{1}{T} + 2Td_i\right)\max_j\{\|Z_j(0)\|\}$ From Assumption 1, we have:

$$\alpha_i(k) \geq \frac{\rho_i}{\left(\frac{1}{T} + 2Td_i\right)\max_j\{\| Z_j(0) \|, \gamma\}}. \qquad (11)$$

Lemma 4: Under Assumptions 1-4. There exist a positive number $h \in \{1,2,\cdots,2m+n\}$ and a positive constant $0 < \hat{\mu} < 1$ such that $[\Gamma(k_{l+\hat{n}} - 1, k_l)] \geq \hat{\mu}$ for all k_i>0 and i, where $\Gamma(k,s) = \prod_{i=s}^{k} \Psi(i), \hat{n} \geq 4(2m+n)$.

Proof: It follows from Lemma 2 that $\Gamma(k,s)$ is a stochastic matrix. Let $\bar{G}(k,s)$ be the directed graph corresponding to $\Gamma(k,s)$, i.e., $\Gamma(k,s)$ is the adjacent matrix of $\bar{G}(k,s)$. It follows from the definition of $\Gamma(k,s)$ and t$\Psi(i)$ hat $[\Gamma(k,k)]_{j_1,m+j_1} > 0, [\Gamma(k,k)]_{m+j_1,j_1} > 0, [\Gamma(k,k)]_{j_2,j_2} > 0$, for all $j_1 \in \{1,2,\cdots,m\}, j_2 \in \{1,2,\cdots,2m+n\}$. From the expression of $\Gamma(k,k)$, it is easy to see that $[\Gamma(k,k)]_{j_2,j_2} > 0$ 0 and $[\Gamma(k,s)]_{j_2,j_2} > 0$. In addition, the joint graph of graphs $\mathcal{G}(k_l),\cdots,\mathcal{G}(k_{l+1}-1)$ has a directed spanning tree, so is the graph $\bar{G}(k_{l+1}-1,k_l)$, whose root node located in $\{1,2,..., 2m+n\}$ for any $c > 0$. Then by imitating the proof of [11, Lemma 2], the proof of this lemma can be completed.

Lemma 5: (1) For any $i \in \{1,2,\cdots,(2m+n)\}$ and $s \geq 0$, there exists $0 \leq \rho_i(s) \leq 1$ such that $\lim_{k\to\infty}[\Gamma(k,s)]_{ji} = \rho_i(s), \forall j \in \{1,2,\cdots,(2m+n)\}$, where $\sum_{i=1}^{2m+n} \rho_{\underset{k}{i}}(s) = 1$;

(2) $\max_i\left|[\Gamma(k,s)]_{ji} - \rho_i(s)\right| \leq C_0(1-\hat{\mu})^{\frac{k}{\hat{n}\eta}}$, where $k \geq s, C_0 = (1-\hat{\mu})^{-2}$.

Proof: The proof of this Lemma is similar to the one of Lemma 3 in Ref. [11], so we omit it.

Theorem 1: Consider the discrete-time heterogeneous multi-agent system (1) and (2) with protocol (3). Under Assumptions 1–4, there exist a state $\bar{x} \in \mathbf{R}^r$ and two constants C>0 and 0<μ<1 such that for any $i \in \{1,2,\cdots,m+n\}$, $k \geq 0, \|x_i(k) - \bar{x}\| \leq C(1-\mu)^k$ and for any $i \in \{1,2,\cdots,m\}, k \geq 0, \lim_{k\to\infty} v_i(k) = 0$, $v_i(k) \in V_i$, i.e., the position states of all agents reach an agreement exponentially and the velocity states of the second-order agents are constrained to stay in their corresponding nonconex constraint sets.

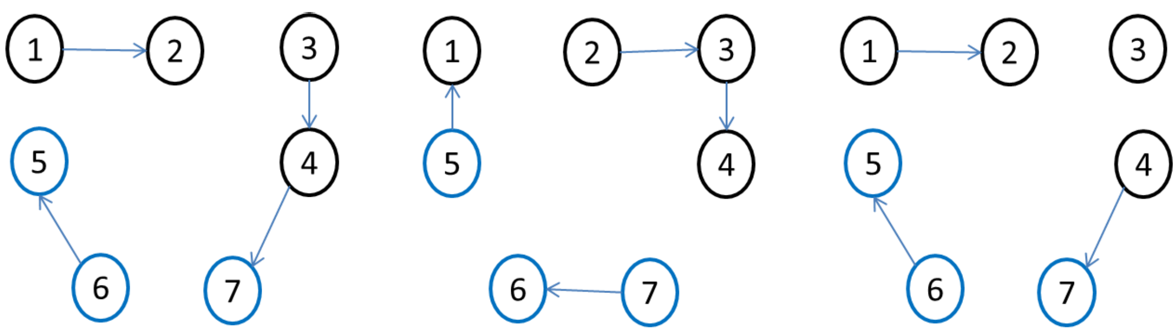

Figure 1. Topological structures between agents.

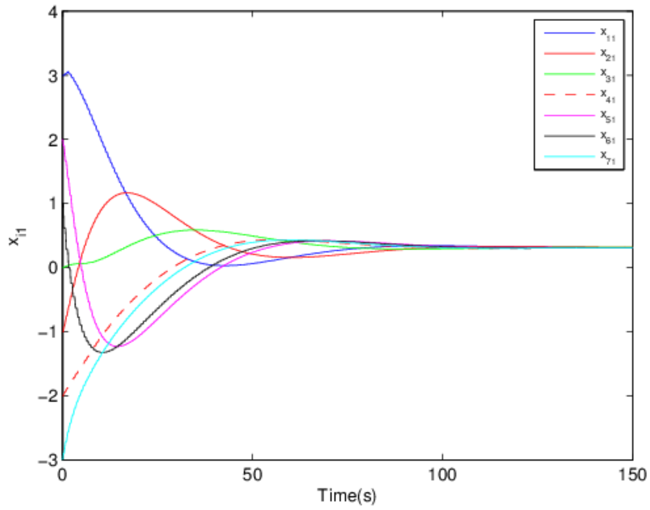

Figure 2. The position states of all agents along abscissa axis.

Proof: Let $\bar{x}=\sum_{j=1}^{2m+n}\rho_j(0)Z_j(0)$. For $i\in\{1,2,\cdots,2m+n\}$, we have $\|Z_i(k)-\bar{x}\|=\|\sum_{j=1}^{2m+n}[\Gamma(k-1,0)]_{ij}Z_j(0)-\sum_{j=1}^{2m+n}\rho_j(0)Z_j(0)\|\le\sum_{j=1}^{2m+n}\|[\Gamma(k-1,0)]_{ij}-\rho_j(0)\|\,\|Z_j(0)\|$. It follows from Lemma 5 that $\|Z_i(k)-\bar{x}\|\le C_0(1-\hat{\mu})^{\frac{k-1}{\bar{n}\eta}}\sum_{j=1}^{2m+n}\|Z_j(0)\|$. Noticing $\sum_{j=1}^{2m+n}\|Z_j(0)\|\le 2\sum_{j=1}^{m+n}\|x_j(0)\|+\frac{2}{T}\sum_{j=1}^{m}\|v_j(0)\|$, we have $\|x_i(k)-\bar{x}\|\le C(1-\mu)^k, i\in\mathcal{I}_s\cup\mathcal{I}_f$, and $\|y_j(k)-\bar{x}\|\le C(1-\mu)^k, j\in\mathcal{I}_s$, where $C=C_0(1-\hat{\mu})^{-1}\left(2\sum_{j=1}^{m+n}\|x_j(0)\|+\frac{2}{T}\sum_{j=1}^{m}\|v_j(0)\|\right), \mu=1-(1-\hat{\mu})^{\frac{1}{\bar{n}\eta}}$ with $C>0$ and $0<\mu<1$.

4. Simulations

In this section, a numerical simulation is given to verify the correctness of the obtained result. Fig. 1 shows the communication graphs with 7 agents in $\mathbf{R}^2$, where agent 1 to 4 are second-order agents and agent 5 to 7 are first-order agent. It is clear that each graph doesn't have a directed spanning tree. Here, suppose the weight of each edge is 0.5 and the sample period T = 0.2. The nonconvex velocity constraint set $V_i=\left\{x|\ \|x\|\le 0.4,\frac{|x[01]^T|}{|x[10]^T|}\le 1\right\}\cup\left\{x|\ \|x\|\le 0.2,\frac{|x[01]^T|}{|x[10]^T|}\ge 1\right\}$. Let $p_i(0)=3$ and $p_i(k+1)=b_i(k)$ for all $k>0$. The initial values $x_1(0)=[3,0]^T$, $x_2(0)=[-1,-2]^T$, $x_3(0)=[0,1]^T$, $x_4(0)=[-2,-1]^T$, $x_5(0)=[2,-3]^T$, $x_6(0)=[1,2]^T$, $x_7(0)=[-3,3]^T$ and $v_i(0)=[0,0]^T$, i = 1,2,3,4. It is easy to verify that Assumptions 1-4 are satisfied. Fig. 2 and 3 show all agents' position states. Fig. 4 shows second-order agents' velocity states in the phase plane. From these simulation results, we can see that the all agents reach an agreement on their position state while the velocity states are constrained to stay in the corresponding nonconvex sets.

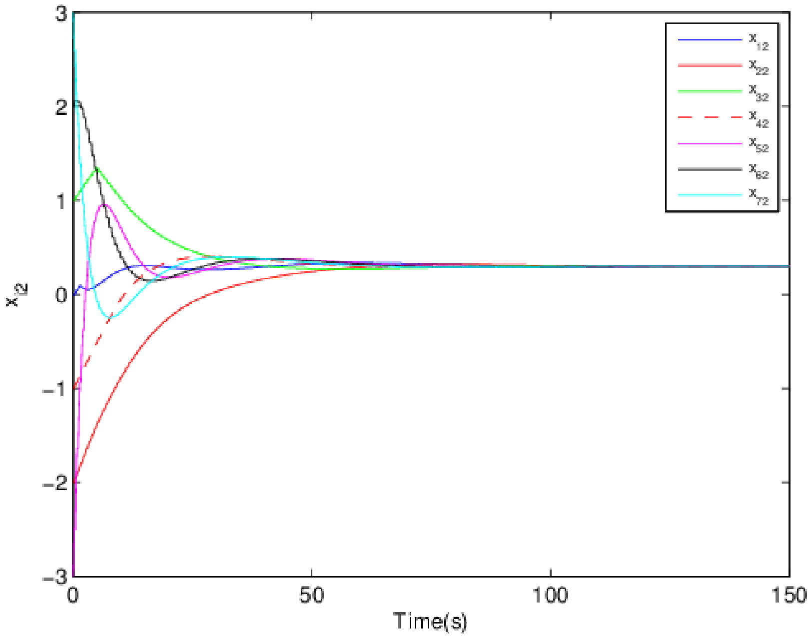

Figure 3. The position states of all agents along longitudinal axis.

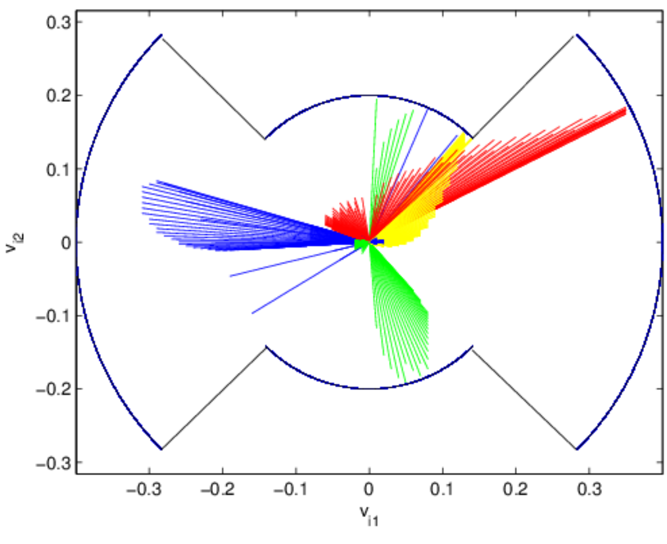

Figure 4. The velocity states of second-order agents.

5. Conclusion

This paper investigated the non-convex velocity-constrained consensus problem of discrete-time heterogeneous multi-agent systems with arbitrarily switching topologies. A distributed constrained consensus protocols was designed to guarantee the position states of all agents converge to a common point while the velocity states of second-order agents stay in some nonconvex constraint sets. Based on a model transformation, the consensus analysis was completed by applying the stochastic matrix theory. Finally, simulation was given to demonstrate the correctness of the proposed protocol.

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References

[1] Moreau L, 2005, Stability of Multiagent Systems with Time-Dependent Communication Links. IEEE Transactions on Automatic Control, 50(2): 169–182.

[2] Ren W, Beard R W, 2005, Consensus Seeking in Multiagent Systems Under Dynamically Changing Interaction Topologies. IEEE Transactions on Automatic Control, 50(5): 655–661.

[3] Lin P, Jia Y, 2009, Consensus of Second-Order Discrete-Time Multi-Agent Systems with Nonuniform Time-Delays and Dynamically Changing Topologies. Automatica, 45(9): 2154–2158.

[4] Zhang B, Jia Y, Matsuno F, 2014, Finite-Time Observers for Multi-Agent Systems Without Velocity Measurements and With Input Saturations. Systems & Control Letters, 68: 86–94.

[5] Lin P, Ren W, Song Y, 2016, Distributed Multi-Agent Optimization Subject to Nonidentical Constraints and Communication Delays. Automatica, 65: 120–131.

[6] Zhao L, Yu J, Lin C, Yu H, 2017, Distributed Adaptive Fixed-Time Consensus Tracking for Second-Order Multi-Agent Systems Using Modified Terminal Sliding Mode. Applied Mathematics and Computation, 312: 23–35.

[7] Mo L, Guo S, 2018, Consensus of Linear Multi-Agent Systems with Persistent Disturbances via Distributed Output Feedback. Journal of Systems Science and Complexity, 32: 835–845.

[8] Jia Y, 2000, Robust Control with Decoupling Performance for Steering and Traction of 4WS Vehicles Under Velocity-Varying Motion. IEEE Transactions on Control Systems Technology, 8(3): 554–569.

[9] Nedic A, Ozdaglar A, Parrilo P A, 2010, Constrained Consensus and Optimization in Multi-Agent Networks. IEEE Transactions on Automatic Control, 55(4): 922–938.

[10] Lin P, Ren W, Farrell J A, 2017, Distributed Continuous-Time Optimization: Nonuniform Gradient Gains, Finite-Time Convergence, and Convex Constraint Set. IEEE Transactions on Automatic Control, 62(5): 2239–2253.

[11] with Nonconvex Constraints, Switching Topologies, and Delays. IEEE Transactions on Automatic Control, 62(11): 5788–5794.

[12] Lin P, Ren W, Yang C, Gui W, 2018, Distributed Consensus of Second-Order Multiagent Systems with Nonconvex Velocity and Control Input Constraints. IEEE Transactions on Automatic Control, 63(4): 1171–1176.

[13] Lin P, Ren W, Gao H, et al., 2018, Distributed Optimization with Nonconvex Velocity Constraints, Nonuniform Posi-

tion Constraints and Nonuniform Stepsizes. IEEE Transactions on Automatic Control, 65(6): 2575–2582.

[14] Mo L, Lin P, 2018, Distributed Consensus of Second-Order Multiagent Systems with Nonconvex Input Constraints. International Journal of Robust and Nonlinear Control, 28(11): 3657–3664.

[15] Zheng Y, Zhu Y, Wang L, 2011, Consensus of Heterogeneous Multiagent Systems. IET Control Theory & Applications, 5(16): 1881–1888.