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Distributed Adaptive Predefined-Time Bipartite Containment Algorithm for Nonlinear Multi-Agent Systems with Actuator Faults

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Abstract: Distributed adaptive predefined-time bipartite containment for a class of second-order nonlinear multi-agent systems are studied with actuator faults. The communication topology of multi-agent systems is fixed and directed. To ensure that followers can reach the convex hull spanned by leaders under the conditions of actuator faults, the sliding mode method is introduced. Control protocol for multi-agent systems demonstrates its effectiveness. Finally, simulations are provided to verify the effectiveness of the proposed algorithm.

Keywords: Actuator faults; Adaptive bipartite containment control; Nonlinear multi-agent systems; Predefined-time

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1. Introduction

During the past years, with the progress of science and technology, multi-agent systems have made great progress due to their wide applications, such as sensor networks, unmanned air vehicles, underwater vehicles, and so on. Compared with centralized control, distributed cooperative control has better advantages and also becomes a research focus, and it was previously solved in some studies [1,2]. Therefore, cooperative control occupies an important position in the study of multi-agent systems.

Containment control, as a crucial research topic, aims to design an appropriate controller to make followers enter into the convex hull spanned by leaders. There have been many research results on containment control. The work in previous research discussed intermittent sampled position date communication containment control under time delay and free time delay [3]. However, most of the mentioned research is mostly based on the study of containment control of cooperative multiagent systems. It is well known that the competitive relationship between multiple agents is ubiquitous, and it is ubiquitous in real life, for example, partisan competition, competition among colleagues or classmates, etc., which has also attracted many researchers to study. Therefore, bipartite containment control with signed graphs has been studied in various ways. In a previous study, authors developed heterogeneous bipartite consensus for linear dynamic systems under state and output feedback control [4]. Authors developed a fixed-time bipartite containment control scheme for command-filter-based nonlinear multi-agent systems in another research [5].

An important metric for evaluating the quality of a designed controller is time. Weighted bipartite containment was proposed previously for the Lagrangian system [6]. Then, some authors considered sampled-date consensus for second-order heterogeneous nonlinear switch dynamic systems [7]. These methods can make all agents reach a common state when $t \rightarrow \infty$. However, for some industrial applications in real life, it is required to reach finite-time bipartite containment for its high control accuracy and strong robustness. Finite-time bipartite heterogeneous output consensus was given for linear dynamic systems under state feedback [8]. Liu *et al.* studied finite-time heterogeneous bipartite containment for linear multi-agent systems with directed topology [9]. However, the settling time to reach a consensus on this method is closely related to the initial value. Thus, the concept of fixed-time consensus was generated, which depends on the initial value. Cai *et al.* fixed-time time-varying output bipartite formation-containment was introduced for heterogeneous linear multi-agent systems [10]. Mu *et al.* investigated fixed-time adaptive bipartite containment control for nonlinear multi-agent systems under unknown disturbances and fuzzy quantized [11]. Unlike the fixed-time containment control, where the convergence time is related to the choice of parameters, the predefined-time containment control is advantageous and the time of convergence can be pre-designed in practice engineering.

So far, many methods have been investigated to solve bipartite containment control of multi-agent systems, for example, adaptive control, robust control, and so on. Sliding mode control has the advantages of good anti-interference performance, robustness, and simple structure. A novel adaptive event-triggered control was proposed by Li *et al.* for a switched nonlinear multi-agent dynamics system with state observer and sampled date control, in which output information can be sampled in sampling instants [12]. A distributed event-triggered output feedback containment controller was put forward by Yang and Qian for nonlinear multi-agent systems under limited communication resources [13].

In practical industrial environment, because of complexity and larger scale, multi-agent systems may encounter faults at any time, which will lead to reduce system performance or even complete system collapse. Therefore, it is significant to consider a fault-tolerant strategy to cope with actuator faults [14]. Recently, a number of different fault-tolerant consensus ways have been investigated for multi-agent systems. In a study by Jin *et al.*, the fault-tolerant consensus of nonlinear multi-agent systems was considered with disturbed and actuator faults based on neural networks and adaptive control strategies. Observer-based finite time leaderless fault-tolerant consensus was studied in for continuous-time linear multi-agent systems under the influence of fractional transformation. When there are unknown bounds, some control methods may not achieve the desired control results. To overcome these shortcomings, the adaptive control strategy is a good method. Adaptive Unmanned Aerial Vehicles (UAVs) event-triggered formation-containment control was studied with external disturbances. In another research, distributed adaptive containment was proposed for uncertain nonlinear multi-agent systems under fixed directed graphs with time delays based on neighbors' output information [15].

Inspired by the mentioned research, in this paper, we focus on designing a predefined-time adaptive bipartite containment for second-order nonlinear multi-agent systems with actuator faults. The main contributions of this paper are as follows:

(1) Compared to discontinuous sliding mode control, we study continuous sliding mode to avoid chattering.

We examine second-order dynamic systems to better align with real-life scenarios. We propose a predefined-time bipartite containment control whose convergence time can be preset, making system control more intelligent.

(2) Semi-globally uniformly predefined time-bounded bipartite containment for nonlinear second-order multi-agent systems is studied using adaptive control by proposed criteria.

(3) Compared with fault-tolerant control investigated in the previous research, the adaptive studied in this paper is simpler and easier to implement in practice. The rest of this paper is organized as follows.

2. Preliminaries

2.1. Graph theory

It denotes $G = (VG, EG, A)$ is a directed graph, where $VG = \{v1, v2, ..., vf+l\}$ represents the set of nodes, $EG \in VG \times VG$ is the set of edges and $A = [aij] \in R(f+l) \times (f+l)$ is the weighted adjacency matrix of the graph G. A directed edge $(vj, vi) \in EG$ in graph G means that agent i can receive information from agent j, but not conversely. Define $aij > 0$ if $(vj, vi) \in EG$, and while $aij = 0$ otherwise. The degree matrix is $D = diag(d1,..., df+l)$, where $di = \sum_{j=1}^{f+l} |aij|$. The Laplacian matrix $L = [lij] \in R(f+l) \times (f+l)$ which is associated with A is defined as e $lii = \sum_{j=1}^{f+l} |aij|$, $lij = -aij$, and $L = D - A$.

Suppose the system consists of l leaders and f followers. And $NF = \{1,..., f\}$ and $ZL = \{f + 1,...,f + l\}$, denote the follower set and the leader set. A signed graph is different from a traditional communication graph. Node set consists of two subgroups $V1$ and $V2$, which $V1 \cup V2 = VG$ and $V1 \cap V2 = \emptyset$. If $(vi, vj) \in EG$, then $aij > 0, (vi, vj \in Vq, q = 1, 2)$, which denotes cooperative interaction, and $aij < 0, (vi \in Vp, vj \in Vq, p, q = 1, 2, p \neq q)$, which denotes antagonistic interaction, and $aij = 0$ otherwise. Besides, signed graph is structurally balanced.

2.2. Some useful lemmas and definitions

Definition 1: Let Ch be a set in a real vector space $R \subseteq RP$. The set is convex if, for any xh and yh in Ch, point $(1 - \alpha h)xh + \alpha hyh \in Ch$ for any $\alpha h \in [0, 1]$. Convex hull for a set of points $\chi p: = \{x1p,..., xmp\}$, denoted by $Co(\chi p)$, that is:

$$Co(\chi p) = \sum_{i=1}^{m} \beta ipxip | xip \in xp, \ \ \beta ip \ \geq 0,. \ \ \sum_{i=1}^{m} 1 \, \beta ip = 1$$

Definition 2: For any initial conditions of followers and leaders, some followers will converge to a convex hull spanned by leaders, and others will converge to a symmetric convex hull, that is, $limt \to T \ dist(xi,Co(xk, k \in Z_l)) = 0, i \in F1$, $lim_t \to T \ dist(vi,Co(v_k, k \in Z_l)) = 0, i \in F2, F1 \cup F2 = F, F_1 \cap F_2 = \emptyset$.

Assumption 1: The communication topology among the followers is directed. For each follower, there exists at least one directed path from leader to follower. Communication topology is structurally balanced.

Lemma 1: For the following system $x = f(t, x(t)), f(0) = 0, x(0) = x_0$, if there exists a positive-definite,

Lyapunov function $V(x) \leq (\alpha V^{\rho}(x) + \beta V^{\iota}(x))^{\kappa} - cV(x) + \delta\rho$, where $Tc, \alpha, \beta, \rho, \iota, \kappa > 0, \kappa\rho < 1, \kappa\iota > 1, c, \delta\rho > 0$ are bounded constants and where , are positive parameters, and Γ(•) is the Gamma function, then the origin is semi-globally uniformly predefined-time-bounded stable equilibrium of the considered system the Tc being the predefined time.

Lemma 2: Let $\eta 1,...,\eta k \geq 0, 0 < q \leq 1, p > 1$, then $\sum_{i=1}^{k} \eta_i^p \leq k^{1-p} \left(\sum_{i=1}^{k} \eta_i\right)^p, \sum_{i=1}^{k} \eta_i^q \geq \left(\sum_{i=1}^{k} \eta_i\right)^q$.

Lemma 3: For all $x, y \in R$, inequality $xy \le \frac{v^{D}}{p}|x|^{p} + \frac{1}{qv^{D}}$ holds, where $v > 0, p, q > 1$, and $(p-1)(q-1) = 1$

2.3. Problem formulation

The dynamics of followers are described by

$$\begin{cases} \dot{x}_i = v_i \\ v_i = u_i + f_{i(x_i, x_i, t) + \delta_i} \end{cases}, i \in N_F \tag{1}$$

where , $v_i \in R^n$, $u_i \in R^n$ are the position, velocity, and control input of *ith* follower, and $fi(x_i, v_i, t)$ is a nonlinear function, and δ_i is an external disturbance.

In earlier articles, the controllers studied were mostly healthy controllers. However, the actuator will inevitably be damaged during long-term operation. In this paper, we considered the actuator faults as loss of effectiveness and bias faults. Assume that the leader is free of actuator faults.

When actuator faults occur on followers, the fault model is defined as $u_i^0 = \rho_i(t)u_i + r_i(t)$, where u_i^0 and u_i are the actual output of the actuator and designed control of the *ith* agent. $\rho_i(t)$ is time-varying partial loss of effectiveness fault severity satisfying $0 \le \tilde{\rho} \le \rho_i \le 1$, $\tilde{\rho}_i$ and $\overline{\rho}_i$ is lower and upper bounds. And r_i is biased fault satisfying $|r_i(t)| \le \overline{\rho}_i$, $\overline{\rho}_i$ is unknown constant.

The dynamics of leaders are described by

$$\begin{cases} \dot{x}_j = v_j \\ \dot{v}_j = u_j + f_j(x_j, v_j, t) \end{cases}, \ j \in Z_L \tag{2}$$

where $x_j \in R^n$ is position, $v_j \in R^n$ is velocity, and $u_j \in R^n$ is control input, correspondingly. And $fj(xj, vj, t)$ are nonlinear functions.

Assumption 2: δi, $i \in Z_L$ satisfies $|\delta i| \le \overline{\delta i}$, where $\overline{\delta i} > 0$ is an unknown constant. u_j, $j \in Z_L$ satisfies $|u_j| \le \overline{u_j}$ where $\overline{u_j} > 0$ is a unknown constant.

Assumption 3: Given $\overline{w_1}, \ldots, \overline{w_l}$ satisfying $\sum_{j=1}^{l} \overline{w_j} = 1$, and $\overline{w_j} \ge 0$, there exist two constants $\rho_1, \rho_2 > 0$ such that for $x, v, x_j, v_j \in R^n$, $j = 1, \ldots, l$, $\left\| f(x,v,t) - \sum_{j=1}^{l} \overline{\omega_j}\, f_j(x_j, v_j, t) \right\| \le \rho_1 \left\| x - \sum_{j=1}^{l} \overline{\omega_j}\, x_j \right\| + \rho_2 \left\| v - \sum_{j=1}^{l} \overline{\omega_j}\, v_j \right\|$

We first define the position and velocity error function

$$e_{xi} = \sum_{j=1}^{f} |a_{ij}| \left(x_i(t) - sign(a_{ij})x_j(t)\right) + \sum_{k=1+f}^{f+l} |a_{ik}| \left(x_i(t) - sign(a_{ik})x_k(t)\right),$$

$$e_{vi} = \sum_{j=1}^{f} |a_{ij}| \left(v_i(t) - sign(a_{ij})v_j(t)\right) + \sum_{k=1+f}^{f+l} |a_{ik}| \left(v_i(t) - sign(a_{ik})v_k(t)\right)$$

$$= \sum_{j=1}^{f+l} |a_{ij}|\, v_i(t) - \sum_{j=1+f}^{f} a_{ij} v_j(t) - \sum_{k=1+f}^{f+l} a_{ik} v_k(t),$$

where $i \in \{1, \ldots, f\}$ and $|aij| sign(aij) = aij$. Let $ex = \{ex1, \ldots, exf\}^T$, $ev = \{ev1, \ldots, evf\}^T$.

Sliding mode is designed as

$$S_i = e_{vi} + \frac{r}{T_s}\left(a e_{xi}^{\alpha} + b e_{xi}^{\beta}\right) \tag{3}$$

where T_s, a, $b > 0$, $0 < \alpha < 1$, $\beta > 1$, r is given as the same as that in **Lemma 1**. And $s = [s1,...,sf]^T$. Time derivative of S_i is

$$\dot{S}_i = \dot{e_{vi}} + \frac{r}{T_s}\left(a\alpha e_{xi}^{\alpha-1} + b\beta e_{xi}^{\beta-1}\right)e_{vi}$$

$$= \sum_{j=1}^{f+l}|a_{ij}|\rho_i u_i - \sum_{j=1}^{f} a_{ij}\rho_j u_j - \sum_{k=1+f}^{f+l} a_{ik}u_k$$

$$+ \sum_{j=1}^{f+l}|a_{ij}|\delta_i - \sum_{j=1}^{f} a_{ij}\delta_j + \sum_{j=1}^{f+l}|a_{ij}|r_i - \sum_{j=1}^{f} a_{ij}r_i$$

$$+ \sum_{j=1}^{f+l}|a_{ij}|f_i(x_i, v_i, t) - \sum_{j=1}^{f} a_{ij}f_j\left(x_j, v_j, t\right)$$

$$- \sum_{k=1+f}^{f+l} a_{ik}f_k(x_k, v_k, t) + \frac{r}{T_s}\left(a\alpha e_{xi}^{\alpha-1} + b\beta e_{xi}^{\beta-1}\right)e_{vi}.$$

According to **Assumption 2**, one has

$$-\sum_{k=1+f}^{f+l} a_{ik}u_k \leq l\overline{u_k},\ \sum_{j=1}^{f+l}|a_{ij}|\,\delta_i - \sum_{j=1}^{f} a_{ij}\delta_j \leq 2(f+l)\overline{\delta}_i,$$

$$\sum_{j=1}^{f+l}|a_{ij}|\,r_i - \sum_{j=1}^{f} a_{ij}r_j \leq 2(f+l)\overline{r}_i.$$

According to **Assumption 3**, one has

$$\sum_{j=1}^{f+l}|a_{ij}|\,f_i(x_i, v_i, t) - \sum_{j=1}^{f} a_{ij}f_j(x_j, v_j, t) - \sum_{k=1+f}^{f+l} a_{ik}f_k(x_k, v_k, t) \leq c\rho_1(x_i -$$

$$\sum_{j=1}^{f+l}|a_{ij}|\sum_{j=1}^{-1} a_{ij}x_j\Big) + c\rho_2\left(v_i - \sum_{j=1}^{f+l}|a_{ij}|^{-1}\sum_{j=1}^{f+l} a_{ij}v_j\right) = l_\rho$$

where $\sum_{j=1}^{f+l}|a_{ij}| = c, \rho_1, \rho_2 > 0$. Define

$$b_i = \frac{r}{T_s}\left(a\alpha e_{xi}^{\alpha-1} + b\beta e_{xi}^{\beta-1}\right)e_{vi} + l_\rho + 2(f+l)\overline{\delta}_i + 2(f+l)\overline{r}_i + l\overline{u_k}.$$

So $\dot{S}_i \leq \overline{\rho}_i\sum_{j=1}^{f+l}|a_{ij}|\,u_i - \overline{\rho}_i\sum_{j=1}^{f} a_{ij}\,u_j + b_i.$

According to **Assumption 2** and **Assumption 3**, one can easily know b_i is unknown bound, and define $b_i^2 \leq l_i$. Define $\widehat{l_i}$ as estimate of b_i^2, then adaption law for estimation b_i^2 is given as

$$\dot{\widehat{l_i}} = -\widehat{l_i} - \frac{\varpi}{T_p}\left(c2^\rho\left|\widehat{l_i}\right|^\rho + d2^\iota\left|\widehat{l_i}\right|^\iota\right)sgn(s_i) + \frac{\|S_i\|^2}{2\varsigma^2} \tag{4}$$

where T_p, c, d, $\varsigma > 0$, $0 < \rho < 1$, $\iota > 1$, $\overline{w}$ is given as **Lemma 1**, and $\widetilde{l_i} = l_i - \widehat{l_i}$.
Control protocol for each agent can be designed as

$$u_i = u_{i1} + u_{i2}$$

$$u_{i1} = \left(\bar{\rho}_i \sum_{j=1}^{f+l} |a_{ij}|\right)^{-1} \left(-\frac{\hat{l}_i s_i}{2\varsigma^2} + \bar{\rho}_i \sum_{j=1}^{f} a_{ij} u_j - \frac{s_i}{2}\right), \quad (5)$$

$$u_{i2} = \left(\bar{\rho}_i \sum_{j=1}^{f+l} |a_{ij}|\right)^{-1} \left(-\frac{\varpi}{2T_p}\left(c|S_i|^{\rho} + d|S_i|^{l}\right) sgn(S_i)\right).$$

3. Main result

Theorem 1: hold **Assumption 2** and **Assumption 3**. Consider the second-order multi-agent systems **(1)** and **(2)** reach semi globally uniformly predefined-time-bounded bipartite containment with the proposed controller **(5)**, sliding mode **(3)**, and adaptive laws **(4)**.

Proof: Design the following Lyapunov function

$V_1 = V_{11} + V_{12}, V_{11} = \sum_{i=1}^{F} S_i^2,\ V_{12} = \sum_{i=1}^{F} \tilde{l}_i^2$. Differentiating $V_{11}(t)$ along the solution of **(3)** yields

$$\dot{V}_{11} = 2\sum_{i=1}^{f} S_i \dot{S}_i$$

$$= 2\sum_{i=1}^{N} S_i \left(\bar{\rho}_i \sum_{j=1}^{f+l} |a_{ij}| u_i - \bar{\rho}_i \sum_{j=1}^{f} a_{ij} u_i + b_i\right)$$

$$= 2\sum_{i=1}^{f} S_i \left(-\frac{\hat{l}_i s_i}{2\varsigma^2} - \frac{s_i}{2} - \frac{\varpi}{2T_p}\left(C|s_i|^{\rho} + d|s_i|^{l}\right) sgn(s_i) + b_i\right)$$

Obviously, $\sum_{i=1}^{f} s_i b_i \le \sum_{i=1}^{f} \frac{\|s_i\|^2 (b_i)^2}{2\varsigma^2} + \sum_{i=1}^{f} \frac{\xi^2}{2}$. One gets

$$\dot{V}_{11} \le \sum_{i=1}^{f} \frac{\|s_i\|^2 (b_i)^2}{\varsigma^2} + \sum_{i=1}^{f} s^2 - \sum_{i=1}^{f} s_i^2$$

$$- \sum_{i=1}^{f} s_i \left(\frac{\varpi}{T_p}\left(c|S_i|^{\rho} + d|S_i|^{l}\right) sgn(S_i)\right) - \sum_{i=1}^{f} \frac{\hat{l}_i s_i^2}{\varsigma^2}$$

$$\le -V_{11} - \frac{\varpi}{T_p} C|V_{11}|^{\frac{\rho+1}{2}} - \frac{\varpi}{T_p} d|V_{11}|^{\frac{c+1}{2}}$$

$$+ \sum_{i=1}^{f} \frac{\|s_i\|^2 (b_i)^2}{\varsigma^2} + \sum_{i=1}^{f} s^2 - \sum_{i=1}^{f} \frac{\hat{l}_i s_i^2}{\varsigma^2}.$$

Otherwise, for function $V_{12}(t)$, it follows from **(4)**

$$\dot{V}_{12} = 2\sum_{i=1}^{f} \tilde{l}_i \dot{\tilde{l}}_i = -2\sum_{i=1}^{f} \tilde{l}_i \dot{\hat{l}}_i \quad (6)$$

$$= 2\sum_{i=1}^{f} \hat{l}_i (l_i - \hat{l}_i) + 2\sum_{i=1}^{f} \frac{2^{\rho} c\varpi}{T_p} |\hat{l}_i|^{\rho} \tilde{l}_i$$

$$+ 2\sum_{i=1}^{f} \frac{2^{\iota} d\varpi}{T_p} |\hat{l}_i|^{\iota} \tilde{l}_i - \sum_{i=1}^{f} \frac{\|s_i\|^2}{\varsigma^2} \tilde{l}_i$$

For the first item of **(6)**, one has $l_i + |\hat{l}_i| \ge l_i - \hat{l}_i$. According to **Lemma 3**,

$$set\ p = 1 + \frac{1}{\rho}, q = 1 + \rho, so\ 2\left|\hat{\hat{l}}_i\right|^{\rho} l_i \le \frac{\rho^{\rho}(2l_1)^{1+\rho}}{(1+\rho)^{1+\rho}} + |\hat{l}_i|^{\rho+1}.$$

Thus, the second item of **(6)** can be written as

$$2\frac{\varpi}{T_p}2^{\rho}c|\hat{l}_i|^{\rho}\tilde{l}_i = 2\frac{\varpi}{T_p}c|\hat{l}_i|^{\rho}2^{\rho}(l_i-\hat{l}_i) \le 2^{\rho}\frac{\varpi}{T_p}c\frac{\rho^{\rho}(2l_1)^{1+\rho}}{(1+\rho)^{1+\rho}} + 2^{\rho}\frac{\varpi}{T_p}c|l_i|^{\rho+1} - 2^{\rho}\frac{\varpi}{T_p}c|l_i|^{\rho+1} - 2^{\rho}\frac{\varpi}{T_p}c|\hat{l}_i|^{\rho+1}$$

From **Lemma 2**, and $l_i + |\hat{l}_i| \ge l_i - \hat{l}_i$, thus

$$-2^{\rho}\left(l_i^{\rho+1} + |\hat{l}_i|^{\rho+1}\right) \le -(l_i + |\hat{l}_i|)^{\rho+1} \le -|l_i - \hat{l}_i|^{\rho+1} = -(\tilde{l}_i)^{\rho+1}.$$

$$So\ 2\frac{\varpi}{T_p}2^{\rho}c|\hat{l}_i|^{\rho}\tilde{l}_i \le 2^{\rho}\frac{\varpi}{T_p}c\frac{\rho^{\rho}(2l_1)^{1+\rho}}{(1+\rho)^{1+\rho}} + 2^{\rho}\frac{\varpi}{T_p}c|l_i|^{\rho+1} - \frac{\varpi}{T_p}c(\tilde{l}_i)^{\rho+1}.$$

Similarly to the second item, one gets from the third item

$$2\frac{\varpi}{T_p}2^{\iota}d|\hat{l}_i|^{\iota}\tilde{l}_i \le \frac{\varpi}{T_p}2^{\iota}d\frac{t(2l_i)^{1+\iota}}{(1+\iota)^{1+\iota}} + \frac{\varpi}{T_p}2^{\iota}d|l_i|^{\iota+1} - \frac{\varpi}{T_p}d(\tilde{l}_i)^{\iota+1}.$$

So **(6)** can become

$$\dot{V}_{12} \le -2\frac{2o-1}{2o}\sum_{i=1}^{f}\tilde{l}_i^2 + o\sum_{i=1}^{f}l_i^2 + \sum_{i=1}^{f}\frac{2^{\rho}c\varpi}{T_p}\frac{\rho^{\rho}(2l_i)^{1+\rho}}{(1+\rho)^{1+\rho}}$$

$$+\sum_{i=1}^{f}\frac{2^{\rho}c\varpi}{T_p}|l_i|^{\rho+1} + \sum_{i=1}^{f}\frac{2^{\iota}d\varpi}{T_p}\frac{(2l_i)^{1+\iota}}{(1+\iota)^{1+\iota}} + \sum_{i=1}^{f}\frac{2^{\iota}d\varpi}{T_p}|l_i|^{\iota+1}$$

$$-\sum_{i=1}^{f}\frac{\varpi d}{T_p}(\tilde{l}_i)^{\iota+1} - \sum_{i=1}^{f}\frac{\|s_i\|^2}{\varsigma^2}\tilde{l}_i - \sum_{i=1}^{f}\frac{c\varpi}{T_p}(\tilde{l}_i)^{\rho+1}$$

$$\le -\frac{2o-1}{o}V_{12} + \Delta_1 - \frac{c\varpi}{T_p}V_{12}^{\frac{\rho+1}{2}} - \frac{\varpi d}{T_p}V_{12}^{\frac{c+1}{2}} - \sum_{i=1}^{f}\frac{\|s_i\|^2}{\varsigma^2}\tilde{l}_i.$$

Where $\Delta_1 = o\sum_{i=1}^{f}l_i^2 + \sum_{i=1}^{f}\frac{2^{\rho}c\varpi}{T_p}\frac{\rho^{\rho}(2l_i)^{1+\rho}}{(1+\rho)^{1+\rho}} +$

$$\sum_{i=1}^{f}\frac{2^{\rho}c\varpi}{T_p}|l_i|^{\rho+1} + \sum_{i=1}^{f}\frac{2^{\iota}d\varpi}{T_p}\frac{t(2l_i)^{1+\rho}}{(1+\iota)^{1+\rho}} + \sum_{i=1}^{f}\frac{2^{\iota}d\varpi}{T_p}|l_i|^{\iota+1}.$$

So we have

$$\dot{V}_1 = \dot{V}_{11} + \dot{V}_{12}$$

$$\le -V_{11} - \frac{\varpi}{T_p}c|V_{11}|^{\frac{\rho+1}{2}} - \frac{\varpi}{T_p}d|V_{11}|^{\frac{c+1}{2}}$$

$$+\sum_{i=1}^{f}\frac{\|s_i\|^2(b_i)^2}{\varsigma^2} + \sum_{i=1}^{f}s^2 - \sum_{i=1}^{f}\frac{\hat{l}_i s_i^2}{\varsigma^2} - \sum_{i=1}^{f}\frac{\|s_i\|^2}{\varsigma^2}\tilde{l}_i + \Delta_1$$

$$-\frac{2o-1}{o}V_{12} - \frac{\varpi c}{T_p}\left((V_{12})^{\frac{\rho+1}{2}}\right) - \frac{\varpi d}{T_p}\left((V_{12})^{\frac{c+1}{2}}\right)$$

$$\le -\frac{\varpi}{T_p}\left(c|V_1|^{\frac{\rho+1}{2}} + d|V_1|^{\frac{c+1}{2}}\right) - \mu V_1 + \Delta_2.$$

where $\mu = min\left\{\frac{2o-1}{o},\ 1\right\}$ and $\sum_{i=1}^{f} \varsigma^2 + \Delta_1 = \Delta_2$. According to **Lemma 1**, one can get

$\varpi_1 = \frac{1-\rho}{\iota-p}, \varpi_2 = \frac{\iota-1}{\iota-p}, \varpi = \frac{\Gamma(\varpi_1)\Gamma(\varpi_2)}{c\Gamma(1)(\iota-p)}\left(\frac{c}{d}\right)\varpi_1$. Sliding mode will reach in predefined time, $t \le Tp$.

When $s_i = e_{vi} + \frac{r}{T_s}\left(ae_{xi}^{\alpha} + be_{xi}^{\beta}\right)$, then $\dot{e}_{xi} = e_{vi} = -\frac{r}{T_s}\left(ae_{xi}^{\alpha} + be_{xi}^{\beta}\right)$.

Set $V_2 = e_x^T e_x$, so

$$\dot{V}_2 = 2e_x^T\dot{e}_x = -2e_x^T\left(\frac{r}{T_s}\left(ae_{xi}^{\alpha} + be_{xi}^{\beta}\right)\right)$$

$$\le -2\frac{r}{T_s}\left(a(V_2)^{\frac{\alpha+1}{2}} + b(V_2)^{\frac{\beta+1}{2}}\right).$$

Sliding mode is predefined-time stable, and settling time is Ts. In summary, the total convergence time to obtain predefined-time bipartite containment is $t \le Tp + Ts$.

Remark 1: Compared with the previous articles, the convergence time of this brief is preset and is independent of the parameters of the system [1,2].

The proof is completed.

4. Simulations

In this section, consider a multi-agent system consisting of 4 followers labeled 1, 2, and 3, with numbers 4 and 2 as leaders labeled 5 and 6. The topology graph is shown in **Figure 1**, where 1 and 3 is a side while 2 and 4 is another side. Nonlinear function of followers are $f_i = -\left(\frac{\dot{g}_1}{\dot{l}_1}\right)\sin(x_i) - \left(\frac{\dot{k}_1}{\dot{m}_1}\right)v_i$, where $\dot{g}_1, \dot{l}_1, \dot{k}_1, \dot{m}_1$ are acceleration of gravity, length, friction coefficient and mass respectively, and let $\dot{g}_1 = 10, \dot{m}_1 = 10, \dot{l}_1 = 10, \dot{k}_1 = 0.1$. Disturbances of followers are $\delta_i = 0.5\cos(0.5it) + \frac{i}{5\pi}$. Initial values are given by:

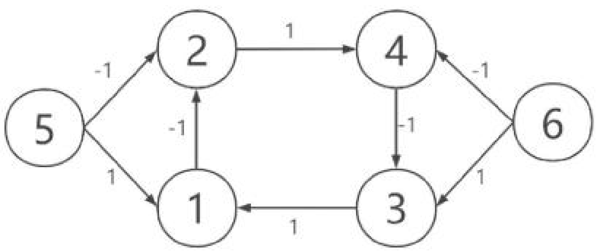

Figure 1. Communication topology of six agents

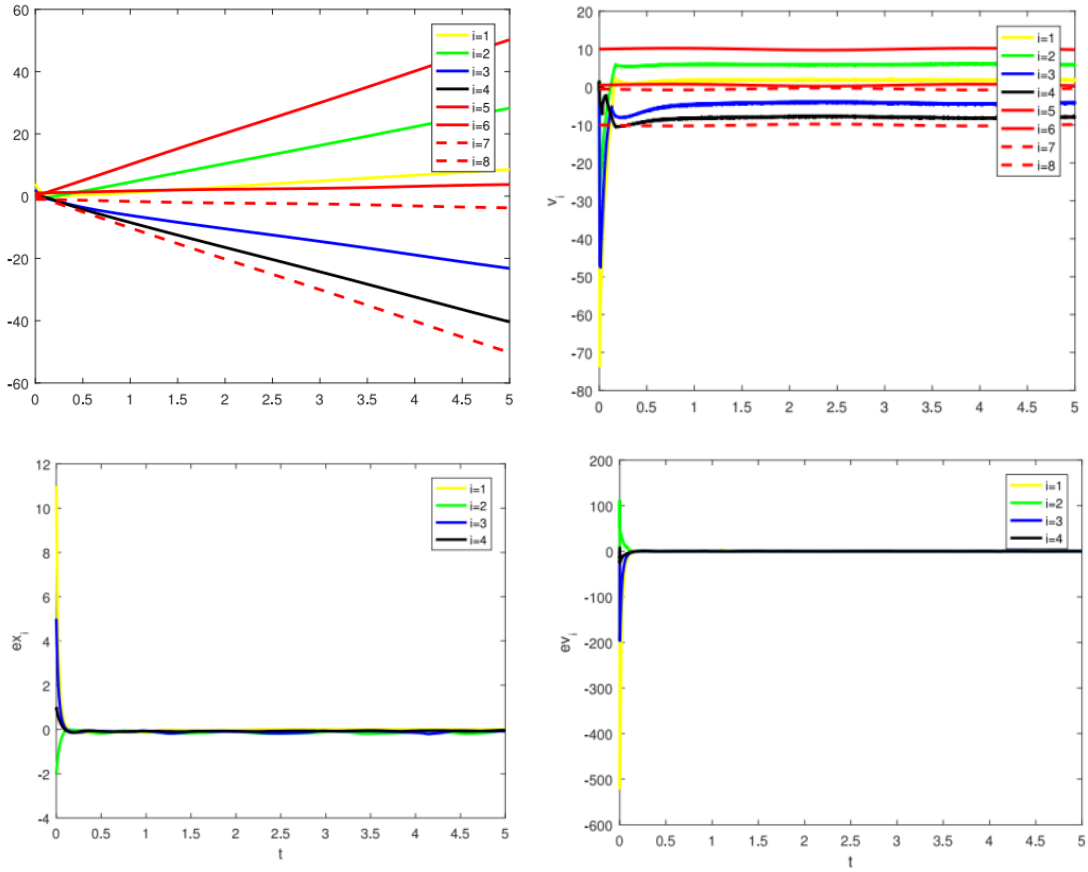

Figure 2. Position and velocity change and error

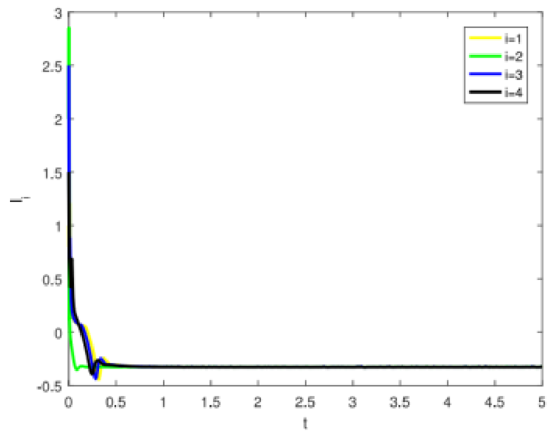

Figure 3. Adaptive change curves

$x_i(0) = (4, 1, 2, 0, 1, 0)$, $v_i(0) = (-1, 2, 0, -1, 0.5, 10)$. System parameters are given by $a = 9$, $b = 2$, $\alpha = 0.5$, $\beta = 1.2$, $Ts = 0.2$, $c = 0.7$, $d = 20$, $\rho = 0.5$, $i = 3$, $Tp = 0.3$, $\varsigma = 5$. And *_x0005_1* = 1/5, *_x0005_ 2* = 4/5, *_x0005_3* = 0.36. Simulation results are described in **Figure 2** and **Figure 3**. Change curves of position and velocity and errors of bipartite containment are depicted in **Figure 2**. Adaptive curves are shown in **Figure 3**. As shown in

Figure 2, followers quickly enter into the geometric region spanned by leaders in predefined time $t = Tp + Ts$ for a given initial value, and bipartite containment control is achieved. From **Figure 3**, a continuous controller with an adaptive method is chattering-free, so it is effective in engineering examples.

5. Conclusion

The distributed predefined-time bipartite containment control for a second-order multi-agent system with actuator faults and adaptive was investigated. Numerical examples are given to show the effectiveness of the proposed method. In future work, combining both finite-time theory and bipartite containment theory can be considered to solve the problem of Denial of Service (DoS) attacks under switching and fixed topologies.

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