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Future Point Estimation Method for Unpowered Gliding Targets

Shoufeng Wang*

Jiangsu Automation Research Institute, Lianyungang 222061, Jiangsu Province, China

***Corresponding author:** Shoufeng Wang, jariwsf@163.com

Abstract: In modern warfare, unpowered glide munitions, represented by JDAM, are widely used. Accurately predicting the future trajectory of such targets is crucial for intercepting them. This paper proposes a future point prediction method for unpowered gliding targets based on attitude computation. By estimating the current state of the target, we derive the target's attitude coordinate system. Subsequently, the paper analyzes the forces acting on the target and updates the state transition matrix, ultimately calculating the future position of the target. Experimental results show that, compared to traditional methods, this approach improves the accuracy of future point predictions by 9% to 45%.

Keywords: Guided bomb; Extended Kalman filter; Maneuver prediction; Fire control calculation; Attitude coordinate system; Future point prediction

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1. Introduction

The unpowered glide-guided bomb represented by JDAM has the characteristics of low cost, high precision and easy mass modification and production. This type of bomb is widely used in modern warfare [1]. The research on the motion characteristics of this type of target and the application prediction of this type of target is very important for the interception efficiency of short-range air defense weapons [2]. From the time sequence of the fire control computer, the process of future point prediction is divided into two stages. The first stage is based on the Bayesian estimation framework of the filter estimation problem. It is mainly a variety of improvement methods of the Kalman filter. A large number of scholars have made achievements in this field [3–5]. Such as Extended Kalman Filter (EKF) [6], Unscented Kalman Filter (UKF) [7] and Cubature Kalman Filter (CKF) [8]. CKF is an excellent method. Compared with EKF and UKF, this method has higher filtering accuracy and stability. The second stage is the future point prediction based on the present point estimate. In the field of maneuvering target tracking, there are many kinds of motion prediction models. Such as the Constant Velocity model (CV), Constant Acceleration model (CA), uniform turning model and so on. The above model is widely used in Kalman filter prediction. However, in the case of maneuvering target prediction, these models are often difficult to obtain satisfactory results. At present, for the prediction of maneuvering targets, excellent methods mainly include the Singer Model [9], Current Statistical (CS) model [10–12] and the Multi Model Interaction (IMM) model [13–15]. However, whether it is singer model, CS model or IMM model, they all need the observed value of maneuvering target in each prediction period. The parameter values of the model need to be updated iteratively. As a result, these models are not suitable for predicting future points over hundreds of cycles.

In order to solve the problem of decreasing prediction accuracy caused by the mismatch between traditional motion model and actual target motion characteristics in future point prediction. In this paper, a method of future point prediction for non-powered glide targets based on attitude solution is proposed. Based on the estimation of the present point state of the target, the attitude of the target is estimated, and then the force of the target is analyzed to achieve the estimation of its motion state.

2. Target state estimation method based on CKF

2.1. State model

Set the target state estimation coordinate system as the northeast sky coordinate system OXYZ. The state of the target at time k is denoted as $X_k = [x, \dot{x}_k, \ddot{x}_k, y, \dot{y}_k, \ddot{y}_k, z, \dot{z}_k, \ddot{z}_k]$. Under the uniformly accelerated motion model (CA), the equation of state is:

$$X_{k+1} = F_k X_k + W_k \tag{1}$$

Among these, F_k represents the state transition matrix of the target at time k. The matrix is obtained from the kinematic characteristics of the target. The matrix here is a 9×9 matrix here:

$$\boldsymbol{F}_k = \begin{bmatrix} 1 & T & \frac{1}{2}T^2 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & T & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & T & \frac{1}{2}T^2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & T & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & T & \frac{1}{2}T^2 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & T \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \tag{2}$$

W_k stands for process noise. The noise follows a Gaussian distribution with variance, Q. T is the filtering period.

2.2. Observation model

The target observation values are azimuth α, pitch β and distance d obtained by observation station (x_o, y_o, z_o). The observation equation of the system is as follows:

$$Z_{k+1} = h(x_{k+1}) + w_{k+1} \tag{3}$$

Z_{k+1} is the measured value at k+1. $h(x_{k+1})$ is the measurement equation. Then, the observation equation is:

$$\alpha(k) = \arctan\frac{x_k - x_0}{y_k - y_0} \tag{4}$$

$$\beta(k) = \arctan\frac{z_k - z_0}{\sqrt{(y_k - y_0)^2 + (x_k - x_0)^2}} \tag{5}$$

$$d(k) = \sqrt{(x_k - x_0)^2 + (y_k - y_0)^2 + (z_k - z_0)^2} \tag{6}$$

Among these, α(k), β(k) and d(k) respectively represent the predicted azimuth Angle, pitch angle and target distance at time k. w_{k+1} represents the measurement noise with variance R.

2.3. State estimation method based on CKF

On the basis of the target estimate at the above time, a state transition matrix is used to achieve one-step state prediction:

(1) Update

Calculation of cubature points:

$$X_{j,k-1} = S_{k-1}\xi_j + \hat{X}_{k-1|k-1} \tag{7}$$

Among which, $S_{k-1} = chol\{P_{k-1}/_{k-1}\}$, where $P_{k-1}/_{k-1}$ represents the covariance matrix of the target state at time k-1. chol{} represents the Cholesky decomposition of the matrix, namely $P_{k-1}/_{k-1} = S_{k-1}S^{T}_{k-1}$. ξ_j represents the j-th volumetric point, where J represents the number of equiweight points. $\xi_j = \sqrt{J/2}[\delta]_j$, $[\delta]_j$ represents the j-th column element of the matrix $[I^{mxm}, I^{mxm}]$. I^{mxm} represents the m-dimensional identity matrix. m = J/2.

Calculation of volume points propagating through the equation of state:

$$X^{*}_{j,k|k-1} = \phi_{k|k-1}X_{j,k-1} \tag{8}$$

State prediction:

$$\hat{X}_{k|k-1} = \sum_{j=1}^{m}\omega_j X^{*}_{j,k|k-1} \tag{9}$$

Covariance prediction:

$$P_{k|k-1} = \sum_{j=1}^{m}\omega_j X^{*}_{j,k|k-1}X^{*T}_{j,k|k-1} - \hat{X}_{k|k-1}X^{T}_{k|k-1} + Q_{k-1} \tag{10}$$

(2) Measurement update

Calculation of cubature points:

$$X_{j,k|k-1} = S_{k|k-1}\xi_j + \hat{X}_{k|k-1} \tag{11}$$

In the equation, $S_{k-1} = chol\{P_{k-1}/_{k-1}\}$.

Calculation of volume points propagating through the equation of state:

$$Z_{j,k|k-1} = h(X_{j,k|k-1}) \tag{12}$$

Measurement prediction:

$$\hat{Z}_{k|k-1} = \sum_{j=1}^{m}\omega_j Z_{j,k|k-1} \tag{13}$$

Estimation of Innovation Covariance:

$$P_{zz,k|k-1} = \sum_{j=1}^{m}\omega_j Z_{j,k|k-1}Z^{T}_{j,k|k-1} - \hat{Z}_{k|k-1}Z^{T}_{k|k-1} + R_k \tag{14}$$

Estimation of cross covariance:

$$P_{xz,k|k-1} = \sum_{j=1}^{m}\omega_j X_{j,k|k-1}Z^{T}_{j,k|k-1} - \hat{X}_{k|k-1}Z^{T}_{k|k-1} \tag{15}$$

Calculation of Kalman filter gain:

$$K_k = P_{xz,k|k-1} P^{-1}_{zz,k|k-1} \tag{16}$$

Status update:

$$\hat{X}_k = X_{k|k-1} + K_k (Z_k - Z_{k|k-1}) \tag{17}$$

Estimation of covariance:

$$P_k = P_{k|k-1} - K_k P_{zz,k|k-1} K_k^T \tag{18}$$

3. Future point prediction based on attitude calculation

3.1. Target attitude coordinate system

O'$X_i Y_i Z_i$ is the target attitude coordinate system, X_t aligned with the direction of the target's velocity, Y_t is perpendicular to the wing plane, Z_t points toward the right wing. It is assumed that the thrust, F_t is aligned with the target's velocity direction, X_t. The lift F_L is aligned with the Y_t, considering that the mass,m can be neglected in the calculations, therefore, the thrust, F_t is used. The lift, F_L and the gravity, F_g represent the target's acceleration.

The transformation matrix of the target state estimation coordinate system to the target attitude coordinate system is set as T_0:

$$T_0 = \begin{bmatrix} T_{11} & T_{12} & T_{13} \\ T_{21} & T_{22} & T_{23} \\ T_{31} & T_{32} & T_{33} \end{bmatrix} \tag{19}$$

The direction of the X_t axis in the target attitude coordinate system O'$X_i Y_i Z_i$ can be determined by the velocity $(\dot{x}_k, \dot{y}_k, \dot{z}_k)$ in $\hat{X}_k$.

$$\begin{pmatrix} T_{11} \\ T_{12} \\ T_{13} \end{pmatrix} = (\dot{x}_k^2 + \dot{y}_k^2 + \dot{z}_k^2)^{-1} \begin{pmatrix} \dot{x}_k \\ \dot{y}_k \\ \dot{z}_k \end{pmatrix} \tag{20}$$

In the state estimation coordinate system O'$X_i Y_i Z_i$, the acceleration of the target is composed of the tension force F_t, lift force F_L and gravity force F_g.

$$A = F_t + F_L + F_g \tag{21}$$

$$A = (\ddot{x}_k, \dot{y}_k, \ddot{z}_k)^T \tag{22}$$

$$F_g = (0, 0, 9.8)^T \tag{23}$$

F_t removes the component of gravity in the X_t direction to form the acceleration in the X_t direction. Then,

$$|F_t| = (\ddot{x}_k, \ddot{y}_k, \ddot{z}_k) \begin{pmatrix} T_{11} \\ T_{12} \\ T_{13} \end{pmatrix} - (0, 0, 9.8) \begin{pmatrix} T_{11} \\ T_{12} \\ T_{13} \end{pmatrix} \tag{24}$$

Simultaneous (20)–(23). The direction of the lift force F_L can be obtained.

$$\begin{pmatrix} F_{Lx} \\ F_{Lx} \\ F_{Lz} \end{pmatrix} = \begin{pmatrix} \ddot{x}_k \\ \ddot{y}_k \\ \ddot{z}_k \end{pmatrix} - \begin{pmatrix} 0 \\ 0 \\ 9.8 \end{pmatrix} - |F_t| \begin{pmatrix} T_{11} \\ T_{12} \\ T_{13} \end{pmatrix} \tag{25}$$

The direction of Y_t can be obtained by lifting force F_L.

$$\begin{pmatrix} T_{21} \\ T_{22} \\ T_{23} \end{pmatrix} = (F_{Lx}^2 + F_{Ly}^2 + F_{Lz}^2)^{-1} \begin{pmatrix} F_{Lx} \\ T_{Lx} \\ T_{Lz} \end{pmatrix} \tag{26}$$

After determining the direction of X_t and Y_t. The direction of Z_t can be calculated by the cross-product formula of vectors.

$$\begin{pmatrix} T_{31} \\ T_{32} \\ T_{33} \end{pmatrix} = \begin{pmatrix} 0 & -T_{13} & T_{12} \\ T_{13} & 0 & -T_{11} \\ -T_{12} & T_{11} & 0 \end{pmatrix} \begin{pmatrix} T_{21} \\ T_{22} \\ T_{23} \end{pmatrix} \tag{27}$$

O'$X_iY_iZ_i$ is determined by T_0.

3.2. State model

Assuming that the target undergoes uniformly accelerated motion during a prediction period.

$$X_{t,k+1} = F_k X_{t,k} + W_{t,k} \tag{28}$$

$$X_{k+1} = T_0^{-1} X_{t,k+1} \tag{29}$$

$X_{t,k}$ representing the state of the target at time k in the attitude coordinate system, $W_{t,k}$ is the state noise, which follows a Gaussian distribution with a variance of Q.

The force relationship acting on the target in the attitude coordinate system is as follows:

$$A_{xt} = F_t - F_g \sin\boldsymbol{\theta} \tag{30}$$

$$A_{yt} = F_L - F_g \cos\boldsymbol{\theta} \cos\boldsymbol{\gamma} \tag{31}$$

$$A_{zt} = F_g \cos\boldsymbol{\theta} \sin\boldsymbol{\gamma} \tag{32}$$

In the formula, θ is the target's angle of attack, γ is the target's roll angle. It is assumed that during the extrapolation period, the roll angle γ remains constant. The calculation method for the angle of attack θ is as follows:

$$\begin{pmatrix} V_x \\ V_y \\ V_z \end{pmatrix} = T_0^{-1} \begin{pmatrix} V_{xt} \\ V_{yt} \\ V_{zt} \end{pmatrix} \tag{33}$$

$$\boldsymbol{\theta} = a\tan\left(\frac{|V_z|}{\sqrt{V_x^2 + V_y^2}}\right) \tag{34}$$

In the formula, V_X, V_Y, V_Z represent the component velocities of the target along each axis in the state estimation coordinate system. V_{xt}, V_{yt}, V_{zt} represent the component velocities of the target along each axis in the attitude coordinate system.

For gliding guided bomb targets, considering that they do not have their propulsion, its axial force F_t、The lift F_L is only related to the drag. According to the formula for air resistance

$$F_t = \frac{1}{2} C_1 \boldsymbol{\rho} S V_k^{\,2} \tag{35}$$

$$F_L = \frac{1}{2} C_2 \boldsymbol{\rho} S V_k^{\,2} \tag{36}$$

In the formula,

$$V_k = \sqrt{V_{x,k}^2 + V_{y,k}^2 + V_{z,k}^2} \tag{37}$$

Assuming that the drag coefficient, C_1 of the target's motion remains constant during the future point prediction time period, the lift coefficient, C_2, air density, p and the frontal area, S remains unchanged, then the axial force F_t, the lift F_L are proportional to the square of the velocity.

$$F_{L,k} = (v_{k+1} / v_k)^2 F_{L,k-1} \tag{38}$$

$$F_{t,k} = (v_{k+1} / v_k)^2 F_{t,k-1} \tag{39}$$

When making future point predictions. The resistance coefficient C_1, lift coefficient C_2, air density, p and windward area, S remain unchanged if the prediction step length is short. During the actual flight of the target, the attitude of the target changes due to the action of force, which leads to the change of aerodynamic characteristics. Therefore, it is necessary to update the target attitude coordinate system. In practical application, the actual predicted step size will be equally divided into several segments. n steps are set to one stage. $X_{t,k+n}$ is converted to X_{k+n} in the northeast sky coordinate system by equation (29). In the calculation of (19)–(27), X_k is replaced by X_{k+n}. The attitude coordinate system has been updated. Calculate n cycles in the new attitude coordinate system. The final result is obtained by iterating the calculation process.

4. Simulation

In this paper, the JDAM-GBU-38 simulation route proposed in the literature was cited for simulation tests [16]. The initial altitude of the route was set at 12,000 m. The starting distance is set to 10,000 m and the initial speed is set to 280 m. The simulation time is 64.4 seconds. The uniformly accelerated linear motion prediction method was set as the control group. The experimental group is the method proposed in this paper. The observation station is at the origin. The data sampling period is 0.01s. The predicted step size is 200 cycles. The number of Monte Carlo tests is 50.

Root Mean Squared Error (RMSE) and Time Average Root Mean Square Error (TRMSE) are used to evaluate simulation performance. RMSE and TRMSE are defined as follows:

$$\text{RMSE} = \sqrt{\frac{1}{M} \sum_{i=1}^{M} \left[\left(\hat{x}_k^i - x_k^i \right)^2 \right]} \tag{40}$$

$$\text{TRMSE} = \frac{1}{L} \sum_{k=0}^{L-1} \sqrt{\frac{1}{M} \sum_{i=1}^{M} \left[\left(\hat{x}_k^i - x_k^i \right)^2 \right]} \tag{41}$$

Where $\hat{x}_k^i$ is the result of the position estimation of the target in a certain direction at the k time in the Monte Carlo experiment. M is the total number of Monte Carlo events. L is the total number of samples taken in a Monte Carlo test.

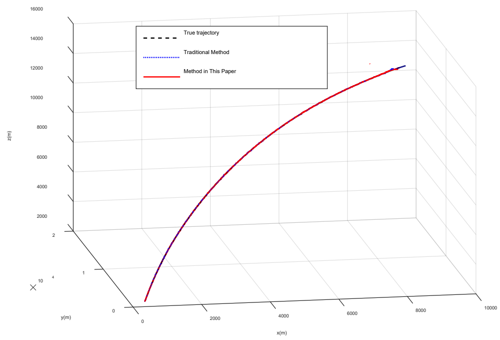

Figure 1. Target route and forecast result chart.

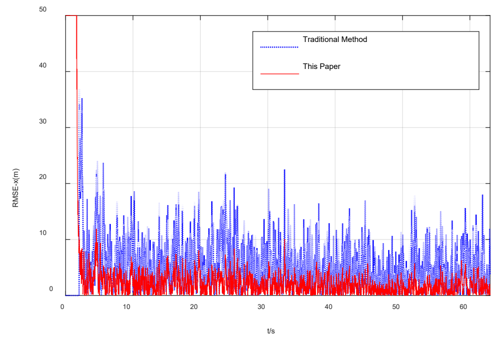

Figure 2. RMSE value in the X- direction.

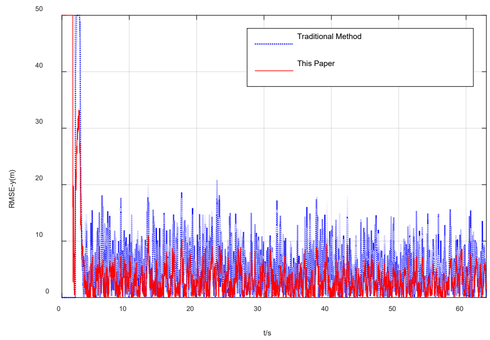

Figure 3. RMSE value in the Y- direction.

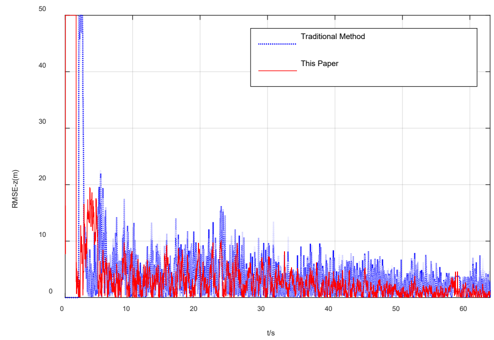

Figure 4. RMSE value in the Z- direction.

As shown in **Figure 1**, both traditional methods and the attitude calculation-based methods effectively reflect the target's actual flight trajectory. From **Figure 2** to **Figure 4**, it can be observed that the future point prediction method based on attitude calculation begins to outperform the traditional methods after approximately 500 cycles. This is because the attitude calculation method is more dependent on the target's acceleration compared to the traditional uniform acceleration prediction method. Only after the acceleration from the Kalman filter converges can the attitude calculation-based method achieve the desired results.

Table 1 reflects the TRMSE values after the predictions of the two methods for the JDAM flight path have stabilized. Compared to the traditional method, the attitude calculation-based method improved the prediction accuracy by 45%, 38%, and 9% in the X, Y, and Z directions, respectively.

5. Conclusion and prospect

In this paper, the Cubature Kalman Filter is used to obtain the present point state of the target, and on this basis, the attitude coordinate system of the target is established to analyze the force state of the target, and the future point is estimated.

It should be noted that since the derivation of the target attitude coordinate system is based on the present point estimation, the accuracy of the present point estimation, especially the estimation accuracy of velocity and acceleration, will directly affect the accuracy of the attitude coordinate system. The next step is to improve the accuracy of the present point estimation by improving the motion model of the unpowered glide target.

Disclosure statement

The author declares no conflict of interest.

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