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Furnace Temperature Curve Optimization Model Based on Differential Evolution Algorithm

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Abstract: When soldering electronic components onto circuit boards, the temperature curves of the reflow ovens across different zones and the conveyor belt speed significantly influence the product quality. This study focuses on optimizing the furnace temperature curve under varying settings of reflow oven zone temperatures and conveyor belt speeds. To address this, the research sequentially develops a heat transfer model for reflow soldering, an optimization model for reflow furnace conditions using the differential evolution algorithm, and an evaluation and decision model combining the differential evolution algorithm with the Technique for Order Preference by Similarity to Ideal Solution (TOPSIS) method. This approach aims to determine the optimal furnace temperature curve, zone temperatures of the reflow oven, and the conveyor belt speed.

Keywords: Furnace temperature curve; Difference equations; Differential evolution algorithms; TOPSIS methods

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1. Introduction

In the recent electronics assembly industry, the quality of soldering directly affects the reliability and performance of electronic products. As a key piece of equipment in the realization of surface mount technology, the precise control of the furnace temperature curve during the welding process is essential to ensure welding quality and production efficiency. The optimization of the furnace temperature curve can not only improve the consistency and quality of welding but also reduce energy consumption and production costs, thereby enhancing the market competitiveness of enterprises.

Traditional furnace temperature curve settings are often empirical and difficult to adapt to changing production conditions and material characteristics. With the development of computing technology, the design method of furnace temperature curves based on mathematical models and algorithm optimization has gradually become a research hotspot. Due to its simplicity, robustness, and efficiency, the differential evolution (DE) algorithm has shown great potential in solving nonlinear, multivariate, and complex system optimization problems.

In this paper, based on the heat transfer model during the welding process of the reflow furnace, an optimization model of the furnace temperature curve based on a differential evolution algorithm is constructed.

The model comprehensively considers the thermal radiation and thermal convection effect during the welding process and the thermal properties of the welding material and realizes the accurate control of the furnace temperature curve by optimizing the temperature of each temperature zone and the speed of the conveyor belt. This study not only provides a scientific method for optimizing the furnace temperature curve but also further considers the symmetry of the furnace temperature curve through the TOPSIS method, to achieve better welding results.

Through this study, we aim to provide an efficient and automated furnace temperature curve design tool for the electronics assembly industry to improve welding quality, reduce production costs, and promote the development of electronic manufacturing technology.

2. Question citation and analysis

When soldering electronic components to a circuit board, the temperature of each part of the reflow oven has a great impact on the quality of the product. In this paper, a reflow oven is taken as the research object, and there are 11 small temperature zones in the reflow furnace, the front and back areas, each small temperature zone is 30.5 cm in length, and there is a gap of 5 cm between the adjacent small temperature zones, and the length of the front and rear areas is 25 cm. In actual production, the product quality is controlled by adjusting the set temperature of each temperature zone and the furnace speed of the conveyor belt. When adjusting, the temperature in the small temperature zones 1–5 is required to be consistent, the temperature in the small temperature zones 8–9 is consistent, and the temperature in the small temperature zones 10–11 is kept at 25 °C. The speed of the conveyor belt can be adjusted in the range of 65–100 cm/min.

The gap between the front of the furnace area, the back of the furnace area, and the small temperature zone is not subject to special temperature control, and its temperature is related to the temperature of the adjacent temperature zone. Additionally, the temperature near the boundary of each temperature zone may also be affected by the temperature of the adjacent temperature zone while the temperature of the production workshop is kept at 25 °C. After the reflow furnace is started, the air temperature in the furnace will reach a stable level in a short time, after which the reflow furnace can be used for welding work. The temperature curve in the center of the welding area is called the furnace temperature curve, and its manufacturing boundary during production is shown in **Table 1**. The research is conducted based on this information.

Table 1. Boundary of furnace temperature curve

Boundary	Minimum value	Maximum value	Unit
Slope of temperature rise	0	3	°C/s
Slope of temperature drop	-3	0	°C/s
Time when temperature rise at 150 °C to 190 °C	60	120	s
Time when the temperature is greater than 217 °C	40	90	s
Peak temperature	240	250	°C

2.1. Mathematical model to describe temperature variation in the welding area

In this study, the temperature of each temperature zone is 173 °C (zones 1–5), 198 °C (zone 6), 230 °C (zone 7), and 257 °C (zones 8–9) respectively with a conveyor belt speed of 78 cm/min. The temperature of the center of the welding zone is listed, the temperature of the center of the welding zone at the end of the small temperature zone 8 is plotted, and the corresponding furnace temperature curve is drawn. The temperature in the center of the soldering area every 0.5 s is stored in the result.csv spreadsheet.

To solve this problem, the analysis method in this article is as follows: Firstly, we set the target temperature of each small temperature zone of the reflow oven. After the furnace reached a steady state, we analyzed the temperature conditions in different areas of the furnace, including the middle of the small temperature zone, near the boundary, and the gap between the small temperature zones. By applying the Fourier theorem in heat transfer science, we derive the functional relationship between temperature and position in the furnace. This analysis allows us to quantify the temperature distribution in the middle of the small temperature zone, near the boundary, and in the interstitial area, and create a picture of the temperature distribution inside the reflow oven. Secondly, the process by which the printed circuit board is heated as it moves in the reflow oven. We can determine the temperature in the reflow furnace as a function of position according to the Fourier theorem in heat transfer science. The temperature in the middle of the small temperature zone, the temperature near the boundary of the small temperature zone, and the temperature of the gap in the small temperature zone were quantified, and the temperature in the reflow furnace was obtained as a function of position. For the process that the printed circuit board is heated when it moves in the reflow furnace in the second step, the relationship between the heat absorbed by the printed circuit board per unit of time and the ambient temperature of the reflow furnace and the relative motion velocity is considered, and the differential equation is listed accordingly. The differential equation is transformed into a difference equation by using the improved Euler formula algorithm. A model of the temperature change in the soldering area of the printed circuit board based on the difference equation was established in the reflow oven welding heat transfer model, and the corresponding furnace temperature curve was drawn, as shown in **Figure 1**.

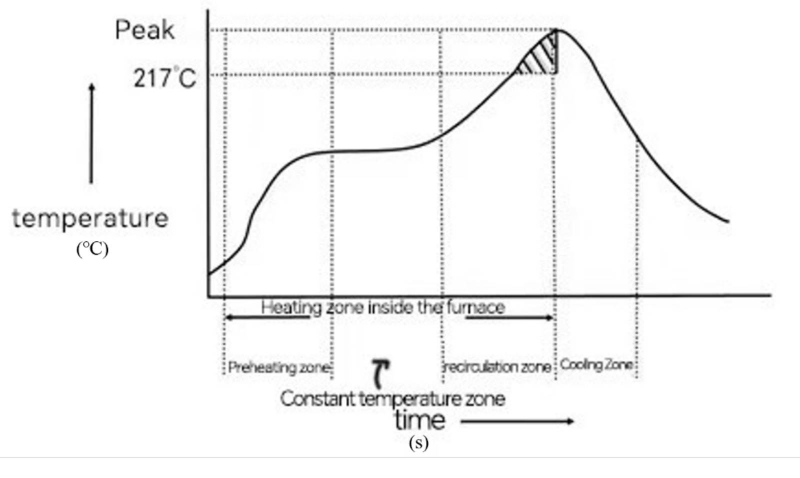

Figure 1. Furnace temperature curve

2.2. Robin Hendrick's view on blended learning

In this phase, we set the temperature of each small temperature zone of the reflow oven to 182 °C (small temperature zones 1–5), 203 °C (small temperature zone 6), 237 °C (small temperature zone 7), and 254 °C (small temperature zones 8–9), respectively. Under these conditions, our main goal was to explore the maximum speed at which the conveyor belt could safely pass through the furnace without compromising the quality of the weld.

To achieve this, we performed the following analysis: When determining the maximum furnace speed of the conveyor belt, the key limiting factor is whether the furnace temperature curve meets the established process limits. Based on this, we adopted a meticulous approach, following the furnace temperature curve model established in the reflow oven welding heat transfer model, and simulating the furnace temperature curve at different speeds by dot drawing technology. We traverse the belt speeds in fine steps to ensure that the furnace temperature profile meets the requirements of the process at each speed.

In this process, we continuously optimize the traversal strategy by gradually increasing the speed while monitoring whether the furnace temperature curve exceeds the preset limits. As soon as we found that the furnace temperature curve was out of bounds, we adjusted the step size and performed a more detailed search to ensure that we found the maximum speed that was closest to the limit and did not cross the boundary. This method not only guarantees the safety of the welding process but also maximizes productivity.

2.3. Optimizing furnace temperature curve to minimize over-temperature areas

Temperature control in the center of the welding area is particularly critical during the welding process. The goal of this paper is to ensure that this temperature exceeds 217 °C for as short a period as possible and that the peak temperature is effectively controlled to avoid thermal damage to the material or affect the weld quality. The ideal furnace temperature curve should minimize the coverage area between 217 °C and the peak temperature while maintaining the welding results, as shown in the shaded section of **Figure 1**.

To solve this problem, this paper analyzes the minimum area of the corresponding part of the furnace temperature curve in a given temperature range and the conveyor belt passing speed range and first gives the formula for calculating the area based on the reflow oven welding heat transfer model. The area covered by the furnace temperature curve exceeding 217 °C to the peak temperature (shaded in **Figure 1**) was taken as the optimization goal, and the environmental parameters (temperature of each temperature zone, conveyor belt passing speed) were used as variables, and the differential evolution algorithm was used to continuously generate offspring through two processes, mutation and evolution, and the minimum value of the area was found.

2.4. Pursue symmetry of the furnace temperature curve and minimize the over-temperature area

During the soldering process, in addition to meeting the process limits, it is also desirable that the furnace temperature curve exceeds 217 °C on both sides of the peak temperature centerline should be as symmetrical as possible (see **Figure 1**).

To solve this problem, the analysis of this paper is as follows: skewness is often used to measure the symmetry of the probability distribution of random variables in probability statistics, and the skewness of sample X is calculated as follows:

$$Sk(X) = E\left[(\frac{X-\mu}{\sigma x})^3 \right] \quad (1)$$

where *μ* and *X* are the mean and standard deviation of sample *X*, respectively. *Sk(X)* can be in the range of (-1,1), and its positive and negative means that the distribution of the observed data of the sample is positively skewed and negatively skewed, and the closer *Sk(X)* is to 0, the probability is dedicated, the more symmetrical the distribution.

We were inspired by this to consider the time of the furnace temperature curve (x-axis) as the observed value of the sample, the temperature (y-axis) is normalized as the probability distribution of the observed values, and the skewness is used to quantify the peak temperature on the furnace temperature curve.

In this study, we not only pursued the minimization of the area covered by the furnace temperature curve exceeding 217 °C to the peak temperature but also sought to achieve the symmetry of the furnace temperature curve on both sides of the peak temperature to ensure uniformity and quality of the welding process. To measure the symmetry of the furnace temperature curve, we introduce the statistic of skewness, the closer its value is to 0, the more symmetrical the furnace temperature curve is.

To comprehensively evaluate the performance of the furnace temperature curve, we used the TOPSIS method to construct an evaluation model. The model combines two key indicators, area and skewness, to generate a comprehensive evaluation index for each furnace temperature curve. This index reflects the performance of the furnace temperature curve in terms of area minimization and symmetry, providing us with comprehensive evaluation criteria.

Under the framework of the differential evolution algorithm, we use the comprehensive evaluation index as the objective function to guide the algorithm in search for the optimal solution. The algorithm starts from the initial population and continuously iterates through mutation and cross-operation to optimize the parameters of the furnace temperature curve until the optimal furnace temperature curve meets the established criteria. This process not only considers the area of the furnace temperature curve but also fully considers the symmetry of the curve to ensure the efficiency of the welding process and the consistency of product quality.

3. Model assumptions, establishment and solving

3.1. Model building

In this study, we focused on analyzing the heating dynamics of printed circuit boards inside a reflow oven. Firstly, given the relative displacement between the printed circuit board and the gas in the furnace as it moves in the furnace, our model mainly considers the influence of two heat transfer mechanisms, heat radiation and heat conduction, on the heating of the circuit board, which is based on the main heat exchange mode under relative motion [1].

Secondly, given that the printed circuit board under study is only 0.15 mm thick, its thin nature means that its surface temperature can reasonably be considered as the temperature at the center of the soldering region, an assumption that simplifies the analysis of soldering hot spots without significant loss of calculation accuracy.

Finally, the model was established without considering the influence of temperature change during the phase transformation of solder paste. This decision is based on a preliminary analysis of the thermal dynamics of each stage of the welding process, and we believe that the effect of the phase transition of the solder paste on the overall temperature distribution is of secondary importance under the specific conditions of this study. This simplification helps us to focus our research on the optimization of furnace temperature curves, thus addressing the main thermodynamic problems in the welding process more directly.

3.2. Establishment and solution of the model

3.2.1. Temperature distribution model in the reflow oven

We divided the reflow furnace into three parts: the middle of the small temperature zone, the vicinity of the boundary of the small temperature zone, and the gap between the small temperature zone. The middle of the small and medium temperature zone and the boundary of the small temperature zone are collectively called the small temperature zone, and the length of each small temperature zone is l = 30.5 cm, and the length of the temperature zone gap is d = 5 cm. In particular, we specify that near the boundary of the small temperature zone is a single strip of length of $0.05l$ on both sides of the small temperature zone, and the length of the middle of the small temperature zone is $0.9l$. The location and dimensions of these three areas are shown in **Figure 2**.

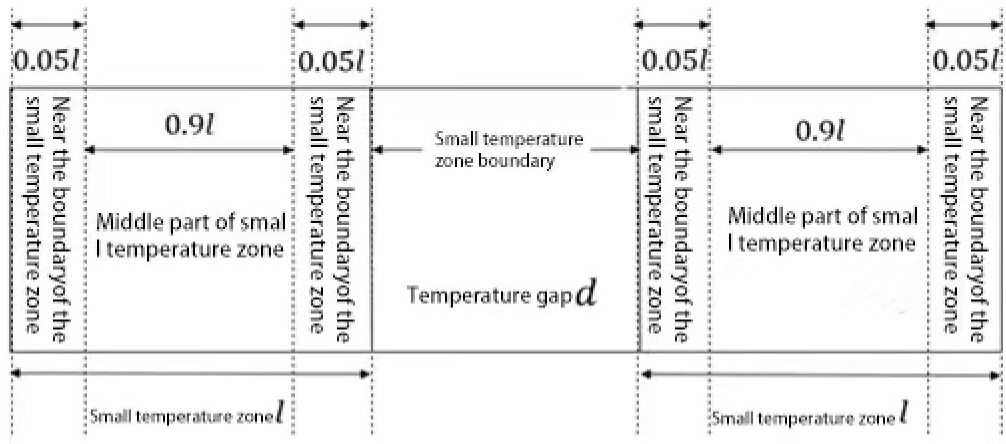

Figure 2. The location of the reflow furnace internal structure and its size

We take the two temperature zones in **Figure 2** as an example to calculate the temperature distribution inside the reflow furnace and note that the set temperature of the left small temperature zone in **Figure 2** is *T1*, and the set temperature of the right small temperature zone is *T2*. We believe that the temperature in the middle of the small temperature zone is the set temperature of the small temperature zone. Considering the system composed of the gap between the small temperature zones and the boundaries of the small temperature zones on both sides of it, after the heat transfer process is stabilized, according to the Fourier heat conduction theorem, for a microelement layer with a thickness of *dx*, the heat flux *q* passing through the layer per unit time is proportional to the temperature change rate *dT/dx*.

$$q = -\lambda \frac{dT}{dx} \tag{2}$$

Among them is the thermal conductivity. In particular, for a thin plane with a thickness of *dx*, when the heat conduction is stable, the heat coming in and out of it per unit time is equal, as shown in **Figure 3**, that is:

$$\frac{dq}{dx} = 0 \tag{3}$$

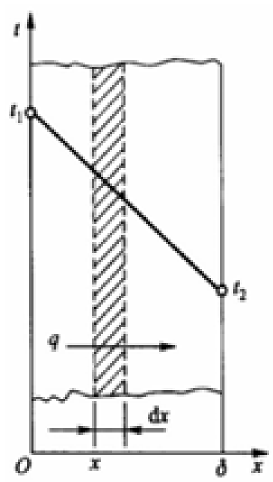

Figure 3. The heat input and output are the same

Simultaneous **Equation (2)** and **Equation (3)**, we get the second-order linear ordinary differential equation:

$$\frac{Td^2}{dx^2} = 0 \tag{4}$$

As shown in **Figure 2**, the width of the system (consisting of the small zone gap and near the boundary of the small zone on either side of it) is $d + 0.1l$. We give the following initial values of **Equation (4)**: $x = 0$, $T = T_1$, $x = d + 0.1l$, $T = T_2$. The left and right sides of **Equation (4)** are integrated twice to obtain the ambient temperature distribution of the system as:

$$T = \frac{T_2 - T_1}{d + 0.1l} x + T_1 \tag{5}$$

Let x be the position coordinates, then the final ambient temperature distribution function relationship is:

$$T_s(x) = \begin{cases} T_1 (0 \le x \le 0.9l) \\ \frac{T_2 - T_1}{d + 0.1l}(x - 0.9l) + T_1 (0.9l \le x \le l + d) \\ T_2 (l + d \le x \le 1.9l + d) \end{cases} \tag{6}$$

The image of the temperature distribution is mapped to each part of the reflow oven to make **Figure 4**, where the thick black line represents the image of the ambient temperature in the reflow oven as a function of position.

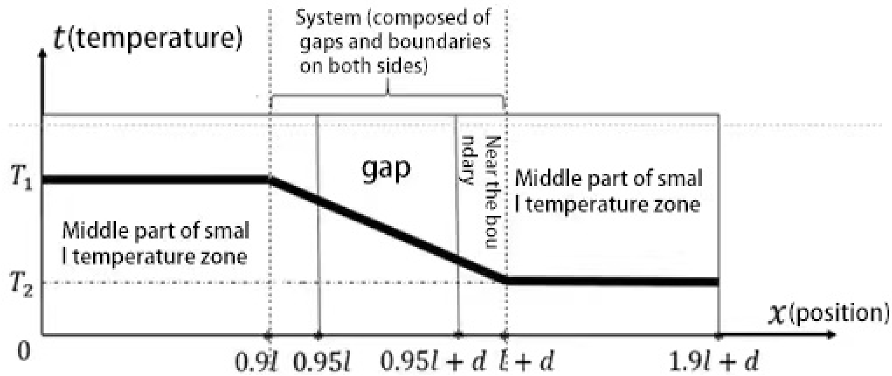

Figure 4. Image of the temperature distribution in two temperature zones

From **Figure 4**, an image of the temperature distribution between the two small temperature zones can be obtained, and as long as the set temperature of the two temperature zones is known, zone 7 can be used to calculate the temperature distribution at different locations in the reflow furnace. There are 11 temperature zones in this problem, and for temperature zones 2–10, the temperature distribution between the local temperature zone and the adjacent gaps can be obtained according to the set temperature of the other temperature zones. In particular, temperature zone 1 and temperature zone 11, are also adjacent to the area in front of the furnace and the area behind the furnace, at this time we can regard the production workshop as a stable heat source of temperature 25 ℃, that is, as the temperature zone with a set temperature of $T = 25$ ℃, and repeat the above calculation. The set values of the temperature of each temperature zone in the reflow oven welding heat transfer model are: the temperature before the furnace and the temperature after the furnace are 25 ℃, 173 ℃ (small temperature zones 1–5), 198 °C (small temperature zone 6), 230 °C (small temperature zone 7), 257 °C (small temperature zones 8–9), 25 °C (small temperature zones 10–11). Substituting these temperatures into **Equation (5)** calculates the relationship between the ambient temperature *Ts* and position *x* in the reflow furnace is shown in **Figure 5**.

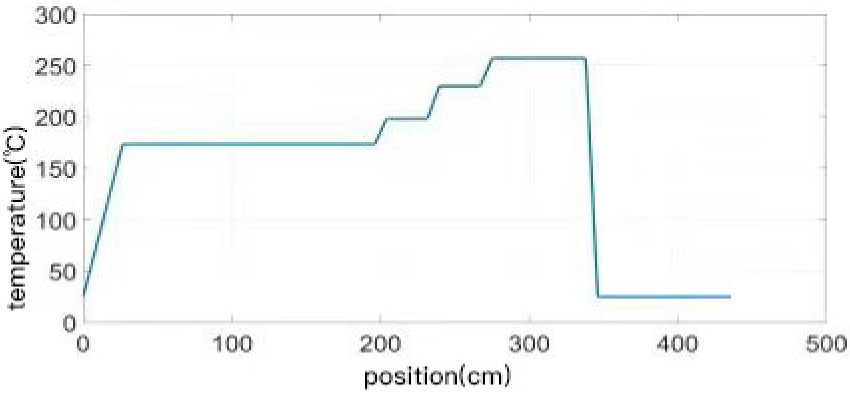

Figure 5. The relationship between the ambient temperature *Ts* of the reflow oven and the position *x*

3.2.2. Model of printed circuit board being heated inside a reflow oven

In the second step, we study the process by which the printed circuit board is heated as it moves inside the reflow oven. From the reflow oven welding heat transfer model, we only need to consider the heat transfer caused by thermal convection and thermal radiation, let the heat transferred by thermal convection of the printed circuit board be Q_1, and the heat transferred by thermal radiation be Q_2, then the total heat Q received by the printed circuit board is:

$$Q = Q_1 + Q_2 \tag{7}$$

Using the double counting method, the total heat absorbed by the printed circuit board in the reflow oven can also be written as:

$$Q = cm\Delta T \tag{8}$$

c is the specific heat capacity of the printed circuit board, *m* is the mass of the printed circuit board, and *T* is the change in the temperature of the printed circuit board, and the units are all in the International System of Units (SI units).

For the process of thermal convection, according to Newton's cooling formula, the power of the printed circuit board to absorb heat is [2]:

$$P_1 = h_c \cdot S\left(T_S - T\right) \tag{9}$$

where *S* is the contact area between the printed circuit board and the gas in the reflow furnace, *Ts* is the ambient temperature in the reflow furnace, *T* is the temperature in the center of the soldering area, and h_c is the convective heat transfer coefficient, which is defined as the heat that can be transferred per second in an area of 1 m^2 when the temperature difference between the solid and the flowing body is 1 °C [3]. According to Boltzmann's law, the heat transfer power during thermal radiation is:

$$P_2 = \varepsilon S\sigma(T_S^4 - T^4) \tag{10}$$

where ε represents the emissivity of the object, S represents the area of the printed circuit board that receives thermal radiation, and σ is the Boltzmann constant. In this problem, we take $\sigma = 5.67 \times 10^{-8}$ W/(m^2· K^4), Ts is the surface temperature of the object, T is the ambient temperature and the units are all in SI units. Substituting **Equation (10)** into **Equation (9)** and deriving it on both sides

$$\frac{dT}{dt} = \frac{1}{cm}\left(P_1 + P_2\right) = \frac{1}{cm}\left(h_c S\left(T_s - T\right) + \varepsilon S \sigma\left(T_s^4 - T^4\right)\right) \tag{11}$$

Let the thickness of the printed circuit board be L, then there is a basic mass relationship:

$$m = \rho SL \tag{12}$$

Substituting **Equation (12)** into **Equation (11)** calculates:

$$\frac{dT}{dt} = \frac{1}{\rho cL}\left(h_C\left(T_S - T\right) + \sigma\varepsilon\left(T_S^4 - T^4\right)\right) \tag{13}$$

The different parameters in **Equation (13)** are identified where $L = 0.15$ mm and the relationship between ambient temperature and position Ts has been given in **Figure 5**. For a printed circuit board, its density ρ, emissivity ε, and specific heat capacity c are all temperature-dependent. We divide the whole heating process into before reflow, during reflow, and after reflow: before reflow refers to the process of the printed circuit board passing through the preheating zone (small temperature zones 1–5) and the constant temperature zone (small temperature zones 6–7), the reflow refers to the process of the printed circuit board passing through the reflow zone (small temperature zones 8–9), and the after reflow refers to the process of the printed circuit board passing through the cooling zone (small temperature zones 10–11). For each phase, the temperature is approximately considered constant, and a given value is substituted for the value of a parameter at that stage, as shown in **Table 2** (the units of the following quantities are SI units).

Table 2. The given numerical value

	Density, ρ (kg/m^3)	**Emissivity, ε**	**Specific heat capacity, c (J/(kg·K)**
Before reflow	7310	0.82	114
During reflow	8218	0.071	238
After reflow	8218	0.071	176

For the convective heat transfer coefficient h_c, the h_c results calculated in previous research in actual industrial production are shown in **Table 3** [4].

Table 3. The h_c calculated in actual industrial production

T **(℃)**	h_c **(W/m^2·k)**
70	48.84
160	48.04
200	48.01
230	48.06
250	48.02

In the same reaction stage (before, during, and after reflow), the five values of convective heat transfer, h_c are 48.84, 48.04, 48.01, 48.06, and 48.02. The values vary slightly, suggesting that the value of h_c remains relatively constant within each stage. The value of the convective heat transfer coefficient of the printed circuit board is set to h_{c1} when it passes through the preheating zone (small temperature zones 1–5) and the constant temperature zone (small temperature zones 6–7). The value of the convective heat transfer coefficient when the printed circuit board passes through the reflow zone (small temperature zones 8–9) is set to h_{c2}. When passing through the cooling zone (small temperature zones 10–11), the value of the convective heat transfer coefficient is set to h_{c3}.

The literature shows that the forced convection coefficient of the gas should be 20–100 W/m^2·K, and the coefficient must be non-negative, so the range is appropriately expanded, taking 0–100 W/m^2·K as a reference range for h_{c1}, h_{c2}, and h_{c3} [5]. To determine the values of h_{c1}, h_{c2}, and h_{c3}, we also need a standard furnace temperature curve as a control. In this study, 709 sets of time and temperature values were used to draw a standard furnace temperature curve. These 709 points are denoted as (t_1, T_1), and so on. For h_{c1}, h_{c2}, and h_{c3}, the first coarse adjustment is carried out, that is, the initial value is 0, the step size is 10, and the last value is 100. For each given set of h_{c1}, h_{c2}, and h_{c3}, the differential equation in **Equation (14)** is changed to a difference equation using the average form of the improved Euler formula:

$$T_P = T_n + \frac{10}{\rho c L}\left(h_C\left(T_{sn} - T_n\right) + \varepsilon\sigma\left(T_{sn}^4 - T_n^4\right)\right) \tag{14}$$

$$T_C = T_n + \frac{10}{\rho c L}\left(h_c\left(T_{S(n+1)} - T_n\right) + \varepsilon\sigma\left(T_{sn}^4 - T_n^4\right)\right) \tag{15}$$

$$T_{n+1} = \frac{1}{2}\left(T_p + T_c\right) \tag{16}$$

The time for the printed circuit board to pass through the preheating zone (small temperature zones 1–5) and the constant temperature zone (small temperature zones 6–7) is t_1 = (272.5 × 60) / 78 = 209.62 s, the time for the printed circuit board to pass through the reflow zone (small temperature zones 8–9) is t_2 = (71 × 60) / 78 = 54.62 s, and the time for the printed circuit board to pass through the cooling zone (small temperature zones 10–11) is t_3 = (89 × 60) / 78 = 68.46 s. By using MATLAB to solve the system of difference **Equations (14)–(16)** with constants σ, c, and ε, we can obtain 709 sets of time and temperature data respectively, denoted as (t_1, T'_1), and so on. The newly obtained 709 points can be used to plot a furnace temperature curve. In particular, the furnace temperature curve also needs to meet the process boundaries in **Figure 1**, that is:

$$\left|\frac{T'_{i+1} - T'_i}{t_{i+1} - t_i}\right| < 3, i = 1,2,3...,708 \tag{17}$$

$$60 \le t_k - t_i \le 90\left(t_i \le t_j \le t_{\max}\right) \tag{18}$$

$$40 \le T'_{\max} \le 250 \tag{19}$$

$$240 \le T'_{\max} \le 250 \tag{20}$$

Equation (17) depicts that the absolute value of the slope of temperature rise (fall) is small, less than 3. **Equation (18)** is to describe the time of 150 °C to 190 °C in the process of temperature rise, the temperature of T_i and T_j to meet the welding center at two moments is T'_i, T'_j, T_k, T_i meets $T'_i \geq$ 150 °C and $T'_{i-1} \leq$ 150 °C, $T'_j \leq$ 190 °C and $T'_{j+1} \geq$ 190 °C. **Equation (19)** is to describe the time when the temperature is greater than 217 °C: Let the time be T_k and let T_1 be the temperature at the welding center of the corresponding T_k. The conditions T_1 = T'_k and T'_1 satisfies $T'_k \leq$ 217 °C and $T'_{k-1} \geq$ 217 °C, $T_i \leq$ 217 °C and T_{i+1} > 127 °C. **Equation (20)** is the peak temperature for T'_{max}. To sum up, the algorithm steps for obtaining h_{c1}, h_{c2}, and h_{c3} are as follows:

(1) Set the traversal conditions of h_{c1}, h_{c2}, and h_{c3}: the initial value is 0, the step size is 10, and the last value is 100.

(2) Generate a set of h_{c1}, h_{c2}, h_{c3}.

(3) For h_{c1}, h_{c2}, and h_{c3} generated in step 2, under the temperature distribution conditions of the reflow oven welding heat transfer model and the temperature distribution of question stem, respectively, check whether the calculated 709 groups (t_i, T_i) meet all the conditions of **Equations (16)–(18)**, return to step 2 if at least one condition is not satisfied, and calculate and record the sum of residuals squares of the 709 groups of data if it satisfies $\sum_{i=1}^{709}\left(T_i - T_i^{'}\right)^2$.

(4) When the traversal is completed, take the group of h_{c1}, h_{c2}, and h_{c3} corresponding to the sum of squares of the smallest residuals.

(5) After the primary traversal of steps 1–4 is completed, adjust the step size to 1, traverse at [h_{c1} -5, h_{c1} + 5], [h_{c2} – 5, h_{c2} + 5], [h_{c3} – 5, h_{c3} + 5], and repeat steps 1–4.

(6) After two iterations, the final result is: h_{c1} = 8, h_{c2} = 65, h_{c3} = 30. As mentioned above, the parameters in the difference **Equation (14)–(16)** will have different values when t is taken with different values, and the final furnace temperature curve is obtained by dotting and plotting with MATLAB, as follows:

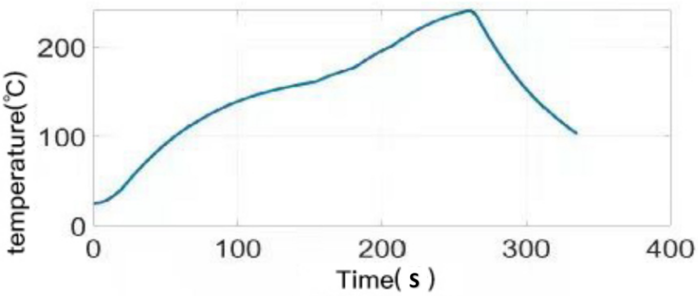

Figure 6. Furnace temperature curve

Among them, the temperatures at the midpoint of 3, 6, and 7 in the small temperature zone and the center of the welding area at the end of the small temperature zone 8 were 128.60 °C, 169.57 °C, 190.88 °C and 226.25 °C, respectively (**Figure 6**).

3.2.3. Determine the maximum conveyor belt furnace speed to solve the model

Based on the model of the temperature distribution of the reflow oven in this study, the temperature distribution in the reflow oven can be obtained by using **Equation (5)** as long as the temperature of the small temperature zone is known. Similarly, the values of h_c, ρ, ε, and c continue to be used. Let v be the speed of the conveyor belt passing through the furnace, since the conveyor belt speed in this study is $65 \leq v \leq 100$ cm/min, to find the maximum conveyor belt pass speed, this study directly traverses with a step size of 0.01, and the specific traversal steps are as follows:

(1) Set the traversal condition of v: the initial value is 65, the step size is 0.01, and the last value is 100.

(2) According to the v generated in step 1, calculate the time t_1 before the reflow of the printed circuit board, the time t_2 when the printed circuit board is reflowed, and the time after the reflow of the printed circuit board is t_3.

(3) Obtains several groups of points (t_i, T_i) according to the temperature distribution of the reflow oven welding heat transfer that has been calculated, and the values of h_{c1}, h_{c2}, h_{c3}, t_1, t_2, t_3, ε, ρ, c in different

stages.

(4) Check whether several groups of points (t_i, T_i) meet all the conditions of **Equations (16)–(19)**, if not, return to step 1. If it is satisfied, record the velocity v at this time, return to step 1, and continue traversal.

(5) After obtaining a plurality of v, take the maximum v as the maximum conveyor belt furnace speed.

In the end, the maximum conveyor belt passing speed was v = 79.52 cm/min.

3.2.4. Optimize the furnace temperature curve to minimize the over-temperature area to solve the model

Let v be the speed of the conveyor belt, and T_1, T_2, T_3, and T_4 are the set temperatures of the small temperature zones (1–5), (6), (7), and (8–9), respectively. To facilitate the representation of unknown quantities, the vector $x = (T_1, T_2, T_3, T_4, v)$, and the value range of each component of x given in the question stem is T_{12} [165, 185], T_{22} [185, 205], T_{32} [225, 245], T_{42} [245, 265], v_2 [65, 100]. Let the furnace temperature curve exceed 217 °C and the area covered by the peak temperature is A, then A is calculated as follows: For any furnace temperature curve, considering that the furnace temperature curve is obtained by the numerical solution of the MATLAB differential equation, we take one point (t_k, T_k), which satisfies $T_k \leq 217$ °C and $T_k \geq 217$ °C. Take one more point to be the highest point of the furnace temperature curve (t_m, T_m), because there are enough points, so the area can be expressed as follows:

$$A = \sum_{i=k}^{m-1} T_i \left(x_{i+1} - x_i\right) \tag{21}$$

Let's first agree that a vector $\vec{x}$ is an individual, $\vec{x}_i$(j) represents the $\vec{j}$ component of the $\vec{i}$ vector and the specific steps of the differential evolution algorithm are as follows:

(1) Establish a population with 6 individuals, which are denoted as $\vec{x}_1$, $\vec{x}_2$, $\vec{x}_3$, $\vec{x}_4$, $\vec{x}_5$, $\vec{x}_6$, where each component of $\vec{x}_i$ should be in the given range, let n be used as the quantity to record the algebra of the population so that $n = 0$.

(2) For the 6 individuals in the population in the first step, firstly, the ambient temperature Ts in the reflow furnace under the corresponding situation is calculated by **Equation (6)**, and then combined with the different parameters given in **Equations (15)–(17)** and temperature distribution model in the reflow oven, several sets of welding center temperature and time data (t_i, T_i) are obtained and these data are recorded.

(3) For a plurality of groups of data (t_i, T_i) obtained in step 2, substitute in **Equations (15)–(19)** to check whether the process limit is satisfied. If satisfied, enter step 4, and return to step 1 if not satisfied.

(4) Using the data in step 2, find the area A corresponding to 6 individuals (6 vectors) according to **Equation (19)**, take the maximum value of area A_b, so that $A_0 = A_b$. Save the vector $\vec{x}_i$ at this time, so that $\vec{x}_0 = \vec{x}_i$.

(5) Any 3 individuals are taken from the population, according to the (t_i, T_i) of each individual recorded in step 2, the area A_i is calculated according to **Equation (20)**, and the area is sorted from small to large A_w, A_m, A_b, and the corresponding vectors are $\vec{x}_b$, $\vec{x}_m$, and $\vec{x}_w$.

(6) Among the three vectors given in step 5, define the fit f_i of a vector $\vec{x}_i$ as:

$$f_i = 1 - \frac{A_i - A_w}{A_m - A_w} \tag{22}$$

Then, according to **Equation (20)**, f_b, f_m, and f_w are obtained, and the fitness of six vectors $f_1, f_2, \ldots, f_6$ is calculated and recorded along with their average $\bar{f}$, maximum f_m, minimum f_w.

(7) Continues from the definition of the variable in step 5, and for $_1$, define $\vec{H}_1 = \vec{x}_b + F_1 (\vec{x}_m - \vec{x}_w)$. where F_1 is the scaling factor, and its expression is as follows:

$$F_1 = 0.1 + 0.8 \cdot \frac{f_m - f_b}{f_m - f_w} \tag{23}$$

and $F_1 \in [0, 2]$. Check whether the components of $\vec{H}_1$ meet the value range given at the beginning of the model in this section, if so, record $\vec{H}_1$ and enter step 8, if not, return to step 5.

(8) Repeat steps 5–7 five more times to ensure that all 6 individuals in the population have corresponding $\vec{H}_i$ and $\vec{F}_i$.

(9) For each set of vectors $\vec{x}_i$, $\vec{H}_i$, define the crossover probability cr_i as:

$$cr_i = \begin{cases} 0.1 + 0.5 \cdot \frac{f_i - f_w}{f_m - f_w}, f_i > \overline{f} \\ 0.1, f_i \le \overline{f} \end{cases} \tag{24}$$

The cr_i of the six vectors is calculated and recorded according to the definition of **Equation (24)**.

(10) For each set of vectors $\vec{x}_i$ and $\vec{H}_i$, take any 1 × 5 row vector $\vec{T}_i$ and leave $\vec{T}_i$ blank as a zero vector, and then reset $\vec{T}_i$ according to the following algorithm:

$$\vec{T}_i(j) = \begin{cases} \vec{x}_i(j), rand(0,1) \le cr_i \\ \vec{H}_i(j), rand(0,1) > cr_i \end{cases} \tag{25}$$

where rand(0,1) represents the random number generated in (0,1) and each component of $\vec{T}_i$ is reset to generate a random number at (0,1), $\vec{T}_i = \vec{x}_i$.

(11) According to the test method in steps 2 to 3, check the corresponding 6 groups of vectors in step 10. Whether the furnace temperature curve in the corresponding case of $\vec{T}_i$ meets the process limit. If satisfied, $\vec{T}_i$ is left unaltered. If not, let $\vec{T}_i = \vec{x}_i$.

(12) Let $\vec{T}_i$ be a new generation of $\vec{x}_i$ so that a new generation of 6 initial individuals will be obtained. For the new generation of individuals, repeat the operation in step 4 to find the maximum value of the area A'_0 in the new generation of individuals and compare it with A_0, and record the vector $\vec{T}_0$ at this time.

(13) If $A'_0 > A_0$, then let $A_0 = A'_0$ and $\vec{x}_0 = \vec{T}_0$, if $A'_0 \le A_0$ leave the value of A_0 unchanged.

(14) Let $n = n + 1$, if $n > 1000$, then terminate the operation, with A_0 and $\vec{x}_0$ as the final result. If $n < 1000$, go back to step 1.

Steps 5 to 7 can also be called mutation, and steps 9 to 11 can also be called crossover, which are the two most core parts of the differential evolution algorithm. After repeating the whole process of the differential evolution algorithm many times, we get the following results: the set temperature T_1 = 177.02 °C in the small temperature zone (1–5), the set temperature T_2 = 196.04 °C in the small temperature zone (6), the set temperature T_3 = 225.08 °C in the small temperature zone (7), the set temperature T_4 = 264.53 c in the small temperature zones (8–9), the set temperature T_5 in the small temperature zones (10–11) is 25 °C, the conveyor belt passing speed v = 89.27 cm/min, and the minimum area A_0 = 366.43 cm^2, the optimal furnace temperature curve is shown in **Figure 7**.

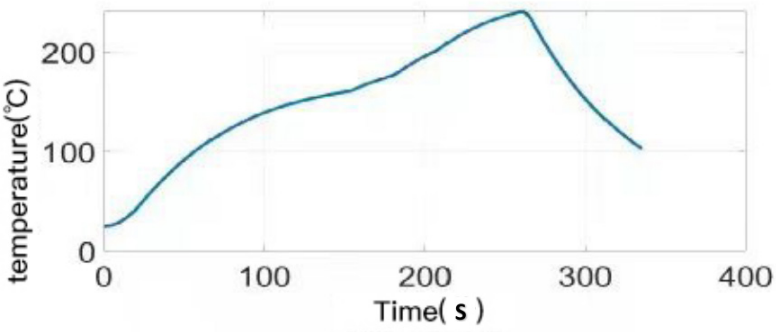

Figure 7. Furnace temperature curve

3.2.5. Pursue the symmetry of the furnace temperature curve and minimize the over-temperature region model

The "optimal furnace temperature curve" in this problem should consider both symmetry and the smallest area. The skewness calculation formula for sample X has been given in the analysis of the model to optimize the furnace temperature curve to minimize the over-temperature region:

$$Sk(X)=E\left[\left(\frac{X-\mu}{\sigma X}\right)^3\right] \tag{26}$$

where μ and X are the mean and standard deviation of sample X, respectively, and E is the mathematical expectation symbol. The skewness of the sample can measure the symmetry of the whole curve, the value range of $Sk(X)$ is (-1,+1), and the positive and negative of $Sk(X)$ means that the symmetry of the whole curve is positive and negative, and the closer $Sk(X)$ is to 0, the more symmetrical the curve. We normalize the skewness and map the skewness from (-1,+1) to (0,1) with the x-function, and we find that the arctangent function has this property. The symmetry of the curve is measured by $1-\frac{2}{\pi}\arctan\left(2\left|Sk(X)\right|\right)$, the range of $1-\frac{2}{\pi}\arctan\left(2\left|Sk(X)\right|\right)$ is between 0 to 1, the closer $1-\frac{2}{\pi}\arctan\left(2\left|Sk(X)\right|\right)$ is to 1, the better the symmetry of the curve, and the closer $1-\frac{2}{\pi}\arctan\left(2\left|Sk(X)\right|\right)$ is to 0, the poorer the symmetry of the curve. The biggest advantage of using the arctan function is that arctan(x) changes very quickly when it is close to 0, and arctan(x) does not change significantly when x is relatively large, which is very consistent with the characteristics of skewness.

Equation (22) defines the fitness degree f_i of the vector $_i$, f_i is a physical quantity located between 0 to 1, the closer f_i is to 1, the smaller the area, that is, the closer to the ideal curve, the closer f_i is to 0, the larger the area, that is, the farther away from the ideal curve. For the $1-\frac{2}{\pi}\arctan\left(2\left|Sk(X)\right|\right)$ and f variables, consider using the TOPSIS method:

$$C=\left(1-\frac{2}{\pi}\arctan\left(2\left|Sk(X)\right|\right),f\right) \tag{27}$$

$C^+ = (1,1)$ is the ideal positive solution and $C = (0,0)$ is the ideal negative solution. Define $S^+ = \|C - C^+\|$ is the distance from an alternative to the ideal positive solution, and $S^- = \|C - C\|$ is the distance from an alternative to the ideal negative solution, and the objective function is defined as:

$$y = \frac{S^-}{S^+ + S^-} \tag{28}$$

In this problem, the larger the value of y, the better, and the differential evolution algorithm is used to find the optimal furnace temperature curve.

(1) Establish a population of 6 individuals, denoted as $\vec{x}_1$, $\vec{x}_2$, $\vec{x}_3$, $\vec{x}_4$, $\vec{x}_5$, $\vec{x}_6$, where each component of $\vec{x}_i$ should be in the given range, let n be the quantity of record population algebra so that $n = 0$.

(2) For the 6 individuals in the population in the first step, firstly calculate the ambient temperature Ts in the reflow furnace under the corresponding situation by using **Equation (6)**, and then combine the different parameters given in **Equations (1)–(15)** and the reflow oven welding heat transfer model to obtain the data (t_i, T_i) of several groups of welding center temperature and time and record these data (the implementation method is the same as the implementation method in the reflow oven welding heat transfer model).

(3) For a plurality of groups of data (t_i, T_i) obtained in step 2, substitute **Equations (18)–(21)** to check whether the process limit is satisfied. If satisfied, enter step 4, and return to step 1 if not satisfied.

(4) Utilize the data in step 2, according to **Equation (28)** and the parameter calculation method in the TOPSIS method, find the value y of the objective function of 6 individuals (6 vectors), take the maximum value of the objective function y_m, let $y_0 = y_b$. Save the vector i at this time, so that $\vec{x}_0 = \vec{x}_i$.

(5) Take any 3 individuals from the population, according to the (t_i, T_i) of each individual recorded in step 2, find the area A_i according to **Equation (19)**, sort the area from small to large A_w, A_m, A_b, and record the corresponding vectors as $\vec{x}_b$, $\vec{x}_m$, $\vec{x}_w$.

(6) In the three vectors given in step 5, f_b, f_m, and f_w are obtained according to **Equation (23)**, and the fitness of the six vectors f_1, f_2, …, f_6, and their average $\bar{f}$, maximum f_m, minimum f_w.

(7) For $\vec{x}_1$, check whether each component of $\vec{H}_1$ satisfies the value range given at the beginning of the model in "optimizing furnace temperature curve to minimize over-temperature areas". If it satisfies, record $\vec{H}_1$ and enter step 8, and return to step 5 if not.

(8) Repeat steps 5–7 five more times to ensure that all 6 individuals in the population have corresponding $\vec{H}_i$, $\vec{F}_i$.

(9) For each set of vectors $\vec{x}_i$, $\vec{H}_i$, calculate and record the cr_i of the six vectors according to the definition of **Equation (24)**.

(10) For each group of vectors $\vec{x}_i$, $\vec{H}_i$, reset $\vec{T}_i$ according to the reset algorithm of **Equation (25)**.

(11) According to the test method in step 2 to step 3, check the corresponding 6 groups of vectors in step 10. Whether the furnace temperature curve in the case of Ti meets the process limit. If satisfied, $\vec{T}_i$ is left unaltered. If not, then let $\vec{T}_i = \vec{x}_i$.

(12) Let $\vec{T}_i$ be $\vec{x}_i$, which gives you a new generation of 6 initial entities. For the new generation of individuals, repeat the operation in step 4 to find the maximum value of the objective function y'_0 in the new generation of individuals and compare it with y_0, and record the vector $\vec{T}_0$ at this time.

(13) If $y'_0 > y_0$, then let $y_0 = y'_0$ and $\vec{x}_0 = \vec{T}_0$, if $y'_0 \leq y_0$ leave the value of y_0 unchanged.

(14) Let $n = n + 1$, if $n > 1000$, then terminate the operation, with y_0 and $\vec{x}_0$ as the final result. If $n < 1000$, go back to step 1.

After repeating the whole process of the differential evolution algorithm many times, we get the following results: the set temperature T_1 = 174.88 °C in the small temperature zones (1–5), the set temperature T_2 = 193.67 °C in the small temperature zone (6), the set temperature T_3 = 229.15 °C in the small temperature zone

(7), the set temperature T_4 = 262.00 °C in the small temperature zones (8–9), the set temperature in the small temperature zones (10–11) is 25 °C, the conveyor belt passing speed v = 85.90 cm/min, and the minimum area A_0 = 401.85 cm^2, the skewness is 0.17.

According to the furnace temperature curve in **Figure 8**, the part of the furnace temperature curve with a temperature higher than 217 °C is not completely symmetrical, the skewness is 0.17, and the area is 401.84 cm^2. The skewness of the furnace temperature curve in "optimizing furnace temperature curve to minimize over-temperature areas" is 0.19, and the area between 217 °C and the peak temperature is 366.02 cm^2. In comparison, the skewness of the furnace temperature curve of the result of "pursue symmetry of the furnace temperature curve and minimize the over-temperature area" has decreased by 10.53%, and the area higher than 217 °C increased by 9.56%. It is in line with the evaluation results of the symmetry and area of the furnace temperature curve above 217 °C. The symmetry of the furnace temperature curve and the furnace temperature curve of the model that minimizes the over-temperature region are affected by the temperature difference between the reflow zone and the cooling zone in the reflow furnace, and the drop in this problem is more than 200, which is difficult to achieve complete symmetry.

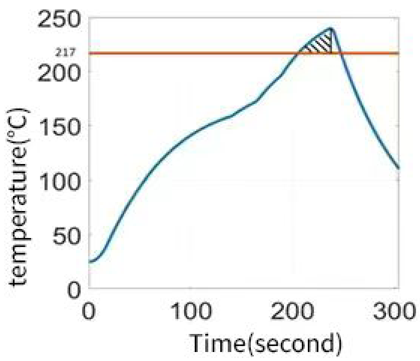

Figure 8. Optimal furnace temperature curve

In the process of optimizing the furnace temperature curve, the differential evolution algorithm is used in this model, which is an effective global optimization method. By simulating the process of natural selection and genetic variation, the algorithm can quickly search for the optimal solution in the parameter space. Compared with the traditional optimization algorithm, the implementation of the differential evolution algorithm in this model has a shorter running time and higher optimization efficiency, which not only accelerates the optimization process but also improves the practicability of solving the problem, so that the model can quickly respond to changes in production and provide timely process parameter adjustment for actual production.

The TOPSIS method, which is a multi-criteria decision-making method based on ideal points, is adopted. In this model, the TOPSIS method is used to comprehensively evaluate the performance of different furnace temperature curves, and the advantages and disadvantages of the furnace temperature curves are effectively quantified by calculating the distance between each alternative and the ideal positive and ideal negative solutions. When dealing with complex problems with multiple evaluation indicators, this method can objectively deal with the weight of decision-making variables, avoid the bias that may be brought about by subjective judgment, and improve the accuracy and reliability of evaluation results. Through the TOPSIS method, the model can provide scientific decision support for the selection of process parameters, and further optimize the control strategy of the welding process.

4. Summary

In this paper, an optimization model of the furnace temperature curve based on a differential evolution algorithm is proposed, which aims to improve the consistency and reliability of welding quality in the electronics assembly industry. The article first introduces the importance of reflow furnace in electronic assembly and points out the necessity of optimizing the furnace temperature curve to improve product quality, and reduce energy consumption and cost. By establishing the heat transfer model, considering the thermal radiation, thermal convection effect, and the thermal properties of the welding material, the furnace temperature curve was optimized with precise control.

In this study, the authors use the Differential Evolution (DE) algorithm for parameter optimization, which shows great potential in solving complex system optimization problems due to its simplicity, robustness, and high efficiency. Through this algorithm, the model not only optimizes the temperature and conveyor speed of each temperature zone but also further considers the symmetry of the furnace temperature curve through the TOPSIS method to achieve better welding results.

The model is described in detail, including a model of the temperature distribution in the reflow furnace and a model of the printed circuit board being heated in the furnace. By solving the model, the optimal furnace temperature curves under different conditions, as well as the corresponding set temperature of each temperature zone and the conveyor belt passing speed are obtained. In addition, it discussed how to determine the maximum conveyor belt speed of the furnace and maximize the production efficiency without sacrificing the quality of the weld.

The model in this study has several advantages, which provide a scientific method for optimizing furnace temperature curves based on the basic laws of heat transfer based on the actual production conditions. The efficiency of the differential evolution algorithm ensures that the model can quickly respond to changes in production, and the application of the TOPSIS method provides objective decision support for the selection of process parameters. These advantages make this model not only innovative in theory but also has high practical value in application, which helps to promote the development of electronic manufacturing technology.

Disclosure statement

The author declares no conflict of interest.

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