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A New Method for Generating Conics

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Abstract: In this paper, based on the mean value theorem of differential, a new method of generating conics such as circles and parabolas is given, and the related algorithm for generating conics is designed.

Keywords: Conic curve; Differential; Generate; Algorithm

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1. Introduction

The curve generation algorithm significantly influences the efficiency of a graph system, as it's repeatedly called during graph rendering. Common methods for generating conic curves include the midpoint and positive-negative approaches. While these methods are reliable, they struggle to directly derive subsequent pixel coordinates from the current pixel's coordinates (x_i, y_i) to (x_{i+1}, y_{i+1}) [2]. To address this, this paper proposes utilizing the differential mean value theorem [3,4]:

$$\frac{f(b)-f(a)}{b-a} = f^{'}(\xi) \tag{1}$$

This theorem asserts that if a function (x) is continuous within $[a, b]$ and differentiable within (a, b), there exists at least one point ξ within $[a,b]$ where Equation (1) holds true. This method simplifies the process by directly calculating (x_{i+1}, y_{i+1}) from (x_i, y_i), eliminating the need for curve equation calculations and reducing computational complexity.

2. Differential median generation algorithm for conics

2.1. Basic method

If the value of ξ in the differential mean value theorem can be determined, then the function (x) value of each node in the interval [a,b] can be calculated from Equation (1).

Let $a=x_0<x_1<x_2< \dots x_{n-1}<x_n = b$ represent $n + 1$ difference points on the interval [a,b]. Then, the function value at each node is given by:

$f(x_{i+1})=f(x_i)+(x_{i+1}-x_i)f'(\xi)$, i=0,1,2,...,n–1

This enables the calculation of the sequence of pixels $(x_0, f(x_0)),(x_1, f(x_1)),\dots,(x_n, f(x_n))$ on $[a,b]$.

2.2. Generation algorithm of conics

2.2.1. Generation method of the curve of the upper half of the parabola

In the first quadrant, the tangent slope m of every point on the upper half of the parabolic curve falls within the range $m \in [0,1]$ [9]. Assuming that the pixel (x_I, y_I) has been obtained, he task now is to compute the next pixel (x_{i+I}, y_{i+I}), that is, as each x pixel is incremented $(x_{i+I}=x_i+1)$. This computation entails determining the value of y_{i+I}. From the mean value theorem of differentials:

$$f^{'}(\xi) = \frac{f(x_{i+1}) - f(x_i)}{x_{i+1} - x_i} = \frac{\sqrt{2px_{i+1}} - \sqrt{2px_i}}{x_{i+1} - x_i} = \sqrt{2px_{i+1}} - \sqrt{2px_i} \tag{2}$$

While $\frac{f(x_{i+1})-f(x_i)}{x_{i+1}-x_i} = f^{'}(\xi)$, that is

$$y_{i+I} = y_I + f^{'}(\xi) \tag{3}$$

Substituting the expression (2) into the expression (3) yields:

$$y_{i+1} = y_i + \sqrt{2px_{i+1}} - \sqrt{2px_i}$$

The formula for generating the differential median of the curve that generates the upper half of the parabola is as follows:

$$\begin{cases} x_0 = \frac{1}{2}p \ , \ y_0 = p \\ x_{i+1} = x_i + 1 \ , \ y_{i+1} = y_i + \sqrt{2px_{i+1}} - \sqrt{2px_i} \ , \quad i = 0, \ 1, \ 2, \ \cdots \end{cases} \tag{4}$$

2.2.2. Generation method of the curve of the lower half of the parabola

In the first quadrant, the tangent slope m of each point on the lower half of the parabolic curve satisfies $\frac{1}{m} \in [0,1]$ [10]. Assuming we have obtained the pixel (x_I, y_I) the task now is to compute the next pixel (x_{i+I}, y_{i+I}), as each y pixel is incremented $(y_{i+I}=y_i+1)$. This computation involves determining the value of x_{i+I}? From the mean value theorem of differentials:

$$f^{'}(\xi) = \frac{f(y_{i+1}) - f(y_i)}{y_{i+1} - y_i} = \frac{\frac{1}{2p}y_{i+1}^2 - \frac{1}{2p}y_i^2}{y_{i+1} - y_i} = \frac{1}{2p}(y_{i+1} + y_i) \tag{5}$$

while $\frac{f(y_{i+1})-f(y_i)}{y_{i+1}-y_i} = f^{'}(\xi)$, that is

$$x_{i+I} = x_I + f^{'}(\xi) \tag{6}$$

Substituting expression (5) into expression (6) yields:

$$x_{i+1} = x_i + \frac{1}{2p}(2y_i + 1)$$

The formula for generating the differential median of the curve of the lower half of the parabola is as follows:

$$\begin{cases} x_0 = 0 \ , \ y_0 = 0 \\ y_{i+1} = y_i + 1 \ , \ x_{i+1} = x_i + \frac{1}{2p}(2y_i + 1) \quad i = 0, \ 1, \ 2, \ \cdots \end{cases} \tag{7}$$

The same can be obtained, the differential median generation method of the circle $x^2 + y^2 = R^2$, that is:

$$\begin{cases} x_0 = 0 \ , \ y_0 = R \\ y_{i+1} = y_i - 1 \ , \ x_{i+1} = x_i + \sqrt{R^2 - y_i^2} - \sqrt{R^2 - y_{i+1}^2} \ , \quad i = 0, \ 1, \ 2, \ \cdots \end{cases} \tag{8}$$

2.3. Parabolic differential median generation algorithm design

2.3.1. Basic steps [11]

(1) Initialize the graphics system and give the coordinates (x_0, y_0) of the origin of the cartesian coordinate

system in the graphics system;

(2) Draw the lower half of the parabola arc:

(i) Set the initial pixel coordinates to(0, 0), that is, x=0, y=0, with the given value *of p*.

(ii) Use the symmetry of the parabola to write pixels ($x_0 + x$, $y_0 + y$) and ($x_0 + x$, $y_0 - y$) in the graphic system;

(iii) Calculate the coordinates of the next pixel: ,.

(iv) Repeat steps (ii) and (iii) until $y > p$

(3) Draw the upper half of the parabola arc:

(i) Use the symmetry of the parabola to plot pixels ($x_0 + x$, $y_0 + y$) and ($x_0 + x$, $y_0 - y$) in the graphic system;

(ii) Calculate the coordinates of the next pixel: x=x+1, y=y+$\sqrt{2p(x+1)} - \sqrt{2px}$;

(iii) repeat steps (i) and (ii) until y exceeds a certain value.

3. Algorithm analysis and discussion

The time complexity of the midpoint method, positive and negative method, and the differential median method for generating conics is *O(n)* [12,13]. The midpoint method involves determining whether the midpoint is inside or outside the conic and then calculating the subsequent pixel value (x_{i+1}, y_{i+1}) [14]. Similarly, the positive and negative method determines whether the current pixel (x_i, y_i) is inside or outside the conic before obtaining the subsequent pixel value (x_{i+1}, y_{i+1}) [15]. In contrast, the differential median method calculates the subsequent pixel value (x_{i+1}, y_{i+1}) directly from the current pixel (x_i, y_i), avoiding complex judgments. Moreover, the differential median method is applicable to various conic curves like ellipses and hyperbolas. Overall, compared to other algorithms, the differential median method offers simpler calculations and easier implementation.

Disclosure statement

The author declares no conflict of interest.

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