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Review Article

Physical Principles of Optimization of the Static Regime of a Cantilever-Type Power-effect Sensor with a Constant Rectangular Cross Section

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Abstract: In this paper an analysis of the physical principles of two-criterion optimization Pareto static mode of operation of power sensors cantilever type of rectangular type with a stable cross-section. The proposed criterion based on the Cauchy number is one of the characteristic numbers of the proportional miniaturization of microsystem technology. It is established that, for a rectangular cantilever with a stable cross-section, the value of the Cauchy does not depend on the width of the microconsole and the material from which it is made.

Keywords: *physical principles; Cauchy number; rectangular cross section*

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0 Introduction

Cantilever (cantilever beam) is a basic element of modern sensors of external force. The physical principle of the operation of the sensors is based on the effect of bending deformation of the cantilever under the action of the applied force to its free end, which is based on the work of biosensors[1,2] and probes for atomic force microscopy[3].

In order to ensure the required stable performance of the sensors, in the process of designing them, it is often necessary to seek a compromise between the projected parameters, for example, to reach the minimum mass of the sensor without decreasing the mechanical strength of its design, ets. Such a multicriteria approach greatly complicates the search algorithm for an Optimy project solution; therefore, when formulating the optimization problem itself, they try to clearly define the control parameters, performance criteria, functional constraints, the scope of finding solutions, and the optimization algorithm itself.

The simplest task is the task of one-criterion optimization. It is formulated as follows[4,5]:

$$f(x) \to \min, \quad x_1, x_2, \ldots, x_n, \qquad (1)$$
$$\textit{subject to} \quad \begin{array}{c} g_i(x) \le 0,\ i = 1, 2, \ldots, p, \\ h_j(x) = 0,\ j = p+1, p+2, \ldots, p+m \end{array}$$

Where the variables x are the vector $X = (x_1, x_2, \ldots, x_n)$, $f(x)$ is of objective or of fitness, the functions, $g_i(x)$ and $h_j(x)$ are the limiting (equality and inequality constraints) function in canonical form and represent the latest condition restrictions on the fitness function.

The formulation (1) is interpreted as the task of searching for the extremum of the target function by the purposeful change of the controlled parameters within the allowable range of changes in the values of the design parameters. However, one-factorial optimization is ineffective, since its solution depends on the constraints imposed on the design parameters. If these restrictions are chosen incorrectly, the solution will not have physical content.

Often, in the optimization problem, many criteria are taken into account and optimization is attempted by all criteria at the same time. However, to achieve optimization of all the criteria taken into account is not possible in principle at the same time; therefore, more attention is paid to a limited number of optimization criteria. Moreover, a certain compromise was achieved by reducing the number of knowingly ineffective solutions, using the Pareto optimization method.

It should be noted that the optimal solution for Pareto means the best solution for Pareto means that it does not contradict the two axioms of Pareto. For the first axiom of Pareto, when estimating one of the two solutions is not worse than estimating another solution for all project parameters or, in the extreme case, one of them is strictly better, then the first solution has an advantage over the other. The second axiom of Pareto states that, when the solution does not fall into any pair that was analyzed under the first axiom, then it cannot appear among the selected and initial set of possible solutions. Thus, the Pareto procedure does not distinguish the only solution but only simplifies the procedure for choosing the most optimal solution, and the improvement of one criterion is accompanied by the deterioration of another criterion[4,5].

In relation to cantilever-type power sensors, the main attention in the literature was given to the study of the problem of two-criterion approximation of the optimization of the operation of power sensors[6-9]. In this paper, using the Nondominated Sorting Genetic Algorithm (NSGA-II) developed in[10], a physical and two-criterion Pareto analysis of the static mode of the cantilever power sensor operation was performed. It is shown that, from the point of view of finding the optimal values of the design parameters of cantilever-type power sensors, the characteristic number (the number of Cauchy) of the proportional miniaturization of the microsystem technology can be considered as one of the criteria of the optimization problem. It is also established that, for a rectangular cantilever with a stable cross-section, the value of the Cauchy number does not depend on the width of the microconsole and the material from which it is made.

1 Basic results and discussion

Figure 1 shows the calculation scheme of the power sensor in the form of a cantilever of a rectangular type with a stable cross section, one end of which is rigidly fixed and the other is free. Here, L, w, and t are the beam lenght, width, and thickness, k is the spring constant, F is normal force action on free beam end, and E is the Young's modulus of elasticity the cantilever material.

In the model of the micromechanical sensor as an inertial system with lumped parameters, the ratio k/m has the dimension of the square of the cyclic frequency ω_0^2, and m is the mass of the cantilever. For rectangular cantilever types with a stable cross section, the

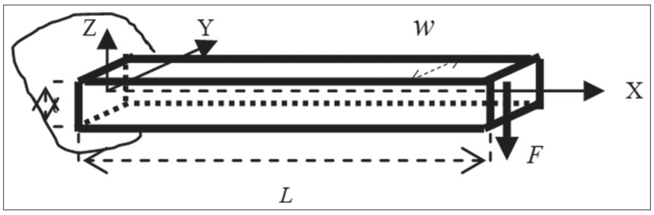

Figure 1. The power sensor in the form of a cantilever of a rectangular type with a stable cross section

coefficient of elasticity $k = \frac{wt^3}{4L^3}E$ [11], therefore, the relationship between inertial and elastic properties of a sensitive element is described by the ratio

$$\omega_0^2 = \frac{t^2 E}{4L^4 \rho} \quad (2)$$

Converting the right-hand side of equation (2) to form

$$\omega_0^2 L^2 \frac{\rho}{E} = \left(\frac{t}{2L}\right)^2 \quad (3)$$

where the left-hand side of equality (3) is a known Cauchy number[12]:

$$Ca = \omega_0^2 L^2 \frac{\rho}{E} \quad (4)$$

as one of the characteristic numbers expressing the laws of proportional miniaturization of elastic-dynamic microsystem technology. Taking into account that during bending of the console, the volumes of local deformation with the speed $\vartheta_S = \sqrt{E/\rho}$ of sound propagate, the Cauchy criterion can be written as

$$Ca = \left(\frac{\omega_0 L}{\vartheta_S}\right)^2$$

In the cantilever with a constant rectangular cross section, the distance from the surface to its neutral axis is equal $c=t/2$, so the right side of the equality (3) can be transformed into a kind:

$$Ca = \left(\frac{t/2}{L}\right)^2 = \left(\frac{c}{L}\right)^2 \quad (5)$$

Consequently, for cantilever-type power sensors with a constant rectangular cross section, the value of the Cauchy number (5) does not depend on the width of the microconsole and the material from which it is made. Therefore, the number of Cauchy is also convenient to choose the optimization criterion when designing power sensors:

$$f_{Ca} = \left(\frac{c}{L}\right)^2 \quad (6)$$

with a minimum number of design parameters c and

L. From a practical point of view, the use of power sensors, for example, in atomic force microscopy, requires the design of sensors with the highest possible value, that is, according to Mastinu *et al.* (4), as short as possible microconsole. Therefore, based on the physical analysis, we substantiate the criterion (6) to be minimized or maximized.

From a practical point of view, the use of power sensors, for example, in atomic force microscopy, requires the design of sensors with the highest possible value, that is, according to (4), as short as possible microconsole. Therefore, based on the physical analysis, we substantiate the criterion (6) to be minimized or maximized. The sensitivity $S = \frac{d\delta}{dF}$ of the working element of the power sensor is determined by the elastic-dynamic characteristics of the cantilever; therefore, the actual criterion for optimizing the sessionors is the amplitude of the bend of the microconsole:

$$f_\delta (L, w, t) = \delta \tag{7}$$

Given the small deviations of the microconsole $\delta << L$, according to the Euler formula[13], the bending amplitude is:

$$\delta = \frac{4FL^3}{Ewt^3} \tag{8}$$

or taking into account the shear deformations described by the shear coefficient (for a rectangular cross section α_{st}=3/2):

$$\delta = \frac{4FL^3}{Ewt^3} + \alpha_{st} \frac{FL}{GS} \tag{9}$$

where $G \quad \frac{E}{2(1 \quad \nu)}$ is Poisson coefficient.

Unlike criterion (6), the optimization criterion (7) for the given parameter values F and E has three design parameters such as L, w, and t.

From the point of view of achieving maximum sensitivity of the sensor, in the optimization problem, the criterion (7) is actually maximized, so when using the minimization algorithm, function (7) should be chosen in the form $f_\delta (L, w, t) = -\delta$. However, if you consider the functional limit on the maximum static deflection of cantilever, then this criterion is written in the form:

$$f_\delta (L, w, t) = \delta_{\max} - \delta \tag{10}$$

and using the minimization algorithm, the criterion (10) will be minimized.

The consideration of the functional limit on the maximum value of the static deflection $\delta_{\max}$ gives the possibility to substantiate the functional constraints in dimensional $\delta_{\max} = \delta$ or in dimensionless (canonical forms $g(x_1, x_2, x_3) \leq 0$

$$g(x_1, x_2, x_3) = \frac{\delta_{\max}}{\delta} - 1 \tag{11}$$

The functional limitation to the static deflection is associated with a limitation to the maximum mechanical stress, which follows from the need to ensure the possibility of multiple deflection of the microconsole for a long time of operation of the sensor. As established in the resistivity of materials, the nature of the distribution of normal stresses $\sigma = \frac{Mc}{I}$ arising from the bending stress moment and the fatigue curve must be taken into account. The strength of the material is defined as $S_y \quad \frac{\sigma_{yield}}{n}$, where σ_{yield} is the material yield strength. The yield strength σ_{yield} corresponds to the stress at which the material gets plastically deformed. The coefficient n is safety coefficient and $n \geq 1$[7], so the limit on the maximum mechanical stress in the cross section of the rigidly fixed end of the microconsole will look like:

$$\sigma_{\max}(t, w, L) = \frac{6FL}{wt^2} - S_y \leq 0 \tag{12}$$

Design parameters such as L, w, and t determine the elastic properties of the microconsole, so for the cantilever with a rectangular cross-sectional cross-section, the following target optimization function will look like:

$$f_k(L, w, t) = 2wECa^{3/2} \tag{13}$$

An important criterion of optimization in a micromechanics is the mass of a sensitive element as such a physical parameter that characterizes the inertia of the external force. For a cantilever-type sensor with a rectangular section, an objective function – the mass criterion has the form:

$$f_m(L, w, t) = m = \rho L w t \tag{14}$$

The other constraints can be written: Bending stress constraint $FLw/2I - \sigma_{max} \leq 0$; deflection constraint $FL^3/3EI - \delta_{max} \leq 0$; length-deflection restriction $L - 10\delta \leq 0$; width-thickness restriction $w - 10t \leq 0$; and dimension restriction $L_{max} - L \leq 0$, $w_{max} - w \leq 0$, $t_{max} - t \leq 0$.

In Figure 2, the two-criterion fronts of Pareto are shown in coordinates (δ, m) (a) and (δ, Ca) (b).

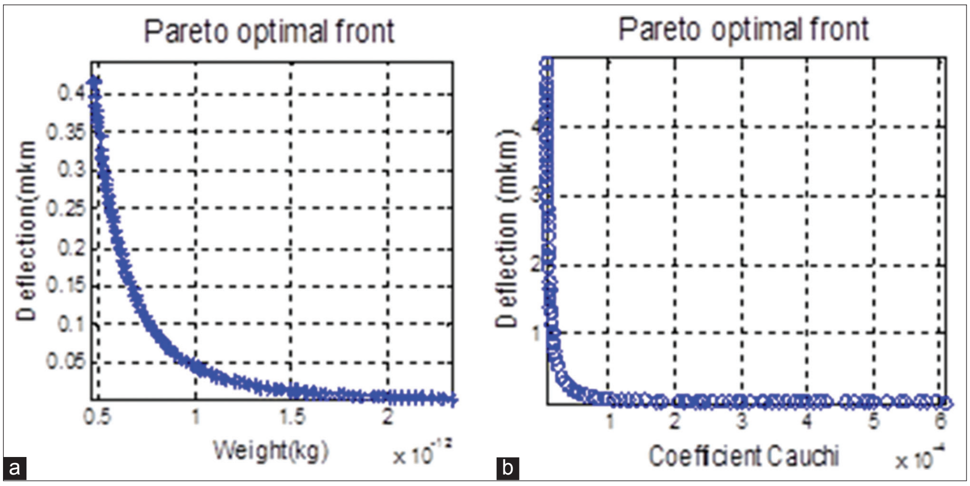

Figure 2. Bi-objective optimization curve of Pareto compromise design in plane coordinate: Deflection mass (a) and deflection-coefficient Cauchi (b)

Here, the material density ρ=2335 kg/m^3, Young modulus=180·10^9 N/m^2, and Yield strength=300·10^9 N/m^2. Target functions are formulated in the form g (L, w, t)={δ(L, w, t), m (L, w, t)} and f (L, w, t) = {δ (L, w, t), Ca (L, w, t)}. The calculations were performed using method NSGA-II by ES-EA procedures for optimization in Matlab codes[15,16]. For calculation, the parameters of optimization used: Number of population = 100 and number of iterations = 100. The birectic fronts of Pareto in coordinates (δ, m),[7,14-16] shown in Figure 2a, is presented as a test for comparison to confirm the correctness of the conclusions as drawn in this paper. As we can see in Figure 2b, in the variations of the geometric parameters of the 0.1 μm <t< 5 μm, the quality of the Pareto-optimization front is high with a uniform distribution of the approximation points by the Cauchy optimization criterion.

2 Conclusions

In this paper an analysis of the physical principles of two-criterion optimization Pareto static mode of operation of power sensors cantilever type of rectangular type with a stable cross-section. The proposed criterion based on the Cauchy number is one of the characteristic numbers of the proportional miniaturization of microsystem technology. It is established that, for a rectangular cantilever with a stable cross-section, the value of the Cauchy does not depend on the width of the microconsole and the material from which it is made.

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