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Research Article

Entropy of Measurements of Electric and Non-eleCtric Systems Fluctuating Parameters

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Abstract: Fluctuations of the measured parameters of the investigated electric and non-electric systems are considered on the thermodynamics basis. We have defined that entropy of the investigated system in relation to environment depends on the correlation between measuring duration T_{meas} of the system's fluctuation parameter and the system's relaxation time τ. Decrement of the energy spectrum of fluctuations with the decrease of frequency is explained by $T_{meas} > \tau$. When $T_{meas} \approx N_{max}\tau$ (where N_{max} is the number of possible ways to create the balanced state of the system), entropy of investigated system tends to zero in relation to the environment, and the measurement error caused by fluctuations becomes the minimal.

Keywords: Entropy; fluctuations; measurement time; relaxation time.

Introduction

Fluctuations have a significant meaning since on one hand they confirm the discrete-statistical nature of macro objects, on the other hand they limit the area of utilization of thermo dynamical notions and laws only for macro systems for which fluctuations of parameters values are small enough comparing with their mean statistical values.

During experiments there is a controversial attitude towards the fluctuations: on one hand, own fluctuations of both voltage and current of the measuring electronic equipment, determine the threshold of sensitivity, especially during measurement of small values of the investigated parameters (it is worth mentioning that the ways to reduce fluctuations are being developed). On the other hand, fluctuations represent independent interest, since they are the basis of numerous physical and chemical processes. In the first and second case for the successful solving of such tasks a clear presentation of the fluctuation process is needed, i.e. what mechanisms cause this process, what is the influential character of such values on the observed process etc. It is clear the vision of the fluctuation process at the microscopic level since, observing from the outside the fluctuations look like Brownian motion of microscopic particles that are components of the system. But this approach does not always explain the observed fluctuations on practice. For instance, fluctuations with energy spectrum (spectral density of power) $W(f) \sim 1/f$, where f – frequency of fluctuations, have lots of hypothesis of possible mechanisms of their generation [1-10]. Most of them are based on the analysis of processes at the microscopic level: fluctuations of concentration of charge carriers [3,10]; fluctuations of mobility of charge carriers [2,7]; capture of charge carriers by the surface energy traps [5] etc. Some hypothesis explain increase of energy spectrum of fluctuations if frequency decreases at the macroscopic level – as a result of nonequilibrium state of the investigated system [11,12].

Let us take a look at the fluctuation process closer. For simplicity sake, let us take an electric macro system that consists of electronic charged micro particles, i.e., electrons, ions, holes, dipoles, etc. As an example of such system let us take a resistor with active resistance R. The latter is located in thermostat to limit the impact of external factors. After some time we can assume that the temperature of the thermostat and the temperature of the resistor are aligned. Now we connect the resistor with the input of spectrum analyzer. Its input resistance is much bigger than the resistance of the resistor (we performed it aiming to ignore the spectrum analyzer's influence).

Resistor as an electric macro system is the source of electric fluctuations: charge fluctuations, current through the resistor, voltage on the resistor's connectors. In turn, the source of those fluctuations is the Brownian movement of the charge carriers in the whole resistor's volume that causes the appearance of unbalanced charges. These charges cause the difference of potentials on the resistor's connectors and the current that equalizes this difference. It is clear that this difference of potentials and its equalizing current fluctuate around their average values that are equal to zero.

During analysis of fluctuation's spectrum let us use the following statement: energy spectrum (spectral density of power) of the system that is in thermodynamic equilibrium state, is same in the whole range of frequencies starting from $f \to 0$ until very high frequencies ($f \to \infty$), where quantum effects could be noticed [13].

Let us analyze the spectrum of fluctuations of the voltage on the connectors of the investigated resistor in the range of frequencies from 1.0 Hz to 100.0 kHz. In most cases (if not in all cases) we will get a different energy spectrum of the investigated fluctuations: spectrum raise if $f \to 0$ and constant value of spectrum starting from a few- tens kHz until 100.0 kHz [14]. If the experiment was performed correctly, the only conclusion that could be made taking into account the applied statement above is that the investigated resistor as an electric macrosystem is not in the thermodynamic equilibrium state. Such conclusion has a big likelihood since "…thermodynamic equilibrium state is only the abstraction because there are no pure equilibrium systems in the nature." [11].

The objective of the paper is analysis of the behavior of fluctuations on the basis of thermodynamics and determination of the condition under which the influence of the fluctuations of the studied system measuring parameter on the measurement error is minimal.

Conducted researches

From the perspective of thermodynamic processes that take place in the systems that are close to the equilibrium state the probability of fluctuations of their parameters is defined by the Einstein's formula [15]:

$$P \sim e^{\frac{\Delta S}{k}}, \tag{1}$$

where ΔS - the deviation of the system's entropy from the maximum value.

$\Delta S = S_N - S_{\max} < 0$, where S_N - entropy value of the system in the unbalanced state, $S_{\max}$ - entropy value of the system in the thermodynamic balanced state. If the system is in balanced state so it's entropy is calculated using the well-known Boltzmann's formula: $S_{\max} = k \cdot \ln N_{\max}$, where $N_{\max}$ – number of possible ways of moving the system in the balanced state.

If $\Delta S = 0$ (system is in balanced state) the probability of appearing fluctuations in the system from (1) $P = 1$. If $\Delta S \neq 0$ in general case the system is in unbalanced state and from formula (1) the probability of fluctuations in such system $P < 1$. Formula (1) is correct with the precision to the constant coefficient that could be assumed to be equal one. Therefore we can write: $P \sim e^{\frac{\Delta S}{k}}$. Formula (1) points only to the possibility of the fluctuations of parameters of the system that deviated from the balanced state but this formula does not connect these fluctuations with their experimental investigations – measurements.

For the balanced system for which macroscopic parameters do not change in time the time value T_{meas} is not the argument of the parameters' values of the system. But for the unbalanced system the values of macroscopic parameters change in time (they are functions of time t) and therefore for such systems it should be taken into account the correlation between measurement time T_{meas} and relaxation time τ that is pointed in [15]: "…correctness of Einstein's formula (formula (1) – author's note) was proved for unbalanced systems, at least for the cases when relaxation time satisfies some defined conditions. These conditions connected with the separation of time scales between fluctuating system and the external environment. Times connected with fluctuating system (relaxation time τ of the system – author's note) should be small comparing with the proper times of external environment (measurement time T_{meas} of fluctuating parameter – author's note) so that the state of the external environment could be considered independently from the instantaneous state of fluctuating system…".

In [16] there is formula for the probability of fluctuations of parameters' values of the system:

$$P_H = 1 - e^{-f-\tau}, \tag{2}$$

where f - fluctuation frequency; τ - relaxation time of the system.

Formula (2) is based on the general considerations that take into account formula (1) when $\Delta S = 0$ ($\tau \to \infty$).

In formula (2) unlike formula (1) there is the separation of time scales between fluctuating system (relaxation time τ) and external environment (frequency f used to measure fluctuations or period $T=\frac{1}{f}$ of fluctuations). Since measurement of parameters of periodic value with period T is performed at the ratio of measurement time T_{meas} and period as $T_{meas} \geqslant T$ so further the period T will be treated as measurement time T_{meas}. Unlike frequency f that could be taken randomly by experimenter (he can take randomly the measurement time T_{meas}) the relaxation time τ depends only on the features of the investigated system – it's inner structure [17] and does not depend on the experimenter. Frequency f used for measurement of fluctuating parameter or the measurement time T_{meas} is connected with the external environment whereas relaxation time τ - with the system itself.

Since formulas (1) and (2) describe the same phenomenon (fluctuations) for the same system but formula (1) shows the proportionality of probability of fluctuations in the system when it deviates from the balanced state on ΔS without measuring these fluctuations and formula (2) defines the probability of fluctuations in the system during measurement of these fluctuations on different frequencies so it is possible to make a logical assumption that:

$$P_H=1-\exp(-f\cdot\tau) = P=\exp\left(\frac{\Delta S}{k}\right),$$

And therefore:

$$\Delta S=k\ln(1-\exp(-f\cdot\tau)), \qquad (3)$$

Since $\Delta S=S_N - S_{\max}$ that means S_N can vary from 0 to $S_{\max}$. Therefore the maximum value of logarithmic function in (3) can be: $\ln(1-\exp(-f\cdot\tau))_{max}=\frac{-S_{\max}}{k}$.

From (3) when $\tau\rightarrow\infty$ then the entropy change $\Delta S\rightarrow 0$ that means that the immense big value of relaxation time corresponds to the balanced state of the system.

From (3) entropy value of the unbalanced system S_N at different frequencies of fluctuations of its parameters (in the relation to the external environment):

$$S_N=k\ln(1-\exp(-f\cdot\tau)) + S_{\max}= S_e + S_{\max} \qquad (4)$$

From (4) S_N is calculated as sum of two components – entropy S_e that appears due to interaction of the investigated system with the external environment and entropy $S_{\max}$ of the balanced state of the system. The interaction of the system with the external environment takes place via measurement of its fluctuating parameters. S_e has negative value and could be called the entropy of «internal order» of the investigated system for convenience. S_e depends first of all from the inner state of the investigated system via relaxation time τ. If the system is in balanced state ($\tau\rightarrow\infty$) then the entropy of the «internal order» of the system $S_e = 0$ despite the measurement conditions (despite the frequency f used for measuring of the fluctuating parameter). Dependence of S_e from the frequency f that is used to measure the fluctuating parameter demonstrates at what frequencies of the measurement (at what times of measurement) the parameters of the «internal order» of the investigated system appear: when $\tau < \infty$ entropy $S_e = 0$ when measurement is performed at high frequencies ($f\rightarrow\infty$) and entropy increases (modulo) when $f\rightarrow 0$. It should be declared that the changes S_e are not connected with the exchange of the substance between the system and external environment (they assume that during measurement the influence of measuring devices on the investigated system is minimum).

In fig.1 there is the dependence of S_e from frequency f used for measuring the fluctuating parameter.

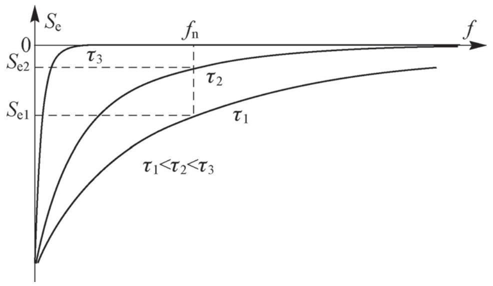

Fig. 1. Dependence of entropy *Se* from frequency *f* used for measuring fluctuating parameter.

From fig.1 depending on the inner parameter τ of the investigated system during measuring of values of its parameters (that look like fluctuations) at the same frequency f_n system may look like unbalanced (for τ_1 and τ_2 $S_e \neq 0$) or like balanced (for τ_3 $S_e = 0$) *in relation to the external environment*.

Balanced or unbalanced state of the investigated system in relation to the external environment depends on the correlation of times – time (τ) related to the investigated system and time related to the external environment (measurement time T_{meas} or frequency f used for measuring of fluctuating parameter): if $\tau \gg T_{meas}$ ($f \gg 1/\tau$) then the macroscopic parameters of the system do not change during measurement time T_{meas} (they are not functions of time) and results of the measurement demonstrate equilibrium values of parameters of the investigated system (S_e=0). For instance, during measurement of spectral density of power (SDP) of fluctuations' parameters of the investigated system at frequencies $f \gg 1/\tau$ the SDP value from [16]:

$$W_H(f) = \frac{e^{f\cdot\tau}}{e^{f\cdot\tau}-1}\cdot W_P(f) \approx W_P(f) = W_0, \qquad (5)$$

where $W_H(f)$ - SDP of unbalanced system; $W_P(f)$ – SDP of balanced system ($W_P(f)$ till ultrahigh frequencies has constant value [13], for instance W_0)

If $\tau < T_{meas}$ then macroscopic parameters of the system change within measurement time T_{meas} (they are functions of time) and measurement results demonstrate different values of parameters of the investigated system for different values $T_{meas} > \tau$ (at different measurement frequencies $fn = 1/T_n < 1/\tau$). For instance, if $f_1 < f_2 < f_3 < 1/\tau$ then SDP of fluctuations of the investigated system from (5): $W_H(f_1) > W_H(f_2) > W_H(f_3)$.

That means if the investigated system is in balanced state ($\tau \to \infty$) then despite the measuring time T_{meas} (despite frequency f used to measure parameters of the system) the results of measurement will register the equilibrium values of parameters of the investigated system ($S_e = 0$). Only if the system is in unbalanced state ($\tau < \infty$) the measurement results will demonstrate the equilibrium values of the measured parameter while $\tau \gg T_{meas}$ ($S_e = 0$) or nonequilibrium values while $T_{meas} > \tau$ ($S_e \neq 0$): value of the measured fluctuating parameter will depend on different values $T_{meas} > \tau$ (will be different at different measurement frequencies $f_n = 1/T_n$).

Entropy S_N of the unbalanced system from (4) could be defined by analogy with Boltzmann's entropy $S_{\max} = k \cdot \ln N_{\max}$ using number of possible combinations N_N like:

$$\begin{aligned} S_N &= k \ln(1-\exp(-f \cdot \tau)) + k \ln N_{\max} = \\ &= k \ln((1-\exp(-f \cdot \tau))\, N_{\max} = k \ln N_N \end{aligned}, \qquad (6)$$

where $N_N = (1-\exp(-f \cdot \tau)) \cdot N_{\max}$ – number of possible combinations to create unbalanced state of the system.

From (6) it is possible to find the correlation between measurement time of the fluctuating parameter $T_{meas\max}$ and relaxation time $\tau$of the investigated system for which the entropy of the unbalanced system with regard to the external environment will be equal $S_N = 0$ – system with regard to the external observation is created by one combination $N_N = 1$ (entropy of "inner order" of the system S_e is maximum $S_{e\max} = -S_{\max}$):

$$\begin{aligned} T_{meas\max} &= \frac{-\tau}{\ln\left[1-\exp\left(-\dfrac{S_{\max}}{k}\right)\right]} = \\ &= \frac{-\tau}{\ln\left[1-\exp\left(-\dfrac{k \ln S_{\max}}{k}\right)\right]} = \qquad (7) \\ &= \frac{-\tau}{\ln(1-\exp(-\ln N_{\max}))} \end{aligned}$$

From (7) for the correlation of measurement time T_{meas} and relaxation time τ like $T_{meas\,\max} \approx N_{\max}\tau$ the entropy of unbalanced system with regard to the external environment (with regard to the external observation) will be equal $S_N = 0$. At the same time the probability of fluctuations of measured parameter with regard to the external observation:

$$P = e^{\frac{\Delta S}{k}} = e^{\frac{S_N - S_{\max}}{k}} = e^{\frac{-S_{\max}}{k}}.$$

When probability of fluctuations of the measured parameter is small the measurement error caused by fluctuations is minimal.

Conclusion

1. While measuring of studied system's fluctuating electric and non-electric parameters it should be taken into account the correlation between its measurement time T_{meas} and relaxation time τ. When $\tau \to \infty$ the system is in equilibrium, and its entropy is maximal S_{max}; energy spectrum of its studied fluctuating parameter is unchangeable for any measurement time T_{meas} (immutable in the particular bandwidth). When $\tau < \infty$ system becomes non-equilibrium, and its entropy S_N with regard to the environment depends on the correlation between T_{meas} and τ: when $\tau \gg T_{meas}$ ($\tau \gg 1/\tau$)$S_N \approx S_{\max}$, the measurement results envisage the equilibrium values of studied parameter; when $\tau < T_{meas}$ ($f < 1/\tau$) $S_N < S_{\max}$, the measurements are fixed different values of studied parameters for the values $\tau < T_{meas}$ (at different measurement frequencies $f_n = 1/T_n < 1/\tau$). The raise of the studied energy spectrum of fluctuations is related to correlation $\tau < T_{meas}$ ($f < 1/\tau$) while frequency decreases as $W(f) \sim 1/f$.

2. If correlation of measurement time of fluctuating parameter and relaxation time of studied system is similar to $T_{meas\,\max} \approx N_{\max}\tau$ the system entropy with regard to the environment tends to zero. Therefore the measurement error caused by the studied parameter's fluctuations is minimal.

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