

# Resilience Investment for Distribution Networks Facing Urban Waterlogging: A Two-Stage Method Combining AHP Zoning and Mixed-Integer Optimization

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**Abstract:** Against the background of frequent urban waterlogging disasters caused by global climate change, the vulnerability of distribution systems has become increasingly prominent. To address the blindness and inefficiency of traditional protective equipment layout, this paper proposes a two-stage optimization method based on risk zoning, aiming to scientifically guide the layout of protective equipment represented by waterproof transformers. First, the method integrates six core indicators including rainfall, elevation, and slope, and uses the Analytic Hierarchy Process (AHP) to conduct refined waterlogging risk zoning of the distribution network, dividing nodes into three risk levels: high, medium, and low. Taking this risk assessment result as a key input, a mixed-integer programming model with the goal of maximizing comprehensive benefits is constructed. Finally, the model is verified through a 25-node numerical example. The results show that through strategic layout, the model protects most loads including high-risk nodes with limited costs, and makes a strategic abandonment of low-value nodes in line with the optimization goal. This study has important practical significance for improving investment efficiency and urban power grid resilience.

**Keywords:** Urban waterlogging; Distribution network resilience; Facility layout optimization; Waterproof transformer

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## 1. Introduction

Global climate change has led to frequent extreme rainfall events, and the vulnerability of distribution systems in disasters is particularly prominent. Once key equipment such as distribution transformers and ring main units are submerged by floods, it is easy to trigger chain failures such as equipment insulation damage, causing significant losses to public safety.

To address this challenge, the adoption of waterproof transformers and the construction of new active protective equipment resistant to waterlogging are considered effective technical paths to improve the resilience of distribution network facilities. For example, in July 2025, China Southern Power Grid's first set

of waterproof medium and low-voltage distribution equipment was completed at Shenzhen Longhua Power Supply Bureau, successfully completing a 48-hour live operation test in a 3m deep water environment <sup>[1]</sup>. However, the cost of such high-performance protective equipment is much higher than that of conventional equipment, and their effectiveness highly depends on a scientific and reasonable spatial layout strategy. Among layout methods, subjective weighting methods represented by the Analytic Hierarchy Process (AHP) are widely used. Li *et al.* also used AHP to integrate multiple influencing factors such as short-term rainfall, cumulative rainfall, GDP, slope, and vegetation coverage when assessing the urban waterlogging risk of transmission lines <sup>[2]</sup>. However, methods such as AHP are highly dependent on expert experience, which may lead to certain subjectivity in results. Wang Deyun *et al.* innovatively used machine learning models such as XGBoost to improve the objectivity of risk assessment by fitting the nonlinear relationship between historical submergence point data and elevation, population density, average annual rainfall, etc. <sup>[3]</sup>.

Facility location and layout optimization is essentially to minimize the full-life cycle comprehensive cost under the premise of meeting specific service needs. Fang Ziwen *et al.* proposed a more refined rolling optimization method with mixed time scales of long and short cycles for continuously evolving disasters such as ice disasters <sup>[4]</sup>. Wang Yongxing *et al.* directly linked facility layout with the battery aging cost of electric buses and found that different station layouts would affect the charging and discharging modes of batteries <sup>[5]</sup>. Mei *et al.* introduced resilience configuration costs or load loss penalties under extreme disaster scenarios as part of the objective function, making facility location decisions take into account their strategic value as distributed energy to support the power grid and reduce key load losses during disasters <sup>[6]</sup>.

Existing studies have made significant progress in fields such as urban waterlogging risk assessment and facility location optimization. However, in these studies, high-precision risk assessment results have not directly guided the precise deployment of resilience resources. Therefore, how to deeply couple the refined urban waterlogging risk assessment results with the resilience layout decisions of key distribution network facilities is an urgent problem to be solved.

## 2. Model construction

### 2.1. Waterlogging risk zoning

The overall technical route of risk zoning based on a multi-index system and AHP adopted in this section focuses on historical rainfall data and topographical and geomorphological characteristics, selecting the following six core indicators as follows:

- (1) Rainfall: Rainfall is the most direct driving factor of waterlogging disasters. In the assessment, areas with greater rainfall have higher disaster risk;
- (2) Elevation: Under the action of gravity, water always flows from high to low. Areas with lower elevation are assigned higher waterlogging risk;
- (3) Slope: Slope directly affects the flow velocity and waterlogging time of surface runoff. Areas with gentle slopes have slow water flow and long rainwater retention time, resulting in higher waterlogging risk;
- (4) Land use/impervious surface ratio: Impervious surfaces convert most rainfall into surface runoff, increasing the burden on drainage pipe networks. Areas with a higher proportion of impervious surfaces have higher waterlogging risk;

- (5) Distance from rivers/water systems: Areas adjacent to rivers usually have higher groundwater levels, which weaken the infiltration capacity of soil. Areas closer to rivers have higher waterlogging risk;
- (6) Drainage pipe network density: Areas with higher drainage pipe network density have higher rainwater discharge efficiency, which can alleviate waterlogging risk to a certain extent.

This study uses AHP for weight calculation. Experts are invited to use the scaling method to compare the importance of the six selected indicators pairwise to construct a judgment matrix A. Calculate the eigenvector of matrix A and normalize it to obtain the weight vector W of each indicator. Consistency check is completed by calculating the Consistency Ratio (CR):

$$CI = (\lambda_{max} - n) / (n - 1) \tag{1}$$

where n is the number of indicators (n=6 in this study). Query the preset Random Index (RI) value (for a 6-order matrix, RI=1.24).

$$CR = CI / RI \tag{2}$$

If  $CR \geq 0.1$ , experts need to re-score.

Standardize all indicator data, converting all indicators into dimensionless, weighted comparable risk contribution scores ( $P_i$ ). A linear weighted summation model is used to calculate the comprehensive risk index (S) of each spatial unit:

$$S = \sum (W_i \times P_i) \tag{3}$$

where  $W_i$  refers to the weight of the i-th indicator, representing the relative importance of the indicator.  $P_i$  refers to the standardized risk contribution score of the spatial unit under the i-th indicator.

Generate a unique comprehensive risk score for each grid unit in the study area, and divide the continuous risk scores into low-risk areas, medium-risk areas, and high-risk areas.

## 2.2. Model problem description

In a power grid composed of multiple key load points, each load point i has its own load demand  $L_i$  and is powered by a local non-waterproof transformer. All waterproof transformers to be updated are of uniform specifications with a fixed maximum power supply capacity  $P_{max}$ , and  $P_{max}$  is greater than the demand of any single load point. The total update cost  $C_i$  of each point is different (Table 1).

Primary protection involves investing in risk point i to upgrade its transformer to a waterproof model, thereby directly reducing its vulnerability to flooding events. In addition to this, a standby redundancy protection mechanism is incorporated: if the transformer at risk point j remains unprotected and fails during a flood, it can receive supporting power from its adjacent neighbor, point i, provided that point i has already been upgraded. Successful protection under this redundancy scheme is defined by a performance threshold, whereby the standby support is considered effective only if the total supporting power supplied to the affected point meets or exceeds 60% of its original load demand. This framework not only enhances system resilience through targeted upgrades but also introduces dynamic benefits by enabling interconnected support between neighboring nodes during disruption events.

**Table 1.** Model variable table

Symbol	Description
$I$	Set of all risk points (also candidate update locations), $i, j \in I$
$M_j$	Set of points physically adjacent to point j in the power grid topology.

Symbol	Description
$C_i$	Total cost of updating the transformer at location i.
$B$	Upper limit of available total investment budget.
$W_j$	Risk level weight of risk point j. 10 for high-risk areas, 5 for medium-risk areas, and 1 for low-risk areas.
$L_j$	Load size of risk point j (kW).
$P_{max}$	Maximum power supply capacity of a single waterproof transformer (kW).
$\lambda$	Objective function weight coefficient, balancing benefits and costs.
$x_i$	0-1 variable: 1 if the transformer at location i is updated, 0 otherwise.
$S_j$	0-1 variable: 1 if the non-updated risk point j achieves “successful protection”, 0 otherwise.
$f_{ij}$	Continuous non-negative variable: supporting power (kW) flowing from the updated neighbor point i to the non-updated disaster-stricken point j.

### 2.3. Objective function

The optimization goal is to maximize the power grid benefit Z, that is:

$$\max Z = \lambda \times P - (1 - \lambda) \times C \quad (4)$$

The total safety benefit P includes primary protection benefit and standby redundancy benefit.

$$P = \sum (W_j \times L_j \times x_j) + \sum (W_j \times \sum \{i \in M_j\} f_{ij}) \quad (5)$$

The total investment cost C includes the sum of the costs of all updated waterproof transformers.

$$C = \sum (C_i \times x_i) \quad (6)$$

### 2.4. Constraints

#### 2.4.1. Budget constraint

The total transformation cost cannot exceed the budget.

$$C = \sum (C_i \times x_i) \leq B \quad (7)$$

#### 2.4.2. Logical correlation constraints for “successful protection”

The conditions are as follows:

- (1) Trigger condition: A point can be identified as “successfully protected” only if the received power reaches the threshold;

$$\sum \{i \in M_j\} f_{ij} \geq 0.6 \times L_j \times S_j \quad (8)$$

- (2) Mandatory correlation: As long as there is power input, it must enter the judgment logic of “successful protection”.

$$\sum \{i \in M_j\} f_{ij} \leq L_j \times S_j \quad (9)$$

## 3. Numerical example analysis

A distribution network in a street in Shenzhen is used as a numerical example for empirical analysis. The distribution network includes 25 devices arranged in a 5×5 matrix, numbered from N\_00 to N\_44. Nodes

are only connected to their physically adjacent nodes. The total budget is 650,000 yuan, and the maximum load of the waterproof transformer is 200 kW. The risk levels, loads, and transformation costs of the 25 distribution nodes are shown in **Table 2**.

**Table 2.** Distribution network node attribute data

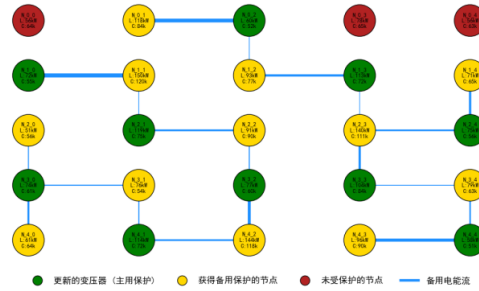
Node Number	Risk Level	Load/kW	Transformation Cost/1000 yuan
N_00	Low	56	64
N_01	Medium	118	84
N_02	Low	60	52
N_03	Low	78	65
N_04	Low	56	63
N_10	Low	72	55
N_11	High	150	120
N_12	Medium	93	77
N_13	Medium	113	72
N_14	Low	71	65
N_20	Low	51	56
N_21	Medium	119	75
N_22	Medium	91	90
N_23	High	140	111
N_24	Low	75	56
N_30	Low	74	61
N_31	Low	76	54
N_32	Low	77	60
N_33	Medium	104	84
N_34	Low	79	63
N_40	Low	61	64
N_41	Medium	114	72
N_42	High	144	118
N_43	Medium	96	90
N_44	Low	58	51

The distribution diagram of the distribution network nodes is shown in **Figure 1**. Among the 25 distribution nodes, there are 3 high-risk nodes, 8 medium-risk nodes, and 14 low-risk nodes.



**Figure 1.** Distribution network node distribution diagram.

The solution result is shown in **Figure 2**. Green nodes represent the locations of updated waterproof transformers, totaling 10 nodes; yellow nodes represent nodes that have obtained standby protection, totaling 12 nodes; red nodes represent nodes that have not received standby protection and have not been updated to waterproof transformers, including 3 nodes; the thickness of the blue line represents the amount of transmitted current.



**Figure 2.** Distribution network optimization diagram.

The optimized power dispatch plan is shown in **Table 3**.

**Table 3.** Standby power dispatch plan

Redundancy Protection Node	Power Transmission Node	Transmitted Load/kW
N_01	N_02	118
N_11	N_10	128
N_12	N_21	22
N_14	N_02	22
N_20	N_13	71
N_22	N_24	71
N_23	N_30	30
N_31	N_21	59
N_34	N_32	32
N_40	N_13	16
N_42	N_24	54
N_43	N_33	70
	N_30	34
	N_41	33
	N_33	26
	N_44	46
	N_30	61
	N_32	91
	N_41	53
	N_44	96

The investment decision of the model is not simply to select nodes with the lowest cost or highest risk. The high-risk nodes N\_11 and N\_23 themselves were not selected for update. Instead, the model chose to invest in their neighbors with lower costs, enabling the protection of high-risk, high-value nodes through standby support

with a smaller budget. Most of the selected nodes are located at positions connecting key areas.

In terms of resilience improvement effect, the key node N<sub>11</sub> received a total of more than 90% of load support from two directions, while node N<sub>23</sub> also received assistance from three different directions, indicating that the model utilized the topological structure of the network to achieve distributed, multi-source redundancy protection.

Finally, 3 nodes remain unprotected. These 3 nodes are all low-risk, low-importance nodes located at the edge of the grid. With a limited budget, resources are concentrated on protecting nodes with higher value and greater risk.

## 4. Conclusion

This study proposes a two-stage layout method integrating refined risk assessment and systematic operational optimization. Through the numerical example analysis of a street distribution network, the results fully verify the scientificity and practicality of the method. The study finds that the optimal layout plan given by the model does not simply reinforce the nodes with the highest risk, but identifies and invests in key locations with investment leverage effect, protecting more high-value targets through standby redundancy at a lower cost, reflecting excellent cost-benefit trade-off capabilities. The theoretical contribution of this study lies in the deep coupling of macro-urban geographical risk assessment and micro-distribution network facility layout optimization, providing a quantifiable and executable decision support tool for improving distribution network resilience.

## Disclosure statement

The authors declare no conflict of interest.

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