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New Iterative Methods for Solving Exponentially General Regularized Nonconvex Variational Inequality

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Abstract: In this paper, we introduce and study a new class of extended exponentially general regularized nonconvex variational inequalities. We showed that the inequalities are equivalent to fixed-point problems through the use of the projection properties. Based on this equivalence, we discuss the existence and uniqueness of solutions to the extended exponentially general regularized nonconvex variational inequalities. We present a new self-adaptive finite p-step iterative projection scheme that uses multiple updates to obtain common solutions of the extended exponentially general regularized nonconvex variational inequality. Furthermore, we analyze the convergence of this algorithm under various suitable conditions.

Keywords: Nonconvex variational inequality; Uniformly prox-regular; Fixed point problems; Self-adaptive finite p-step projection algorithm

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1. Introduction

Variational inequalities, introduced by Stampacchia, arise from the extension of classical variational problems and provide a unified framework for solving a wide range of complex problems [1–5]. In the late 20th century, Clarke and Poliquin developed the theory of prox-regular sets, extending variational inequalities to the nonconvex sets and significantly advancing nonconvex variational analysis [6–8]. Meanwhile, the work of Poliquin and Rockafellar established fundamental properties of nonsmooth and prox-regular functions, providing a theoretical basis for algorithm design and convergence analysis [9–11]. In this paper, we investigate a new class of extended exponentially general regularized nonconvex variational inequalities. Using projection techniques, we establish their equivalence with fixed-point problems. Furthermore, by combining adaptive strategies with a finite p-step iteration, we propose an adaptive projection algorithm that enhances efficiency and stability, particularly for complex applications.

2. Preliminaries and basic results

Throughout this article, we let H denote a real Hilbert space that is equipped with an inner product $\langle\cdot,\cdot\rangle$and corresponding norm$\|\cdot\|$. We also let K be a closed subset of H . We recall the following definitions and results of nonsmooth analysis [7].

Definition 1. Let $x \in H$ be a point that is not lying in K . The distance from x to $u \in K$ is called closest distance or a projection of x onto k , it is expressed in the following formula:

$$d_K(x) := \inf_{u \in K} x - u .$$

The set of all the closest points is denoted by:

$$P_K(x) := \{x \in H : x - u = d_K(x)\} .$$

Definition 2. The proximal normal cone of k is denoted by:

$$N_K^P(u) := \{\xi \in H \,|\, u \in P_K(u + \alpha\xi), \alpha\rangle 0\} .$$

Lemma 1. Let K be a nonempty closed subset in H Then $\xi \in N_K^P(u)$ if and only if there exists a constant $\alpha = \alpha(\xi, u) > 0$ such that the following proximal normal inequality holds with all$v \in K$,

$$\langle \xi, v - u \rangle \leq \alpha v - u^2 .$$

Definition 3. For any$r \in [0, +\infty]$, the subset $K_r \subset H$ is called uniformly r-prox-regular. If every nonzero proximal normal to K_r can be implemented by an r -ball. This means that for all $x, \overline{x} \in K_r$ and$0 \neq \xi \in N_K^P(\overline{x})$.

$$\langle \zeta, x - \overline{x} \rangle \leq \frac{1}{2r} x - \overline{x}^2 \cdot$$

Lemma 2. Let $r > 0$ and K_r be a nonempty closed and uniformly r-prox-regular subset H . Set$U(r) = \{u \in H : 0 < d_{K_r} < r\}$. Then the following statements hold:

(1) For all $x \in U(r)$, $P_{K_r}(x) \neq \varnothing$;

(2) For all$r' \in (0, r)$, P_{K_r} is Lipschitz continuous with constant $L_P = \frac{r}{r - r'}$ on$U(r') = \{u \in H : 0 < d_{K_r} < r'\}$.

Definition 4. The mapping $N_{K_r}^B : H \rightrightarrows H$ is defined for $r > 0$ by

$$N_{K_r}^B(u) = \begin{cases} N_{K_r}^P(u) \cap \operatorname{int} B(0, r) & \text{if} \quad u \in K_r, \\ \varnothing & \text{if} \quad u \notin K_r. \end{cases}$$

Here $B(0,r)$denotes the closed ball of center 0 and radius r .

3. Extended exponentially general regularized nonconvex variational inequality

Let$T, f, g : H \to H$ be three nonlinear single-valued operators with $\rho > 0$ such that $K_r \subseteq f(H)$ We consider the problem of finding $u \in H$ such that $g(u) \in K_r$ and

$$\langle \rho e^{T(u)} + g(u) - f(u), f(v) - g(u) \rangle + \frac{1}{2r} \| f(v) - g(u) \ ^2 \geqslant 0, \forall v \in H : f(v) \in K_r,$$

Equation (1) is called extended exponentially general regularized nonconvex variational inequality involving three different nonlinear operators (EEGRNVI).

Lemma 3. If K_r is a uniformly prox-regular set, then the Equation (1) is equivalent on $N_{K_r}^B(g(u)$ to that of finding $u\in H$ such that $g(u)\in K_r$ and

$$0\in\rho e^{T(u)}+g(u)-f(u)+N_{K_r}^B(g(u)),$$

Proof: Let $u\in H$ with $g(u)\in K_r$ be a solution of the Equation (1). Since the vector zero always belongs to any normal cone, if $\rho e^{T(u)}+g(u)-f(u)=0$, we have:

$$0\in\rho e^{T(u)}+g(u)-f(u)+N_{K_r}^B(g(u)).$$

If $\rho e^{T(u)}+g(u)-f(u)\neq 0$, for all $f(v)\in K_r$ with $v\in H$, we have

$$-\left\langle \rho e^{T(u)}+g(u)-f(u),f(v)-g(u)\right\rangle \leqslant \frac{1}{2r}\| f(v)-g(u)\ {}^2.$$

By **Definition 3**, we conclude that:

$$0\in\rho e^{T(u)}+g(u)-f(u)+N_{K_r}^B(g(u)).$$

Converse is also clear.

Lemma 4. Let T,f,g be the same as in problem (1), and let $0<\rho<\frac{r'}{1+\| e^{T(u)}}$. Then $u\in H$ with $g(u)\in K_r$ is a solution of the problem (1) if and only if:

$$g(u)=P_{K_r}\left(f(u)-\rho e^{T(u)}\right).$$

Proof: since $0<\rho<\frac{r'}{1+\| e^{T(u)}}$,

$$0\in\rho e^{T(u)}+g(u)-f(u)+N_{K_r}^B(g(u))\Leftrightarrow -\rho e^{T(u)}+f(u)\in g(u)+N_{K_r}^B(g(u))$$
$$\Leftrightarrow f(u)-\rho e^{T(u)}\in\left(I+N_{K_r}^B\right)(g(u))\Leftrightarrow g(u)=P_{K_r}\left(f(u)-\rho e^{T(u)}\right).$$

Theorem 1. Let T,f,g and ρ be the same as in the **Equation (1)** such that:

(1) T is ν-strongly monotone with respect to f and λ-Lipschitz continuous;

(2) g is τ-strongly monotone and ι-Lipschitz continuous;

(3) f is ϕ-Lipschitz continuous.

If the constant $\rho>0$ satisfies the following condition:

$$\begin{cases}\left|\rho-\dfrac{\nu}{\lambda^2}\right|<\dfrac{\sqrt{r^2\nu^2-\lambda^2(r^2\phi^2-(r-r')^2\left(1-\mu\right)^2)}}{r\lambda^2},\\ r\nu>\lambda\sqrt{r^2\phi^2-(r-r')^2(1-\mu)^2},\\ r\phi>(r-r')(1-\mu),\mu=\sqrt{1-\left(2\tau-\iota^2\right)}<1,\\ 2\tau<1+\iota^2.\end{cases}$$

Where $r'\in(0,r)$, then **Equation (1)** *admits a unique solution.*

Proof: Define the mapping $\Psi:H\to H$ by

$$\Psi(x)=x-g(x)+P_{K_r}\left(f(x)-\rho e^{T(x)}\right),\forall x\in H:g(x)\in K_r,$$

where Ψ is a contraction mapping.

Let $x, x' \in H$ with $g(x), g(x') \in K_r$ be given. It follows from **Lemma 2** that:

$$\| \Psi(x) - \Psi(x') \leqslant x - x' - \left(g(x) - g(x')\right)$$

By using τ -strongly monotonicity and ι -Lipschitzian continuity of g, we have

$$\| x - x' - \left(g(x) - g(x')\right) ^2 \leqslant \left(1 - 2\tau + \iota^2\right) \; x - x' \; ^2.$$

$$\| f(x) - f(x') - \rho\left(T(x) - T(x')\right) ^2 \leqslant \left(\phi^2 - 2\rho\nu + \rho^2\lambda^2\right) \; x - x' \; ^2.$$

Substituting **Equation (8)** and **Equation (7)** for **Equation (6)**, we obtain

$$\| \Psi(x) - \Psi(x') \leqslant \gamma \; x - x' ,$$

$$\gamma = \sqrt{1 - 2\tau + \iota^2} + \frac{r}{r - r'}\sqrt{\phi^2 - 2\rho\nu + \rho^2\lambda^2}.$$

4. Self-adaptive finite p-step projection algorithm

In this section, we introduce the self-adaptive finite p-step projection algorithm. Let T, f, g and ρ be the same as the **Equation (1)** and suppose the inverse of the operator g exists. From **Lemma 4**, it has that u is a solution of (1) is equal to u is a zero of the function, we have:

$$w(u,\rho) = g(u) - P_{K_r}\left[f(u) - \rho e^{T(u)} \right].$$

Theorem 2. For all $u \in H$ and $\rho' \geqslant \rho > 0$, it holds that

$$\| w(u,\rho') \geqslant w(u,\rho) ,$$

$$\frac{\| w(u,\rho')}{\rho'} \leqslant \frac{w(u,\rho)}{\rho}.$$

Theorem 3. $u \in H, u^* \in H$ and $\rho > 0$, then:

$$\left\langle g(u) - g(u^*), D(u,\rho) \right\rangle \geqslant J(u,\rho),$$

$$D(u,\rho) = \rho\left(e^{T\left(g^{-1}\left(P_{K_r}\left[f(u^*) - \rho e^{T(u^*)}\right]\right)\right)} - e^{T(u)} \right) + g(u + u^*) + f(u - u^*) - w(u,\rho),$$

$$J(u,\rho) = \left\langle \rho\left(e^{T\left(g^{-1}\left(P_{K_r}\left[f(u^*) - \rho e^{T(u^*)}\right]\right)\right)} - e^{T(u)} \right) + g(u + u^*) + f(u - u^*), w(u,\rho) \right\rangle$$

Theorem 4. If u is not a solution of the **Equation (1)**, then there exist $\delta \in (0,1)$ and $\dot{o}_1 > 0$, for all $\rho \in (0, \dot{o}_1]$, such that:

$$\rho \| e^{T(u)} - e^{T\left(g^{-1}\left(P_{K_r}\left[f(u) - \rho e^{T(u)}\right]\right)\right)} \leqslant \delta \; w(u,\rho) .$$

By **Theorem 2** and **Theorem 3**, we have $\left\langle g(u) - g(u^*), D(u,\rho) \right\rangle \geqslant J(u,\rho) \geqslant (1-\delta) \| w(u,\rho) ^2$.

Let $S:K_r \to K_r$ be a nearly uniformly Lipschitzian mapping. Here the set of all fixed points of S denoted by $Fix(S)$ and the set of all **Solutions (1)** of the problem denoted by EEGRNVI with K_r, T, f and g. If $u \in Fix(S) \bigcap EEGRNVI(K_r, T, f, g)$ then by **Lemma 4**, for every $n \geq 0$,

$$u = S^n u = u - g(u) + P_{K_r}\left[f(u) - \rho e^{T(u)}\right] = S^n u - g(u) + P_{K_r}\left[f(u) - \rho e^{T(u)}\right].$$

$\{\alpha_{n,i}\}_{n=0}^{\infty}$, $\{\beta_{n,i}\}_{n=0}^{\infty}$ $(i = 1,2,...,p)$ are two p sequences in interval [0,1], such that $\Sigma_{n=0}^{\infty}\alpha_{n,i} < \infty$, $\Sigma_{n=0}^{\infty}\beta_{n,i} < \infty$. Also $\{q_{n,i}\}_{n=0}^{\infty}$, $\{l_{n,i}\}_{n=0}^{\infty}$, $\{\gamma_{n,i}\}_{n=0}^{\infty}$ $(i = 1,2,...,p)$ are three p sequences in H to take into account a possible inexact computation of the resolvent operator point. Among them, the following conditions should be satisfied: $\{l_{n,i}\}_{n=0}^{\infty}$ are p bounded sequences in H and $\Sigma_{n=0}^{\infty} q_{n,i} < \infty$, such that:

$$\begin{cases} q_{n,i} = q'_{n,i} + q''_{n,i}, n \geqslant 0, i = 1,2,...,p \\ \lim\limits_{n\to\infty} \| q'_{n,i} = 0, i = 1,2,...,p, \\ \Sigma_{n=0}^{\infty} q''_{n,i} < \infty, \Sigma_{n=0}^{\infty}\gamma_{n,i} < \infty. \end{cases}$$

Algorithm 1. Let T, f, g and ρ be the same as the **Equation (1)**. For any chosen initial point $x_0 \in K_r$, compute the approximate solution u_{n+1} for a given $u_0 \in H$ by the iterative schemes.

Step 0. Given $\epsilon > 0$, $\gamma \in [0,1)$, $\mu \in [0,1)$, $\rho > 0$, $\delta \in \left(\frac{1}{2}, 1\right)$ and $\delta_0 \in \left(\frac{1}{2}, 1\right)$. Here $u_0 \in H$, set $n = 0$.

Step 1. Stopping criteria: Set $\rho_n = \rho$. If $w(u_n, \rho_n) < \epsilon$, then stop; otherwise, find the smallest non-negative integer m_k, such that $\rho = \rho\mu^{m_k}$, satisfying:

$$(1-\delta) w(u_n, \rho_n) \leq \rho_n e^{T(u_n)} - e^{T(l_n)} \leq \delta w(u_n, \rho_n),$$

$$l_n = g^{-1}\left(P_{K_r}\left[f(u_n) - \rho e^{T(u_n)}\right]\right).$$

Step 2. Compute.

$$d(u_n, \rho_n) = \rho\left(e^{T(l_n)} - e^{T(u)}\right)$$

$$\alpha'_n = \frac{\delta \| w(u_n)^{\ 2}}{\| d(u_n, \rho_n)^{\ 2}}.$$

Step 3. Finite p-step iteration. Set $y_{n,1} = u_{n+1}$, and for $i = 1,2,\ldots,p-2,$ compute:

$$y_{n,i} = \left(1 - \alpha_{n,i+1} - \beta_{n,i+1}\right) g(u_n) + \alpha_{n,i+1}\left(S^n \Phi(y_{n,i+1}) + q_{n,i+1}\right) + \beta_{n,i+1} l_{n,i+1} + r_{n,i+1},$$

where

$$\Phi(y_{n,i}) = y_{n,i} - g(y_{n,i}) + P_{K_r}\left[f(y_{n,i}) - \rho e^{T(y_{n,i})}\right].$$

Step 4. Get the next iterate.

$$g_n(u_{n+1}) = P_{K_r}\left[f(u_n) - \gamma\alpha'_n d(u_n, \rho_n)\right].$$

Step 5. If:

$$(1-\delta_0) \| w(u_n, \rho_n) \leq \rho_n \quad e^{T(u_n)} - e^{T(l_n)} \leqslant \delta_0 \quad w(u_n, \rho_n) ,$$

then set $\rho = \frac{\rho}{\mu}$, else set $\rho = \rho_n$. Set $n := n+1$, and go to **Step 1**.

Theorem 5. Let $\{a_n\}$, $\{b_n\}$ and $\{c_n\}$ be three nonnegative real sequences satisfying the following condition: there exists a natural number n_0 such that:

$$a_{n+1} \leqslant (1-t_n) a_n + b_n t_n + c_n, \forall n \geqslant n_0,$$

where $t_n \in [0,1]$, $\Sigma_{n=0}^{\infty} t_n = \infty$, $\lim_{n\to\infty} b_n = 0$, $\Sigma_{n=0}^{\infty} c_n < \infty$.

Then $\lim_{n\to\infty} a_n = 0$.

Proof: The proof directly follows from **Proposition 2** [12].

Theorem 6. Let T, f, g and ρ be the same as in **Theorem 1**. Suppose that $S: K_r \to K_r$ is a nearly uniformly L-Lipschitz mapping such that $Fix(S) \bigcap EEGRNVI$ and $(Kr, T, f, g) \neq \varnothing$. In addition, let $L\gamma < 1$, where γ is the same as in **Theorem 1**. If there exists a constant $\alpha > 0$ such that $\Pi_{i=1}^{p} \alpha_{n,i} > \alpha$ for each $n \geq 0$, then the iterative sequence $\{u_n\}_{n=0}^{\infty}$ generated by **Algorithm 1** converges strongly to the only element of $Fix(S) \bigcap EEGRNVI$ and $(Kr, T, f, g) \neq \varnothing$. Afterward, the iterative scheme yields an approximate solution u_{n+1}, which ultimately converges to the exact solution u of the variational inequality. And let $C_3 < 1$.

Proof. From the definition of u_{n+1} and u^*,

$$u_{n+1} = (1-\alpha_{n,1} - \beta_{n,1}) u_n + \alpha_{n,1} \left(S^n \Phi(y_{n,1}) + q_{n,1} \right) + \beta_{n,1} l_{n,1} + r_{n,1},$$

$$u^* = (1-\alpha_{n,1} - \beta_{n,1}) u^* + \alpha_{n,1} \left(S^n \Phi(y_{n,1}) + q_{n,1} \right) + \beta_{n,1} l_{n,1} + r_{n,1}.$$

Then

$$\| u_{n+1} - u^* \leq (1-\alpha_{n,1} - \beta_{n,1}) \ u_n - u^* + \alpha_{n,1} \ S^n \{ y_{n,1} - g(y_{n,1})$$
$$+ P_{K_r} \left(f(y_{n,1}) - \rho T(y_{n,1}) \right) \} - S^n \left\{ u^* - g(u^*) + P_{K_r} \left(f(u^*) - \rho T(u^*) \right) \right\} \|$$
$$+ \beta_{n,1} \| \ l_{n,1} - u^* + \alpha_{n,1} \ q_{n,1} + \ r_{n,1} .$$

Let $\Gamma = sup_{n \geqslant 0} \| \ l_{n,i} - u^* \ : i-1, 2, ..., p$, then

$$\| u_{n+1} - u^* \leq (1-\alpha_{n,1} - \beta_{n,1}) \ u_n - u^* + \alpha_{n,1} L\gamma \ y_{n,1} - u^*$$

Similarly, following the previous proof, we can also establish the follow inequality that for each $i = 1, 2, \ldots, p-2$,

$$\| y_{n,i} - u^* \leq (1-\alpha_{n,i+1} - \beta_{n,i+1}) \ u_n - u^* + \alpha_{n,i+1} L\gamma \ y_{n,i+1} - u^*$$

$$\| y_{n,p-1} - u^* \leq (1-\alpha_{n,p} - \beta_{n,p}) \ u_n - u^* + \alpha_{n,p} L\gamma \ u_n - u^*$$

For $y_{n,p-2}$, we have

$$\| y_{n,p-2} - u^* \leq (1-\alpha_{n,p-1} - \beta_{n,p-1}) \ u_n - u^* + \alpha_{n,p-1} L\gamma \ y_{n,p-1} - x^*$$

By using **Equation (23)** and **Equation (24)**, we have:

$$\| y_{n,p-2} - u^* \leq (1 - \alpha_{n,p-1} - \beta_{n,p-1} + \alpha_{n,p-1} (1-\alpha_{n,p} - \beta_{n,p}) L\gamma$$

In turn, using the inequalities that we derived earlier, we end up with an inequality for $y_{n,i}$,

It follows from **Equation (21)** and **Equation (26)**, we get:

$$\| u_{n+1}-u^* \| \le \left(1-\alpha_{n,1}-\beta_{n,1}\right)\| u_n-u^* \| +\alpha_{n,1}L\gamma \| y_{n,1}-u^* \| +\alpha_{n,1}\| q'_{n,1}\| + \| q''_{n,1}\| + \| r_{n,1}\| +\beta_{n,1}\Gamma$$

$$\le\left[1-\left(1-L\gamma\right)\prod_{i=1}^{p}\alpha_{n,i}L^{p-1}\gamma^{p-1}\right]\| u_n-u^* \| +\sum_{i=1}^{p}\prod_{j=1}^{i}\alpha_{n,j}L^{i-1}\gamma^{i-1}\| q'_{n,i}\| + \| q''_{n,1}\| +\sum_{i=2}^{p}\prod_{j=1}^{i-1}\alpha_{n,j}L^{i-1}\gamma^{i-1}\| q''_{n,i}\| + \| r_{n,1}\|$$

$$+\sum_{i=2}^{p}\prod_{j=1}^{i-1}\alpha_{n,j}L^{i-1}\gamma^{i-1}\| r_{n,i}\| +\left(\beta_{n,1}+\sum_{i=2}^{p}\prod_{j=1}^{i-1}\alpha_{n,j}\beta_{n,i}L^{i-1}\gamma^{i-1}\right)\Gamma =\left[1-\left(1-L\gamma\right)\prod_{i=1}^{p}\alpha_{n,i}L^{p-1}\gamma^{p-1}\right]\| u_n-u^* \|$$

$$+\left(1-L\gamma\right)\prod_{i=1}^{p}\alpha_{n,i}L^{p-1}\gamma^{p-1}\frac{\sum_{i=1}^{p}\prod_{j=1}^{i}\alpha_{n,j}L^{i-1}\gamma^{i-1}\| q'_{n,i}\|}{\alpha\left(1-L\gamma\right)L^{p-1}\gamma^{p-1}}+\sum_{i=2}^{p}\prod_{j=1}^{i-1}\alpha_{n,j}L^{i-1}\gamma^{i-1}\| e'_{n,i}\| +\sum_{i=2}^{p}\prod_{j=1}^{i-1}\alpha_{n,j}L^{i-1}\gamma^{i-1}\| r_{n,i}\|$$

$$+\| e'_{n,1}\| + \| r_{n,1}\| +\left(\beta_{n,1}+\sum_{i=2}^{p}\prod_{j=1}^{i-1}\alpha_{n,j}\beta_{n,i}L^{i-1}\gamma^{i-1}\right)\Gamma =\left[1-\left(1-L\gamma\right)\prod_{i=1}^{p}\alpha_{n,i}L^{p-1}\gamma^{p-1}\right]\| u_n-u^* \| +A.$$

Since $L\gamma<1$ and $\lim_{n\to\infty}\| q'_{n,i}\| =0$, so $\| u_n-u^* \| \to 0 (n\to\infty), \lim_{n\to\infty}u_{n+1}=u^*$.

$$\| g_n\left(u_{n+1}\right)-g_n\left(u_n\right)\|^2 = \| P_{K_r}\left[f\left(u_{n+1}\right)-\gamma\alpha'_{n+1}d\left(u_{n+1},\rho_{n+1}\right)\right]-P_{K_r}\left[f\left(u_n\right)-\gamma\alpha'_n d\left(u_n,\rho_n\right)\right]\|^2$$

$$\le L_P^2\left(\| f\left(u_{n+1}\right)-f\left(u_n\right)\|^2+\gamma^2\| w_{n+1}-w_n\|^2\right)\le L_P^2(\phi^2\| u_{n+1}-u_n\|^2+\gamma^2(\| g_n\left(u_{n+1}\right)-g_n\left(u_n\right)\|^2$$

$$+L_P^2\left(\phi^2\| u_{n+1}-u_n\|^2+L_e^2\| u_{n+1}-u_n\|^2\right)).$$

Then

$$\| g_n\left(u_{n+1}\right)-g_n\left(u_n\right)\|^2\le\frac{L_P^2(\phi^2+\gamma^2\left(L_P^2\left(\phi^2+L_e^2\right)\right)}{\left(1-\gamma^2\right)}\| u_{n+1}-u_n\|^2\le C_3\| u_{n+1}-u_n\|^2.$$

Since $\lim_{n\to\infty}u_{n+1}=u^*$, then $g_n\left(u_{n+1}\right)-g_n\left(u_n\right)$ is convergent. Thus the sequence $\{u_n\}_{n=0}^{\infty}$ generated by **Algorithm 1** converges strongly to the only element of $Fix(S)\bigcap EEGRNVI\left(K_r,f,g\right)$. The result demonstrate that the general solution u_{n+1} of Algorithm 1 converges to the approximate solution u. Additionally, it can be inferred from **Equation (17)** and **Equation (18)** that the sequences $y_{n,i}$ strongly converge to u in H. These outcomes imply that a solution exists under certain circumstances and is unique.

5. Conclusion

We studied the exponentially generalized regularized nonconvex variational inequality involving multiple nonlinear operators. By means of projection techniques, an equivalence with fixed point problems is established enables the analysis of existence and uniqueness of solutions. Based on this framework, a self-adaptive finite p-step projection algorithm is proposed by combining adaptive strategies with a p-step iterative scheme. Under suitable conditions, the convergence of the algorithm is rigorously proved, and the results extend and improve existing work in nonconvex variational inequality theory.

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