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Dynamic Memristor-Inspired Zeroing Neural Network with Hybrid Activation for Finite-Time Convergence and Remote Sensing Image Fusion

Jiaqi He, Mu Li*

School of Information and Electrical Engineering, Hunan University of Science and Technology, Xiangtan 411100, Hunan, China

**Corresponding author:* Mu Li, mli@hnust.edu.cn

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Abstract: Solving time-varying nonlinear equations in real time is a significant challenge in modern computing. Dynamic Memristor-Inspired Zeroing Neural Networks (DMZNN) have shown strong performance in this field, but their convergence speed and robustness heavily rely on the design of the activation function. This paper proposes a novel hybrid activation function inspired by the nonlinear characteristics of memristors. By integrating cubic and sublinear terms, the proposed function facilitates multi-stage error decay, effectively addressing the slow convergence and poor noise resistance of traditional activation functions. Theoretical analysis shows that the DMZNN model, built upon this activation function, can converge in finite time. Robustness under parameter perturbations and additive noise is rigorously proven using Lyapunov theory. Simulation results demonstrate that the convergence speed of the DMZNN model is obviously faster than that of traditional ZNN models when solving second-order, third-order, and fourth-order time-varying nonlinear equations. Additionally, in the application of remote sensing image fusion, DMZNN outperforms traditional gradient-based methods in both fusion quality and processing speed, demonstrating its practical effectiveness and superiority in real-world applications.

Keywords: Zeroing neural network; Finite-time convergence; Memristor; Time-varying nonlinear equations; Remote sensing image fusion

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1. Introduction

Solving time-varying nonlinear equations in real-time is a critical challenge in many fields such as robotics, signal processing, and optimization[1]. Traditional numerical methods, like Newton's iteration, often face limitations such as slow convergence and high computational costs when dealing with dynamic systems[2]. Zeroing neural networks (ZNNs) have emerged as an efficient alternative, leveraging dynamic error equations and activation functions to track solutions in real time[3,4]. However, conventional activation functions frequently demonstrate slow convergence and inadequate noise resistance[5]. Recent research has centered on the development of enhanced activation functions to enhance the performance of ZNNs. Memristor-based designs offer smooth nonlinearity but may still lack finite-time convergence and robustness. The proposed hybrid activation function integrates cubic and sublinear terms, thereby facilitating accelerated finite-time convergence and enhanced noise immunity[6].

To address these challenges, we propose the Dynamic Memristor-Inspired Zeroing Neural Network (DMZNN). Inspired by the nonlinear characteristics of memristors, the DMZNN integrates a novel hybrid activation function that combines cubic and sublinear terms. This new function accelerates error decay and enhances convergence speed, robustness, and noise resistance, surpassing traditional ZNNs.The DMZNN model is proven to converge in finite time, with Lyapunov stability theory ensuring its robustness under parameter perturbations and noise. Simulations show that DMZNN achieves significantly faster convergence compared to traditional ZNN models for second- to fourth-order time-varying nonlinear equations. Furthermore, we apply the DMZNN model to remote sensing image fusion, where it outperforms traditional gradient-based methods in both fusion quality and processing speed. This demonstrates the potential of DMZNN for real-time, high-efficiency image fusion applications.

2. Problem formulation and evolution of the ZNN model

2.1 Problem formulation and traditional ZNN models

Consider the time-varying nonlinear equation:

$$f(x(t),t)=0 \qquad x(t)\in \mathbf{R} \tag{1}$$

Define the error e(t)=f(x(t), t). The ZNN dynamic is:

$$\frac{\partial f}{\partial x}\cdot\frac{dx}{dt}=-\frac{\partial f}{\partial t}-\gamma f\left(x\left(t\right),t\right) \tag{2}$$

where γ is a convergence factor greater than zero.

2.2 Dynamic Memristor-Inspired Hybrid Activation Function

This paper uses a memristor with a cubic smoothing curve. For a cubic function model, the resistor exhibits a smooth, monotonic, and increasing nonlinear characteristic curve[7].

$$\mathrm{q}\left(\varphi\right)=a\varphi+b\varphi^3 \tag{3}$$

Where a, $b > 0$. To achieve finite-time convergence, we augment it with a sublinear term:

$$M\left(e\right)=ae+be^3+c\tanh\left(\frac{e}{\delta}\right)\left|e\right|^{\varepsilon} \tag{4}$$

With a, b, $c > 0$, $0 < \varepsilon < 1$. The resulting DMZNN dynamic is as follows:

$$\frac{\partial f}{\partial x}\cdot\frac{dx}{dt}=-\gamma\left(ae+be^3+c\cdot\tanh\left(\frac{e}{\delta}\right)\left|e\right|^{\varepsilon}\right)-\frac{\partial f}{\partial t} \tag{5}$$

2.3 Finite-Time Convergence Mechanism

When the error $e(t)$ is large, the cubic term be^3 dominates, the error decays according to $O(1/\sqrt{t})$.

$$\int_{e_0}^{\mathrm{e}}\frac{1}{e^3}de=-\gamma b\int_0^t dt \Rightarrow e\left(t\right)=\frac{e_0}{\sqrt{1+2\gamma b e_0^2 t}} \tag{6}$$

When error e(t) is small, the linear term *ae* dominates and the error decays exponentially according to the formula $e(t) \propto e^{-\gamma at}$, and the sublinear term has been shown to increase the absolute value of the derivative, thereby contributing to additional attenuation power[8].

$$\frac{de}{dt} \approx -\gamma c|e|^{\varepsilon} \Rightarrow e(t) = \left[e(0)^{1-\varepsilon} - \gamma c(1-\varepsilon)t\right]^{\frac{1}{1-\varepsilon}} \tag{7}$$

It converges to zero within a finite time, $T = \frac{\left|e(0)^{1-\varepsilon}\right|}{\gamma c(1-\varepsilon)}$.

3. Theoretical Analysis

Theorem 1 (Finite-time convergence)

For the error dynamics specified above, the error converges to zero within a finite time T, which is constrained by the following bounds:

$$\mathrm{T} \leq \frac{|e(0)|^{1-\varepsilon}}{\gamma c(1-\varepsilon)} \tag{8}$$

Proof:

Construct Lyapunov function $V(e) = \frac{1}{2}e^2$ and apply finite-time stability theory.

$$\dot{V}(e) = e \cdot \frac{de}{dt} = -\gamma\left(ae^2 + be^4 + c|e|^{\varepsilon+1}\right) \tag{9}$$

If $\alpha = \frac{\varepsilon+1}{2}$, then 0.5 < α < 1, the system satisfies the finite-time stability condition. According to the finite-time Lyapunov theorem, a convergence time exists.

$$\mathrm{T} \leq \frac{V(e(0))^{1-\alpha}}{\gamma c(1-\alpha)} = \frac{|e(0)|^{1-\varepsilon}}{\gamma c(1-\varepsilon)} \tag{10}$$

Therefore, error *e*(*t*) converges to zero in a finite amount of time, *T*.

Theorem 2 (Robust finite-time stability)

Assume that the parameters *a*, *b*, *c* are subject to bounded perturbations $\Delta a, \Delta b, \Delta c$ that satisfy $|\Delta a| \leq \delta_a, |\Delta b| \leq \delta_b, |\Delta c| \leq \delta_c$ and that the upper bounds of the perturbations satisfy $\delta_a < a, \delta_b < b, \delta_c < c$. In this case, the improved error dynamics equation still maintains finite-time stability[9].

Proof:

Error dynamics after disturbance:

$$\frac{de}{dt} = -\gamma\left((a+\Delta a)e + (b+\Delta b)e^3 + (c+\Delta c)sign(e)|e|^{\varepsilon}\right) \tag{11}$$

Perform a Lyapunov derivative analysis and calculate $\dot{V}(e)$ after the perturbation.

$$\dot{V}(e) \leq -\gamma\left((a-\delta_a)e^2 + (b-\delta_b)e^4 + (c-\delta_c)|e|^{\varepsilon+1}\right) \tag{12}$$

Since $a - \delta_a > 0$, $b - \delta_b > 0$, $c - \delta_c > 0$, the original Lyapunov function still satisfies:

$$\dot{V}(e) \leq -k|e|^{\varepsilon+1}, k = \gamma(c - \delta_c) \tag{13}$$

The convergence time is updated to $T_{robust} \leq \dfrac{|e(0)|^{1-\varepsilon}}{k(1-\varepsilon)}$.

4. Numerical simulation results

4.1 Experiment on Solving Second-Order Time-Varying Nonlinear Equations

This section will compare the ZNN and DMZNN models' ability to solve TVNE computer numerical simulations. This comparison will demonstrate the DMZNN model's superiority to the ZNN model, particularly in solving high-order nonlinear equations. Without loss of generality, consider the following second-order TVNE:

$$f(x(t),t) = 0.1x^2 - 0.1\cos^2(2t) - 0.6\cos(2t) - 0.9 \tag{14}$$

In the initial state x(0) ∈ [-5,5], the solution states and residuals of the ZNN and DMZNN are depicted in **Figure 1** and **2**, respectively.

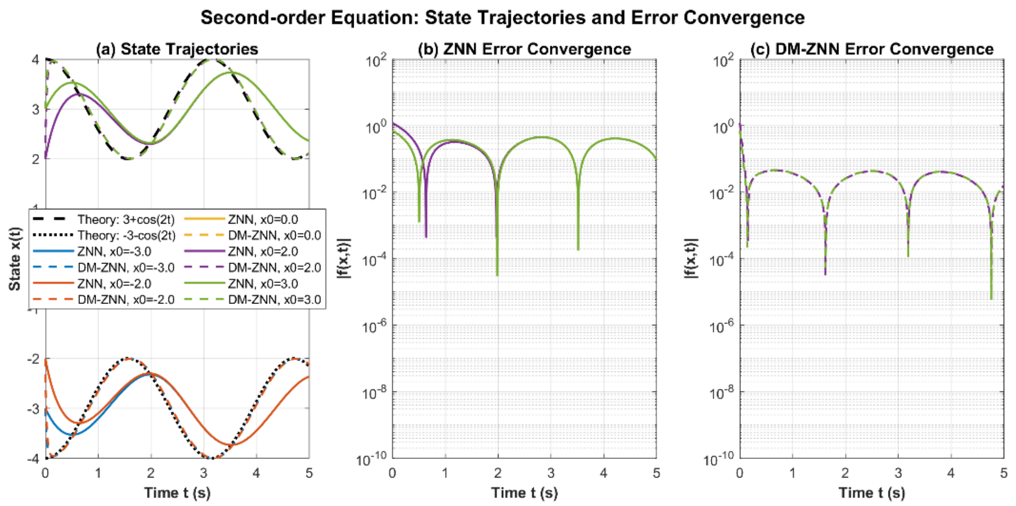

Figure 1. The neural state solution x(t) of SOTVNE in equation (14) for the ZNN and DMZNN models

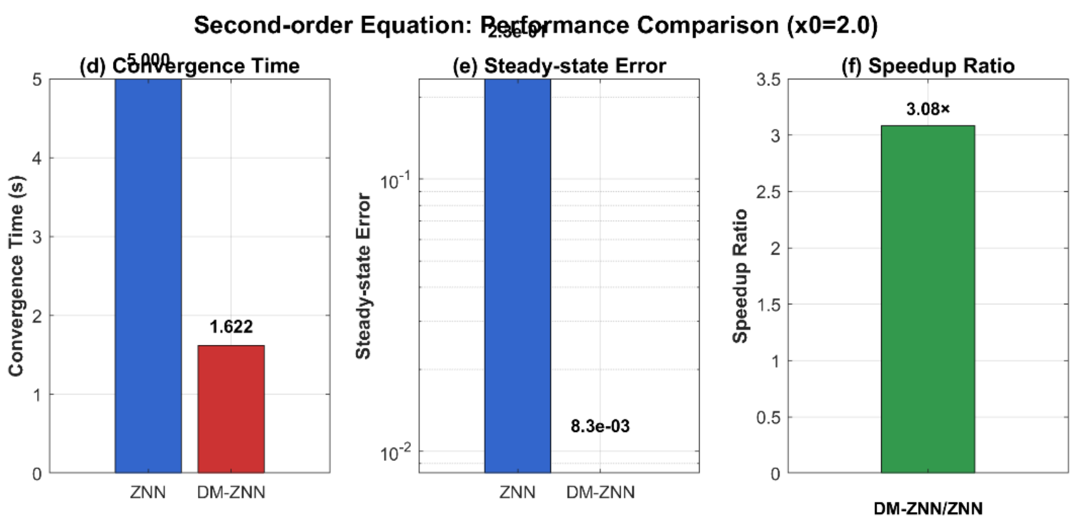

Figure 2. The residual |f(x(t),t)| of SOTVNE in equation (14) for the ZNN and DMZNN models

As **Figure 1** and **2** show, the ZNN model converges in approximately 1 s, while the DMZNN model converges in approximately 0.4 s when solving the SOTVNE in (14). Therefore, the DMZNN model is more efficient than the ZNN model at solving the SOTVNE problem.

4.2 Experiment on solving third-order time-varying nonlinear equations

In order to validate the model's effectiveness for higher-order TVNEs, the third-order TVNE was solved. The simulation results are displayed in **Figure 3–4**, respectively.

$$f\left(x(t),t\right)=0.01\left(x-\cos 2t-5\right)\left(x+\cos 2t+5\right)\left(x-\cos 2t\right) \tag{15}$$

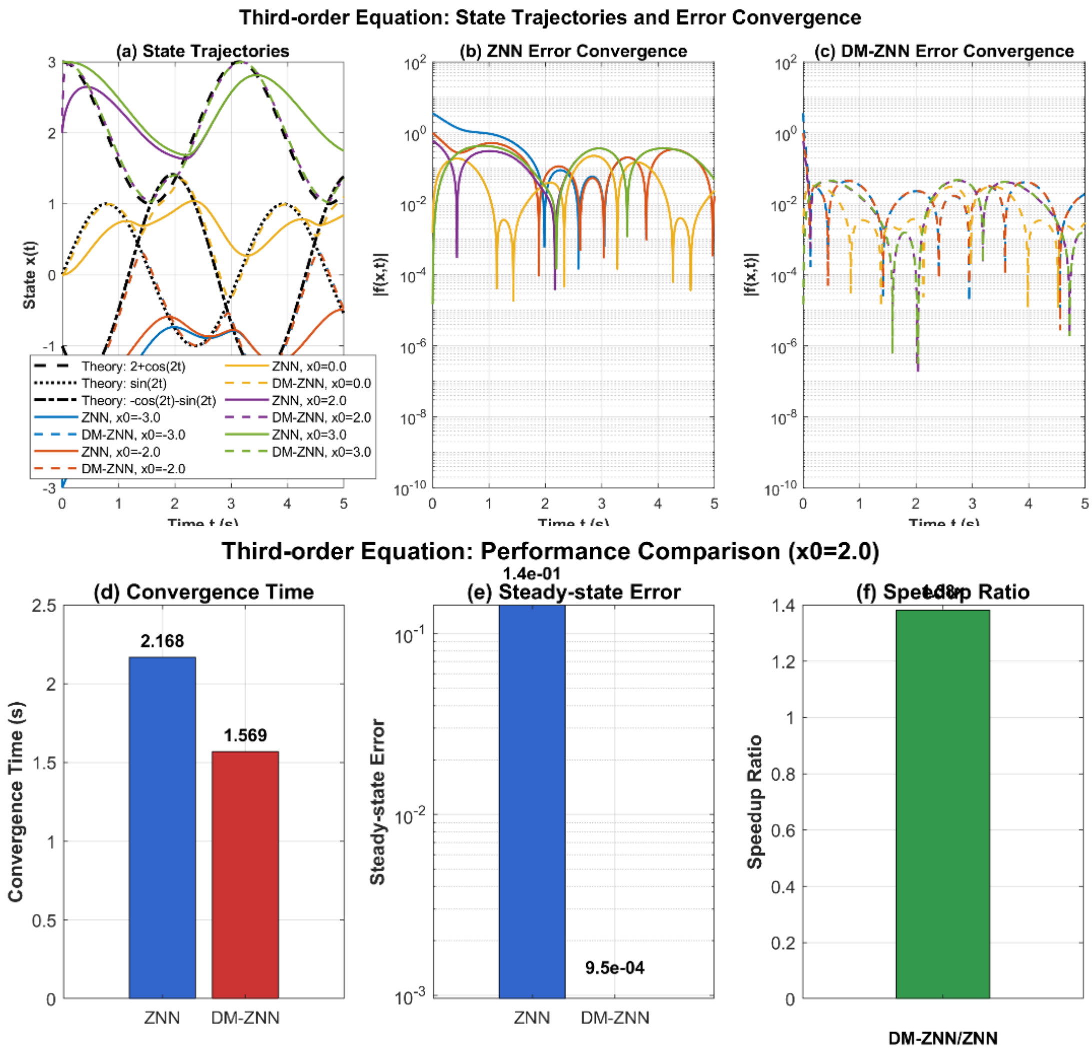

Figure 4. The residual |f(x(t),t)| of SOTVNE in equation (15) for the ZNN and DMZNN models

As demonstrated in the case of traditional ZNN models, significant oscillations and path overlap are exhibited during the solution of high-order problems. Additionally, residuals demonstrate an inability to converge to zero. Conversely, the DMZNN model demonstrated rapid and stable convergence, devoid of oscillations in all instances. Due to space limitations, only the solution data for the fourth-order equation is provided.

In summary, the comparative simulation experiments confirm the efficiency of the proposed DMZNN model for solving time-varying nonlinear equations, particularly high-order ones.

The aim of remote sensing image fusion is to integrate complementary information from multiple images in order to generate a composite image with high spatial and spectral fidelity[10].

This paper uses the QuickBird remote sensing dataset from the National Cryosphere Desert Data Centre. Subjective evaluation is predicated on visual comparison, while objective evaluation employs the calculation of multiple metrics by referencing reference images. Due to space constraints, the main text only presents a set of typical fusion results (**Figure 5**).

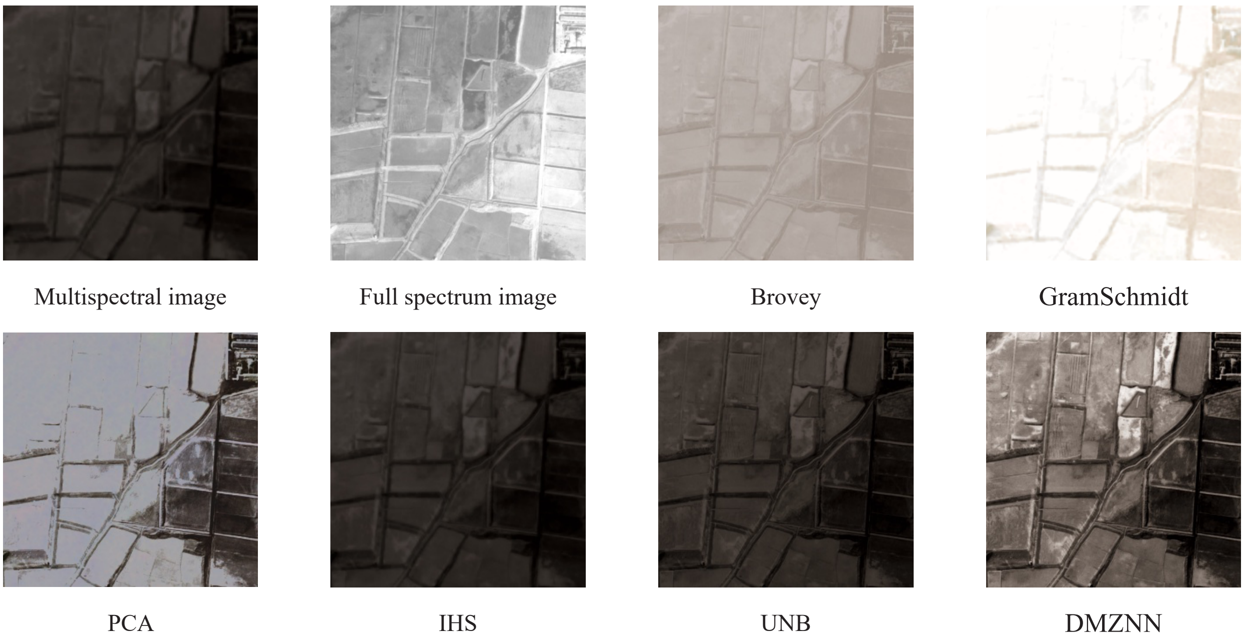

Figure 5 Simulation Experiment Fusion Results Based on the QuickBird Remote Sensing Dataset

The results of the fusion experiment, which are based on the QuickBird remote sensing dataset, are presented in **Table 1**. A review of the extant literature reveals that traditional methods generally exhibit suboptimal performance. While certain methodologies exhibited superior performance in specific metrics, they did not consistently demonstrate effectiveness in overall evaluation indicators. In contrast, the DMZNN method proposed in this paper achieved optimal or near-optimal results across all key metrics, exhibiting comprehensively superior fusion performance.

Table 1. Evaluation of the Fusion Results of the Simulation Experiments Based on the SPOT Remote Sensing Datasets

Method	SSIM ↑	PSNR ↑	CC ↑	ERGAS ↓	SAM ↓	RMSE ↓
Brovey	0.4167	4.82	0.9551	76.76	54.44	17.22
Gram-Schmidt	0.1247	2.59	0.6434	99.18	66.70	19.16
PCA	0.3870	6.75	0.8844	61.97	13.35	11.82
IHS	0.8834	42.31	0.9706	3.71	1.29	9.42
UNB	0.9023	38.32	0.9540	6.59	1.15	9.12
DMZNN	0.8974	41.65	0.9826	3.62	1.04	8.89

5. Summary

This paper presents the Dynamic Memristor-Inspired Zeroing Neural Network (DMZNN), a novel model for solving time-varying nonlinear equations in real-time. DMZNN is based on a hybrid activation function inspired by memristors, which combines cubic and sublinear terms to achieve multi-stage error decay and finite-time convergence. The proposed model addresses the challenges of slow convergence and poor robustness seen in traditional Zeroing Neural Networks (ZNN), especially when dealing with noise and perturbations. We rigorously prove the finite-time convergence and robustness of DMZNN using Lyapunov theory. Experimental results show that DMZNN outperforms traditional ZNN models in solving second- to fourth-order time-varying nonlinear equations, with faster convergence. Additionally, DMZNN is successfully applied to remote sensing image fusion tasks, where it significantly improves both fusion quality and processing speed compared to traditional gradient-based methods. This demonstrates DMZNN's potential for real-time, high-efficiency image fusion applications. The contributions of this work are twofold: first, the introduction of a dynamic, memristor-inspired hybrid activation function to improve convergence and robustness; second, the development and application of the DMZNN model to a practical real-world problem—remote sensing image fusion. The results show that DMZNN provides a robust, efficient, and theoretically grounded solution for solving dynamic optimization problems in real-time applications.

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Disclosure statement

The author declares no conflict of interest.

Author contributions

Jiaqi He: Investigation, Methodology, Soft-ware, Writing – original draft. Mu Li: Funding acquisition, Resources, Supervision, Writing – review & editing.

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