Journal of Electronic Research and Application, 2025, Volume 9, Issue 3
http://ojs.bbwpublisher.com/index.php/JERA
ISSN Online: 2208-3510
ISSN Print: 2208-3502

Optimization of Multi-Vehicle Routing Problem with Time Windows and Simultaneous Pickup and Delivery

Biao Wang*

School of Maritime Transport, Ningbo University, Ningbo 315211, Zhejiang, China

**Author to whom correspondence should be addressed.*

Abstract: This paper addresses the Multi-Vehicle Routing Problem with Time Windows and Simultaneous Pickup and Delivery (MVRPTWSPD), aiming to optimize logistics distribution routes and minimize total costs. A vehicle routing optimization model is developed based on the operational requirements of the KS Logistics Center, focusing on minimizing vehicle dispatch, loading and unloading, operating, and time window penalty costs. The model incorporates constraints such as vehicle capacity, time windows, and travel distance, and is solved using a genetic algorithm to ensure optimal route planning. Through MATLAB simulations, 34 customer points are analyzed, demonstrating that the simultaneous pickup and delivery model reduces total costs by 30.13%, increases vehicle loading rates by 20.04%, and decreases travel distance compared to delivery-only or pickup-only models. The results demonstrate the significant advantages of the simultaneous pickup and delivery mode in reducing logistics costs and improving vehicle utilization, offering valuable insights for enhancing the operational efficiency of the KS Logistics Center.

Keywords: Vehicle routing problem; Time windows; Multi-vehicle types; Simultaneous pickup and delivery; Genetic algorithm

Online publication: 6 June, 2025

1. Introduction

The Vehicle Routing Problem (VRP) was first introduced by Dantzig and Ramser in their seminal paper "The Truck Dispatching Problem" in 1959 [1]. Since then, extensive research has been conducted to address the challenges in real-world distribution and route planning, leading to the development of various models and algorithms. Specifically, for the simultaneous pickup and delivery problem, Ai *et al.* employed a Particle Swarm Optimization (PSO) algorithm to solve the vehicle routing problem with simultaneous pickup and delivery [2]. Zhang *et al.* combined an Ant Colony Algorithm with the MMAS algorithm to address single-warehouse reverse logistics, while Jun and Kim proposed a comprehensive heuristic algorithm involving route construction, improvement, and perturbation steps [3,4]. Zhou and Shen introduced an adaptive parallel genetic algorithm using the C-W saving method and dual-demand heuristic initialization [5]. Christopher *et al*. applied Tabu Search and Variable Neighborhood Descent (VND) algorithms to solve the VRPSDP. Li and Chen incorporated time windows and pickup-delivery demands into the PCVRP model, solving it with multiple algorithms [6,7]. Yan *et al*. developed a multi-vehicle low-carbon VRP model using a Quantum Evolutionary Algorithm (QEA) and proposed a multi-objective QEA for optimizing multi-distribution center problems [8].

Based on this, this paper investigates the multi-vehicle simultaneous pickup and delivery routing problem with time windows. Taking the KS Logistics Center as the research object, we establish a model for its existing one-way distribution mode and suboptimal route design, and propose a genetic algorithm to solve these problems. Simulations and case analyses are conducted using MATLAB.

2. Modelling

2.1. Description of the problem

The MVRPTWSPD problem involves optimizing vehicle routes for a logistics center serving multiple customers with simultaneous pickup and delivery requirements. Key assumptions include: (1) a single logistics center serves all customers; (2) mixed delivery and pickup shipments are allowed; (3) pickup and delivery demands for each customer are known; (4) vehicles have identical models and maximum load limits; (5) distances between customers and the logistics center are known; (6) each customer and vehicle is serviced once; (7) the same vehicle can handle both pickup and delivery; (8) total goods at each customer must not exceed vehicle capacity; and (9) factors like goods deterioration, driver working hours, road accessibility, and transport regulations are not considered. The objective is to minimize total costs, including fixed and variable distribution costs, while maximizing vehicle load rates and reducing the number of trips.

2.2. Description of parameters and variables

Based on the above problem description and assumptions, the parameters and variable symbols used in the thesis are described as follows to facilitate the mathematical modelling of MVRPTWSPD [9–12]:

U: Customer Point Aggregation, is the distribution center, $i,j = 1,2,3,...,n$;

C: The collection of vehicles available to the distribution center, $C = \{k_v\}$, $k = 1,2,3,...,m$, $v = 1,2,3$, k_v represents one of the v models, for a total of three models;

Q_{k_i}: Type v distribution vehicle k rated load square;

g_v: Self-weight of distribution vehicle type v;

d_i: Number of delivery demand parties at customer point i;

p_i: Number of pickup demand parties at customer point i;

$[E_i, L_i]$: The customer point i specifies the time frame for the arrival of the delivery vehicle, where E_i is the earliest specified point of arrival of the delivery vehicle and L_i is the latest specified point of arrival of the delivery vehicle;

$[ET_i, LT_i]$: the range of vehicle arrival times tolerated by the customer point i, where ET_i: The earliest arrival time tolerated by the customer point i; LT_i: The latest arrival time tolerated by the customer point i;

t_i: The point in time when the vehicle starts service at the customer's point i;

t_{ij}: Vehicle travelling time from a customer point i to customer point j;

s_{ij} : The distance between the customer point *i* and the customer point *j*;

S : Maximum mileage of the vehicle;

C_s : Cost of loading and unloading per unit of cargo;

N_{k_v} : Type *v* distribution vehicle call-out costs;

z_{ijk_v} : Costs incurred per kilometer per kilogram by the distribution vehicle *k* of type *v* travelling from a customer's point *i* to the customer's point *j*;

q_{ijk_v} : The number of cargo load squares for the distribution vehicle *k* of type *v* when travelling directly from a customer point *i* to customer point *j*;

x_{ijk_v} : A value of 1 indicates that the *v* type vehicle *k* directly serves customers *i* and customer *j*, otherwise its value is 0;

x_{0jk_v} : A value of 1 indicates that the *v* type distribution vehicle *k* is departing from the distribution center, otherwise its value is 0;

x_{j0k_v} : A value of 1 means that *v* type distribution vehicle *k* returns to the distribution center, otherwise its value is 0;

2.3. Model building

In this paper, we focus on the four aspects of vehicle loading and unloading costs, running costs, vehicle fixed costs, and time window penalty costs, to minimize the total system cost [13–15], to establish a mathematical model of VRPSDPSTW for multiple vehicles, as shown in Equation 1:

$$Min(i,j,v,k)=\sum_{v=1}^{3}\sum_{k=1}^{m}\sum_{j=0}^{n}x_{0jk_v}N_{k_v}+2\sum_{v=1}^{3}\sum_{k=1}^{m}\sum_{i=0}^{n}\sum_{j}^{n}C_s(p_i+d_i)x_{ijk_v}+ \tag{1}$$
$$\sum_{v=1}^{3}\sum_{k=1}^{m}\sum_{i=0}^{n}\sum_{j=0}^{n}z_{ijk_v}[g_v+(d_{ij}+p_{ij})\rho]s_{ij}x_{ijk_v}+\theta_1\sum_{i=0}^{n}\max(E_i-t_i,0)+$$
$$\theta_2\sum_{i=0}^{n}\max(t_i-L_i,0)$$

Constraints:

Vehicle deployment constraints:

$$\sum_{v=1}^{3}\sum_{k=1}^{m}\sum_{i=0}^{n}x_{ijk_v}=1,\ i,j\in R,i,j\in R,1\le j\le n,i\ne j \tag{2}$$

$$\sum_{j=0}^{n}x_{0jk_v}\le 1,\quad k_v\in C \tag{3}$$

$$\sum_{j=0}^{n}x_{j0k_v}\le 1,\quad k_v\in C \tag{4}$$

Vehicle load constraints:

$$q_{ijk_v}\le Q_{k_v}X_{ijk_v},i,j\in R,k_v\in C,i\ne j \tag{5}$$

$$\sum_{j=1}^{n}q_{0jk_v}=\sum_{i=0}^{n}\sum_{j=1}^{n}d_ix_{ijk_v},i,j\in R,k_v\in C,i\ne j \tag{6}$$

$$\sum_{i=1}^{n}q_{i0k_v}=\sum_{i=0}^{n}\sum_{j=1}^{n}p_ix_{ijk_v},i,j\in R,k_v\in C,i\ne j \tag{7}$$

$$\sum_{i=0}^{n}q_{ijk_v}+(p_i-d_i)\sum_{i=0}^{n}x_{ijk_v}=\sum_{i=0}^{n}q_{ijk_v},i,j\in R,k_v\in C,i\ne j \tag{8}$$

$$\sum_{j=1}^{n}q_{0jk_v}\le Q_{k_v}x_{ijk_v},i,j\in R,k_v\in C,i\ne j \tag{9}$$

$$\sum_{j=1}^{n}q_{i0k_v}\le Q_{k_v}x_{ijk_v},i,j\in R,k_v\in C,i\ne j \tag{10}$$

$$q_{ijk_v}\ge 0,i,j\in R,k_v\in C,i\ne j \tag{11}$$

$$p_i \geq 0, d_i \geq 0, Q_{k_v} \geq 0, i \in R \tag{12}$$

Service time constraints:

$$t_j \geq max\{ET_i, (t_j + s_j + t_{ij})x_{ijk}\}, i, j \in R \tag{13}$$

$$t_j \leq LT_i, j \in R \tag{14}$$

Vehicle mileage constraints:

$$\sum_{i=0}^{n} \sum_{j=0}^{n} s_{ij} x_{ijk_v} \leq S,\ i, j \in R, k_v \in C, i \neq j \tag{15}$$

Decision variables:

$$x_{ijk_v} \in \{0,1\},\ i, j \in R, k_v \in C, i \neq j \tag{16}$$

$$x_{j0k_v} \in \{0,1\},\ j \in R, k_v \in C, i \neq j \tag{17}$$

$$x_{i0k_v} \in \{0,1\},\ i \in R, k_v \in C, i \neq j \tag{18}$$

In the mathematical model, the objective function (Equation 1) minimizes the total distribution cost, including vehicle call-up, loading/unloading, running, and time window deviation penalty costs. Constraints (Equation 2) ensure each customer node is visited exactly once. Constraints (Equations 3,4) restrict each vehicle to one delivery trip. Constraint (Equation 5) ensures the vehicle load at any point does not exceed its maximum capacity. Constraints (Equations 6,7) define the vehicle's load upon departure from and return to the distribution center, equating it to the total delivery and pickup demands, respectively. Constraint (Equation 8) governs load changes along the route. Constraints (Equations 9,10) enforce that the vehicle's load at departure and return does not exceed its maximum capacity. Constraint (Equation 1) ensures non-negative load capacity, while Constraint (Equation 12) requires positive maximum load limits if pickup or delivery demands are positive. Constraints (Equations 13,14) specify service start time requirements, ensuring vehicles arrive and serve within acceptable time windows. Constraint (Equation 15) limits the vehicle's travel distance to its maximum mileage. Constraints (Equations 16–18) define decision variables for the model.

3. Example analysis

3.1. Simulation data and parameter settings

Thirty-four customers in the KS logistics center were selected as distribution demand points, and the demand, time window, and loading and unloading time for each customer point are shown in **Table 1**. Similarly, three common models in KS logistics center are selected as distribution vehicles, and there are differences in the number of loaded parties, calling cost, and vehicle weight among the three models. The details are shown in **Tables 2** and **3**.

Table 1. Distribution center and customer points related information

Customer point	Latitude and longitude coordinates	Pick-up volume (m^3)	Deliveries (m^3)	Customer tolerance time window	Customer's ideal time window	Unloading time (min)	Loading time (min)
0	(120.799,31.302)	-	-	[7:00–22:00]	[7:00–22:00]	-	-
1	(120.529,31.402)	31	3	[10:00–17:30]	[12:00–16:50]	25	7
2	(121.087,30.94)	36	3	[12:30–17:50]	[13:00–17:00]	30	9
3	(121.274,31.256)	60	6	[8:00–12:30]	[9:00–11:50]	52	16
4	(120.345,30.118)	48	5	[14:00–16:30]	[14:30–16:00]	42	13
5	(121.135,31.171)	62	6	[7:20–12:00]	[8:00–11:40]	55	17
6	(121.447,30.965)	43	4	[15:00–18:40]	[15:30–15:00]	39	12
7	(121.276,31.072)	23	2	[7:40–11:30]	[8:00–11:00]	19	6
8	(121.317,31.362)	11	0	[10:20–13:30]	[10:30–12:30]	9	1
9	(121.752,31.218)	43	4	[13:40–15:20]	[14:00–15:00]	39	11
10	(119.941,31.803)	39	4	[12:00–18:30]	[13:30–17:00]	35	11
11	(121.018,31.264)	64	7	[10:30–14:20]	[10:50–14:00]	56	18
12	(121.315,31.225)	29	2	[15:10–20:10]	[16:00–18:30]	26	7
13	(120.73,30.685)	19	1	[8:30–12:30]	[9:30–12:00]	15	5
14	(121.22,31.199)	13	1	[14:10–19:10]	[15:30–19:10]	11	3
15	(121.132,30.905)	32	3	[12:40–16:40]	[13:30–16:00]	26	7
16	(120.915, 31.931)	30	3	[9:00–17:30]	[9:30–17:10]	26	8
17	(120.95,31.5955)	33	3	[13:30–19:00]	[15:30–18:40]	27	8
18	(120.244,31.848)	30	3	[8:00–14:30]	[9:40–13:30]	26	8
19	(120.234,31.591)	24	2	[15:00–20:30]	[15:30–18:00]	19	5
20	(121.233,30.995)	25	2	[7:20–12:00]	[8:00–11:00]	20	5
21	(119.853,31.291)	19	1	[15:30–21:40]	[14:00–19:40]	16	4
22	(121.392,31.467)	32	3	[7:30–14:30]	[8:00–12:00]	28	7
23	(120.22,30.522)	16	1	[10:40–15:30]	[11:10–13:50]	12	3
24	(121.429,31.382)	22	2	[12:30–15:20]	[13:00–15:00]	17	4
25	(120.957,31.407)	24	2	[12:30–21:30]	[13:40–20:30]	18	5
26	(121.468,29.754)	17	1	[8:30–14:30]	[9:10–13:30]	13	4
27	(121.319,31.062)	29	2	[11:10–16:10]	[11:30–15:20]	25	7
28	(120.73,31.48)	27	2	[8:30–19:40]	[9:20–19:10]	22	6
29	(120.596,31.301)	22	2	[14:10–19:10]	[15:00–18:30]	18	6
30	(119.913,31.006)	17	1	[10:40–20:50]	[11:30–19:10]	13	3
31	(120.625,30.854)	20	1	[8:40–20:30]	[9:00–19:00]	16	4
32	(121.502,30.263)	21	2	[9:20–16:00]	[10:30–15:10]	17	4
33	(121.175,29.811)	29	3	[9:30–14:40]	[10:00–12:00]	24	8
34	(120.538,31.147)	29	2	[10:50–19:30]	[11:30–18:00]	25	8

Table 2. Fixed parameters for the model

Parameters	Unit (of measure)	Numerical value
Cargo vehicle transport rates	Yuan/ton-kilometer	2
Fuel feed	Kilometers per hour	60
Maximum travelling distance S	Kilometer	500
Loading and unloading costsC_S	Yuan/m^3	0.2
Time penalty costs for early arrival of distribution vehicles θ_1	Yuan/minute	0.1
Time penalty costs for the delayed arrival of distribution vehicles θ_2	Yuan/minute	0.2

Table 3. Parameters related to the three types of vehicles

Serial number	Vehicle type	Maximum number of loaded parties Q_k (m^3)	Invocation cost N_k (Yuan /vehicle-times)	Vehicle weight g_v (kg)
Type A	9.6 meter van/high-bar trucks	90	50	6000
Type B	12.5 meters van/high-bar trucks	120	150	8000
Type C	17.5 meters trailer (van)	160	200	12000

3.2. Analysis of solution results

In this study, the genetic algorithm (GA) was implemented using MATLAB 2021b to solve the model. Due to the GA's iterative convergence toward optimal solutions, the algorithm was executed 10 times to ensure solution reliability. The average runtime was 91.6 seconds, with each run's path and cost recorded. The path with the lowest distribution cost was selected as the optimal solution. The distribution route is illustrated in **Figure 1**. The algorithm demonstrated rapid convergence as iterations increased, indicating its effectiveness in solving the model and its strong convergence performance.

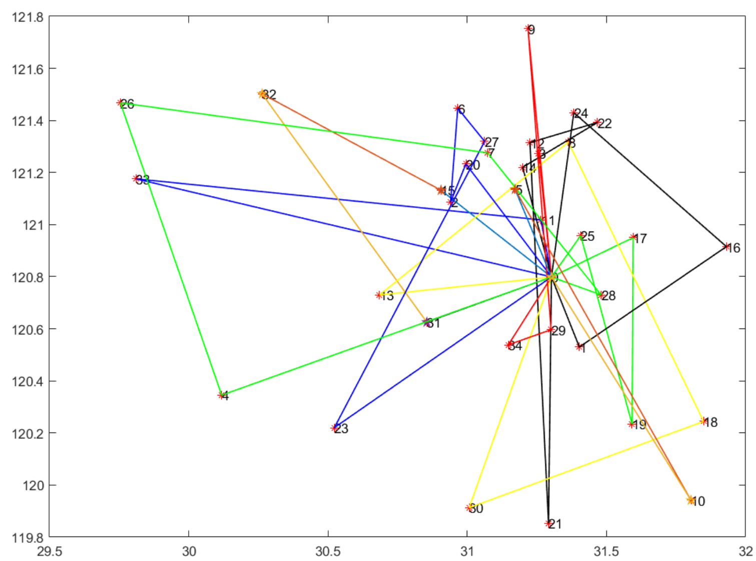

Figure 1. Optimal driving path for simultaneous pickup and delivery

As shown in **Table 4**, there are 11 distribution paths and 11 vehicles are called, of which 4 are type A vehicles, 6 are type B vehicles, and 1 is a type C vehicle. The optimal total distribution cost is 86,794.172 yuan, of which 84,850.7093 yuan is the vehicle running cost, 1,350 yuan is the calling cost, 443.2 yuan is the loading and unloading cost, and 150.263 yuan is the time window penalty cost. The total distance travelled by the vehicle was 3317.605 km. The average loading rate of the vehicle on departure was 81.72%, and on return was 7.12%.

Table 4. Optimal cost distribution options

Vehicle usage	Client service sequence	Length of route (Km)	Total cost per unit (Yuan)	Vehicle loading rate at departure (%)	Vehicle loading rate on return (%)
Type A 1st	0–3–0	90.832	1653.201	66.67%	6.67%
Type A 2nd	0–17–19–25–0	194.132	5381.285	90.00%	7.78%
Type A 3rd	0–15–32–31–0	293.116	6812.260	81.11%	6.67%
Type A 4th	0–13–8–18–30–0	467.079	11588.171	85.56%	5.56%
Type B 1st	0–11–33–0	353.299	6095.911	77.50%	8.33%
Type B 2nd	0–14–22–12–21–0	332.597	8463.513	77.50%	5.83%
Type B 3rd	0–34–29–9–0	249.413	6219.510	78.33%	6.67%
Type B 4th	0–28–7–26–4–0	491.642	15288.390	95.83%	8.33%
Type B 5th	0–1–16–24–0	235.555	6136.012	69.17%	6.67%
Type B 6th	0–5–10–0	266.815	6504.567	84.17%	8.33%
Type C 1st	0–20–2–6–27–23–0	343.127	12651.352	93.13%	7.50%

3.3. Optimization effect analysis

After the research above, it is proven that the distribution mode of simultaneous pickup and delivery is more economical and has the advantage of reducing the distribution cost compared with the distribution mode of simple delivery or simple pickup. Therefore, to make the results more rigorous and considerate, this paper will be solved for simple delivery and simple pickup respectively, in the case of the objective function, constraints, customer point of demand is the same, the same run ten times to select the value of the smallest cost, and then simultaneous pickup and delivery and the two for comparison and analysis. The results obtained from simultaneous pickup and delivery and simple delivery and simple pickup are summarized and represented by **Table 5** below.

Table 5. Comparison of solution results for different pickup modes

Distribution model	Length of route (km)	Loading rate (%)	Number of vehicles (vehicles)	Running costs (Yuan)	Call cost (Yuan)	Loading and unloading costs (Yuan)	Time penalty cost (Yuan)	Total cost per unit (Yuan)
Simple delivery	3317.605	81.72 %	11	81450.13	1350	407.6	150.263	83357.99
Simple pickup	2855.391	24.72 %	4	40086.88	200	35.6	545.053	40867.53
Single delivery + single pickup	6172.996	53.22 %	15	121537.01	1550	443.2	695.316	124225.52
Simultaneous pick-up and delivery	3317.605	64.42 %	11	4850.709	1350	443.2	150.263	86794.17

As can be seen from **Table 5**, the total number of delivery vehicles for simple delivery and simple pickup is 15, the average loading rate of vehicles is 53.22%, the cost of time deviation penalties is 695.316 yuan, the cost of operation is 121537.01 yuan, and the total cost of the two distribution modes is as high as 124225.52 yuan, which is higher compared to simultaneous pickup and delivery modes, where the vehicle call is 4 more vehicles, and the cost is higher by 37431.3539 yuan, 43.13% higher, the vehicle loading rate increased by 20.04%, optimizing the route 2855.3912 km.

4. Conclusion

This study validates the effectiveness of the simultaneous pickup and delivery distribution model through simulation. Using actual data from the KS Logistics Center, 34 customer points were analyzed, and genetic algorithms were employed to optimize distribution routes, comparing them with delivery-only and pickup-only modes. The results demonstrate that the simultaneous pickup and delivery model outperforms traditional models in terms of distribution cost, vehicle loading rate, and travel distance, reducing total costs by 30.13%. These findings underscore the practical value of the simultaneous pickup and delivery model for enhancing the operational efficiency of the KS Logistics Center.

Disclosure statement

The author declares no conflict of interest.

References

[1] Dantzig GB, Ramser JH, 1959, The Truck Dispatching Problem. Management Science, 6(1): 80–91.

[2] Ai TJ, Kachitvichyanukul VA, 2009, Particle Swarm Optimization for the Vehicle Routing Problem with Simultaneous Pickup and Delivery. Computers & Operations Research, 36(5): 1693–1702.

[3] Zhang T, Tian W, Zhang Y, et al., 2008, Improved Ant Colony System for VRPSPD with Maximum Distance Constraint. Systems Engineering-theory & Practice, 28(1): 132–140.

[4] Jun Y, Kim BI, 2012, New Best Solutions to VRPSPD Benchmark Problems by a Perturbation-based Algorithm. Expert Systems with Applications, 39(5): 5641–5648.

[5] Zhou R, Shen W, 2018, Adaptive Parallel Genetic Algorithm for the Path Problem of Loading and Unloading Integrated Vehicles. China Mechanical Engineering, 29(22): 2740–2749.

[6] Li L, Chen Y, 2021, ALNS Algorithm for Solving PCVRP Problem with Soft Time Window for Simultaneous Delivery. Journal of Shenyang University of Aeronautics and Astronautics, 38(03): 78–85.

[7] He M, Yang M, Han X, et al., 2024, Research on Time-dependent Simultaneous Pickup and Delivery Vehicle Path Problem with Time Window. Transportation Systems Engineering and Information, 24(04): 231–242 + 262.

[8] Yan J, Chang L, Wang L, et al., 2021, Algorithm for Solving the Simultaneous Pickup and Delivery Vehicle Path Problem with Time Windows. Industrial Engineering, 24(05): 72–76.

[9] Zhang QH, Wu S, 2020, Modelling of Simultaneous Pickup and Delivery Vehicle Path Problem with Time Windows and Modelling Factor Solution Algorithm. Computer Applications, 40(04): 1097–1103.

[10] Zhang W, 2019, Research on Path Optimisation of Vehicles Picking Up and Delivering Goods at the Same Time, dissertation, Changsha University of Science and Technology.

[11] Ma Z, Jiao H, Zhang Z, et al., 2024, Vehicle Path Optimisation Algorithm for Urban Logistics and Distribution. Journal of System Simulation, 1–10, viewed April 3, 2025.

[12] Ge N, Chen J, Wu L, 2023, Optimisation of Material Distribution Path of AGV Trolley in Flexible Manufacturing Workshop. Equipment Manufacturing Technology, (05): 70–73.

[13] Wang S, Xiang Y, Wang Y, 2023, Electric Vehicle Distribution Path Planning Method Considering Temporary Parking Charging and Discharging Management. Power Automation Equipment, 43(07): 94–101.

[14] Wang Y, Li H, Luo S, et al., 2023, Path Optimisation Problem of Electric Vehicles for Multi-center Co-distribution under Resource Sharing Model. Journal of System Management, 32(06): 1119–1141.

[15] Zhu F, Cao T, 2023, Optimisation Method of Workshop Material Distribution Path based on Time Window Constraint. Mechanical Design and Manufacturing, (01): 136–139 + 144.