

# Analysis of the Impact of Inductive Reasoning on the Mathematical Thinking Style of Deaf and Hard-of-Hearing Students

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**Abstract:** In this paper, we combine the teaching and learning situation of deaf and hard-of-hearing students in the Linear Algebra course of the Computer Science and Technology major at the Nanjing Normal University of Special Education. Based on the cognitive style of deaf and hard-of-hearing students, we apply example induction, exhaustive induction, and mathematical induction to the teaching of Linear Algebra by utilizing specific course content. The aim is to design comprehensive teaching that caters to the cognitive style characteristics of deaf and hard-of-hearing students, strengthen their mathematical thinking styles such as quantitative thinking, algorithmic thinking, symbolic thinking, visual thinking, logical thinking, and creative thinking, and enhance the effectiveness of classroom teaching and learning outcomes in Linear Algebra for deaf and hard-of-hearing students.

**Keywords:** Cognitive style; Mathematical thinking style; Deaf university students; Inductive reasoning; Linear Algebra

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## 1. Introduction

The British philosopher Francis Bacon proposed the view that “cognition determines thinking, thinking determines behavior, and behavior determines outcomes” in his work *Novum Organum*, which emphasizes the close connection between cognition, thinking, and behavior. Cognition refers to the ability of humans to perceive and understand external information. Perception is the process through which we acquire external stimuli through sensory organs, including vision, hearing, touch, taste, and smell. Understanding, on the other hand, is the process of processing and interpreting information based on perception. Through perception and understanding, we are able to acquire knowledge and understanding about the world. Therefore, for deaf and hard-of-hearing students, their cognitive processes are lacking in auditory perception, which prevents them from directly relying on hearing to obtain information.

This lack of auditory perception channels determines the cognitive style of deaf and hard-of-hearing

students. In 1967, Fiebert<sup>[1]</sup> explored the cognitive style of deaf individuals, revealing that their cognitive style differs from that of individuals with normal hearing. They rely more on sensory channels such as vision and touch to obtain information and perceive the world. Additionally, Fiebert's study found that gender also influences the cognitive style of deaf individuals, with female deaf individuals being more susceptible to environmental and socio-cultural influences. Furthermore, cognition determines thinking. In 1997, Sternberg<sup>[2]</sup> proposed thinking styles based on his theory of mental self-government. Different thinking styles can affect an individual's problem-solving skills, decision-making, and creative thinking, among other aspects. As a result, more and more scholars have studied the thinking styles of deaf or hearing-impaired individuals. In 2002, Sternberg<sup>[3]</sup> and his team released a revised thinking style inventory. In the same year, Zhang<sup>[4]</sup> explored the relationship between thinking styles and thinking modes based on Sternberg's thinking style inventory and further investigated the relationship between thinking styles and cognitive development. Zhang<sup>[5]</sup> argued that students with higher levels of cognitive development tend to use a wider range of thinking styles, while those with lower levels of cognitive development use fewer types of thinking styles. In 2008, Zhang<sup>[6]</sup> further discussed the relationship between thinking styles and emotions, and in 2009, revealed the relationship between anxiety and thinking styles. In 2013, Zhang<sup>[7]</sup> discussed the plasticity factors of thinking styles and the impact of thinking styles on student development, highlighting the important role of family education and the learning environment in changing thinking styles. In 2014, Cheng and Zhang<sup>[8]</sup> explored the differences in thinking styles between deaf, hard-of-hearing, and hearing students using the discipline of art and design as an example, and further elucidated the changing trends of thinking styles within one year in 2015<sup>[9]</sup>. In 2016, Cheng *et al.*<sup>[10]</sup> explored the relationship between thinking styles and self-efficacy among deaf or hearing-impaired university students; and in 2019<sup>[11]</sup>, Cheng investigated the contribution of learning concepts among deaf or hearing-impaired university students to thinking styles. In 2021, Cheng<sup>[12]</sup> examined the changes in thinking styles among Chinese deaf university students and suggested that measures should be taken by universities to cultivate the creative thinking and complex cognitive abilities of deaf and hard-of-hearing students in order to better promote their learning and development. In 2022, Hammad and Awed<sup>[13]</sup> explored the relationship between thinking styles and self-efficacy among deaf and hard-of-hearing students. In 2023, Kambara *et al.*<sup>[14]</sup> discussed the impact of positive interventions on thinking styles.

According to the literature review, it is found that due to the limited access to auditory information, deaf and hard-of-hearing students may have certain limitations in understanding non-verbal information in oral communication, such as intonation and timbre. Additionally, communication difficulties may arise when they interact with hearing individuals, leading to misunderstandings and information asymmetry. However, deaf and hard-of-hearing students can rely on visual information to acquire information, and visual enhancement can partially compensate for the lack of auditory information. As a result, their cognitive style differs slightly from that of hearing individuals due to sensory differences, as they tend to have a more visual cognitive style, which includes strong visual and spatial thinking. From the series of papers by Cheng, it can be seen that we can promote a change in the thinking style of deaf or hard-of-hearing college students through the external environment. By creating a more inclusive environment, we can encourage the thinking style of deaf or hard-of-hearing college students to be more similar to that of hearing college students.

Therefore, in this paper, we combine the cognitive style of deaf or hard-of-hearing college students and utilize the classic method of induction in mathematics to analyze the impact on their mathematical thinking styles, such as quantitative thinking, algorithmic thinking, symbolic thinking, visual thinking, logical

thinking, and creative thinking in the teaching of Linear Algebra.

## 2. Mathematical induction

The idea of mathematical induction can be traced back to the ancient Greek mathematician Eudoxus of Cnidus in the 4th century BC, who proposed a proof method called the “method of exhaustion” or “method of extrema,” which closely resembles the philosophy of modern mathematical induction. Over time, this approach gradually gained attention and application among mathematicians. In the 10th century, the Persian mathematician Al-Samawal, an Arab, described a method of induction to prove mathematical propositions. He introduced the concepts of the “first step” and “inductive step” (also known as “characteristic of the step and the generalization”) for the first time and integrated these into a general “concept of recognition.” In the centuries that followed, this idea was absorbed and applied by other Arab and European mathematicians, such as Euler and Stirling. In the 19th century, the British mathematician De Morgan introduced the term “successive recursion” in his work *Encyclopaedia Metropolitana* and later proposed the name “mathematical induction”<sup>[15]</sup>. As time passed, mathematical induction was increasingly applied across various fields of mathematics, becoming an important tool for solving mathematical problems<sup>[16-18]</sup>.

In this paper, we explore example induction, exhaustive induction, and mathematical induction in the teaching of Linear Algebra according to the learning situation of students with hearing impairments majoring in Computer Science and Technology at Nanjing Normal University of Special Education. The goal is to enhance the effectiveness of classroom instruction and learning outcomes in linear algebra for these students. Additionally, we aim to reinforce the mathematical thinking styles of students with hearing impairments, such as understanding quantity relationships, algorithmic thinking, symbolic thinking, visual thinking, logical reasoning, and creative thinking, through the learning of Linear Algebra courses.

## 3. The application of example induction in Linear Algebra teaching

For deaf and hard-of-hearing students, the concept of Matrix and Matrix Operations may be abstract and difficult to understand. However, they are significant for studying computer science and technology because of their widespread applications in the field. Therefore, a teaching method suitable for the cognitive style of deaf and hard-of-hearing students is adopted to help them understand the concepts of matrix and master their operations. In this section, we will use the concept of matrix and matrix addition as an example to explain the impact of example induction on deaf and hard-of-hearing students’ learning of abstract concepts and operations of matrix.

### 3.1. Teaching the concept of matrix

First, the concept of a numerical table is established through examples. For example, the expenses for breakfast, lunch, and dinner of two students, Student A and Student B, in September 2023 can be recorded in a table as shown in **Table 1**.

**Table 1.** Expenses for Monday’s meals (in yuan)

|           | Breakfast | Lunch | Dinner |
|-----------|-----------|-------|--------|
| Student A | 4         | 10    | 8      |
| Student B | 5         | 12    | 6      |

Then, the table is abstracted into symbolic mathematical language, that is, a 2x3 numerical table:

$$A = \begin{bmatrix} 4 & 10 & 8 \\ 5 & 12 & 6 \end{bmatrix}$$

Next, students are divided into groups of three or four to complete the expense tables for the day before the class and abstract them into symbolic mathematical language. They are encouraged to think about the number of elements in the table, and what the rows, columns, and elements represent.

Finally, the concept of a matrix is induced through examples:

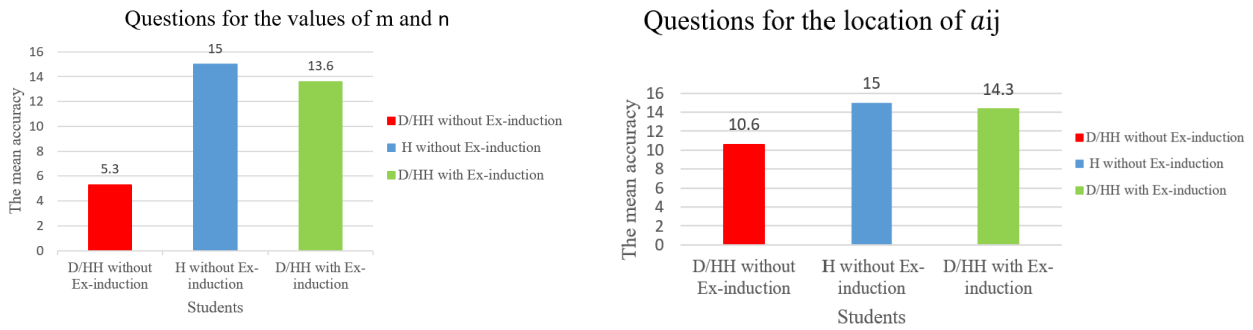
Matrix: A table consisting of  $m \times n$  numbers arranged in  $m$  rows and  $n$  columns

$$\begin{bmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \vdots & \vdots & \dots & \vdots \\ a_{m1} & a_{m2} & \dots & a_{mn} \end{bmatrix}$$

is called an  $m \times n$  matrix. Here,  $a_{ij}$  refers to the element at the intersection of the  $i$ -th row and the  $j$ -th column. The row index  $i$  comes before the column index  $j$ .

After introducing the concept of matrix, it is important to emphasize to deaf and hard-of-hearing students that  $m$  represents the number of rows and  $n$  represents the number of columns. The values of  $m$  and  $n$  can change depending on the specific matrix, and  $m$  and  $n$  can be assigned values through the previous group work on expense tables, this will reinforce their symbolic thinking. Furthermore, the position of  $a_{ij}$  can be well understood by deaf and hard-of-hearing students due to their strong visual cognition style. The relationship between  $i$  and  $j$  is understood as  $i$  comes “before”  $j$ , and  $a_{ij}$  represents the element at the intersection of the  $i$ -th row and the  $j$ -th column. At this point, the understanding of the concept of matrices and their symbolic representation is completed. To demonstrate the effectiveness of example induction, we perform an analysis of their homework. The homework is composed of 30 questions, where 15 questions are for the values of  $m$  and  $n$ ; 15 questions are for the location of  $a_{ij}$ . The students are composed of deaf, hard-of-hearing students, and hearing students, the deaf and hard-of-hearing students are divided into two groups: one group learns the concept of matrix by example induction (13 people), one group learns the concept of matrix same as the hearing students (13 people), without example induction. The number of hearing students is 13 as well. The mean accuracy of the three kinds of homework can be found in **Figure 1**.





**Figure 1.** The effectiveness of example induction

From **Figure 1**, we can see that the mean accuracy of homework with the deaf and hard-of-hearing students learned by example induction is similar to the hearing students, and outperforms the deaf and hard-of-hearing students learned without example induction. In contrast, deaf and hard-of-hearing students who do not undergo example induction learning show lower assignment accuracy rates for the two kinds of homework. It appears that example induction has a positive impact on the learning of deaf and hearing-impaired students, helping them better understand and apply the knowledge they have learned, and enhancing their quantitative thinking and symbolic thinking for deaf and hard-of-hearing students.

This form of example induction can also be used in teaching the coefficient matrix, augmented matrix, constants vectors, matrix representation of equations, vector representation of equations, elementary matrix, and matrix representation of quadratic forms in Linear Algebra. Example induction helps hearing-impaired students understand these concepts better.

### 3.2. Matrix addition

Matrix addition is a fundamental matrix operation, and it is widely used in various fields such as statistics, machine learning, etc. For hearing students, this operation is relatively simple. Under the condition of having matrix of the same size, adding two matrices is equivalent to adding the corresponding elements at each position. However, for deaf and hard-of-hearing students, we need to start by using inductive examples to help them understand the concept of matrix of the same size. We approach this from the perspective of square matrix and rectangular matrix, allowing these students to understand the conditions that square matrix and rectangular matrix must satisfy in order to have the same size. By only providing examples of rectangular matrices, students may have difficulty understanding the concept of square matrices having the same size.

For example, consider:

$$A_1 = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}, \quad A_2 = \begin{bmatrix} 2 & 4 & 6 \\ 8 & 10 & 12 \end{bmatrix},$$

These are two  $2 \times 3$  matrices of the same size. Here, we emphasize that we only focus on the number of rows and columns, and the elements inside can be changed. Through exercises, we reinforce students' understanding of the concept of the same-sized rectangular matrix and further abstract the concept. For instance,  $A_3$  is a  $5 \times 8$  matrix, and  $A_4$  is also a  $5 \times 8$  matrix. The task is to determine if these two matrices are of the same size. Observing the students' performance in exercises helps us assess their grasp of the concept of same-sized matrices. Once students can judge correctly, we further introduce symbolic abstractions, such

as  $A_5$  being an  $m \times n$  matrix,  $A_6$  being an  $n \times m$  matrix, and  $A_7$  being an  $m \times n$  matrix. Students must determine which two matrices among these three are of the same size. Through repeated examples and inductive reasoning, students get a better understanding of the concept of the same-sized rectangular matrix. We also emphasize that the number of rows and columns can change for individual matrices, but as long as two matrices have the same number of rows and columns, they are the same size matrices. After students comprehend the concept of the same-sized rectangular matrix, we introduce the concept of the same-sized square matrix as a supplement to their understanding of same-sized matrices. Otherwise, deaf and hard-of-hearing students might struggle to determine if square matrices are the same size.

Once students understand the concept of same-sized matrices, they can easily grasp the concept of adding the corresponding elements. In actual teaching practice, we use inductive examples to guide deaf and hard-of-hearing students toward a better understanding of matrix addition with different numbers of rows and columns. For example, adding  $A_1$  and  $A_2$ ,  $A_3$  and  $A_4$ ,  $A_5$  and  $A_7$ , etc. Additionally, we use exercises or assignments involving the addition of same-sized square matrices to reinforce their understanding. To demonstrate the effectiveness of example induction, we perform an analysis of their homework. The homework is composed of 20 questions, where 10 questions are for rectangular matrix addition and 10 questions are for square matrix addition. The rectangular matrix addition comprises three pairs of  $2 \times 3$  matrices, three pairs of  $3 \times 2$  matrices, two pairs of  $4 \times 5$  matrices, and two pairs of  $8 \times 5$  matrices. The square matrix addition comprises four pairs of  $3 \times 3$  matrices, four pairs of  $5 \times 5$  matrices, and two pairs of  $8 \times 8$  matrices. The students are composed of deaf, hard-of-hearing students, and hearing students, the deaf and hard-of-hearing students are divided into two groups as well. One group learns the matrix addition by example induction (13 people). One group learns the matrix addition similar to the hearing students (13 people), without example induction. Specifically, the process of adding two matrices is detailed under the same size condition, adding two matrices is equivalent to adding the corresponding elements at each position, with a pair of  $2 \times 3$  matrices and a pair of  $3 \times 3$  matrices as examples to demonstrate. The number of hearing students is 13 as well. The result of the homework can be found in **Table 2**.

**Table 2.** The accuracy of the homework

| Types of matrices           | Sizes of matrices          | D/HH without Ex-induction | H without Ex-induction | D/HH with Ex-induction |
|-----------------------------|----------------------------|---------------------------|------------------------|------------------------|
| Rectangular matrix addition | $2 \times 3$ matrices pair | 100%                      | 100%                   | 100%                   |
|                             | $3 \times 2$ matrices pair | 9.21%                     | 100%                   | 98.01%                 |
|                             | $4 \times 5$ matrices pair | 8.01%                     | 100%                   | 98.65%                 |
|                             | $8 \times 5$ matrices pair | 6.01%                     | 100%                   | 98.97%                 |
| Square matrix addition      | $3 \times 3$ matrices pair | 100%                      | 100%                   | 100%                   |
|                             | $5 \times 5$ matrices pair | 9.71%                     | 100%                   | 100%                   |
|                             | $8 \times 8$ matrices pair | 8.67%                     | 100%                   | 100%                   |

From **Table 2**, we can see that deaf and hard-of-hearing students without example induction get a good performance for  $2 \times 3$  matrices pair and  $3 \times 3$  matrices pair, which is the same size as the examples, but showed a significant decrease in performance for others matrices that are not covered in the examples. Conversely, deaf and hard-of-hearing students who underwent example induction exhibited better performance across a wider range of matrix sizes. They showed a strong understanding of the abstract operation of matrix

addition, as they were able to apply their knowledge to matrices that varied in size, even if they were not covered in the examples. Overall, the results highlight the significant impact of example induction in promoting a comprehensive understanding of matrix addition among hard-of-hearing students, ultimately leading to improved performance across a range of matrix sizes and a better grasp of the abstract nature of the operation. Example induction can help the deaf and hard-of-hearing students enhance their mathematical understanding of matrix addition through inductive reasoning and practice, and enhancing their symbolic thinking, algorithmic thinking, and creative thinking.

#### 4. Application of exhaustive induction method in Linear Algebra education

The inverse of a matrix is an essential part of linear algebra, making it crucial to master methods for computing matrix inverses. For students with hearing impairments, however, these methods can be too abstract. For example: Assume  $A$  is an invertible matrix, then  $A^{-1}(A, E) = (E, A^{-1})$ . This relationship can be demonstrated by using elementary row operations to find the inverse of a matrix, which can be represented as:

$$(A, E) \xrightarrow{\text{elementary row operations}} (E, A^{-1})$$

The above symbolic language represents the three steps of finding the inverse of a matrix using elementary transformations:

Step 1: Augmenting the matrix  $A$  with an identity matrix of the same order on its right side.

Step 2: Applying elementary row operations to the augmented matrix  $(A, E)$  until  $A$  is transformed into an identity matrix.

Step 3: The matrix that was originally  $E$  now becomes the inverse matrix of  $A$ .

Having clarified these three steps, we begin with a concrete example of a  $2 \times 2$  invertible matrix  $B_1$ . Through elementary row transformations, we illustrate the process of finding the inverse  $B_1^{-1}$ , emphasizing that at this point  $A$  is the  $2 \times 2$  matrix  $B_1$ , and  $E$  is the  $2 \times 2$  identity matrix corresponding to the same order as  $B_1$ . Following this, students practice finding the inverses of  $2 \times 2$  matrix to understand the concepts involved.

Once students are proficient with a  $2 \times 2$  matrix, we move on to the concrete representation of a  $3 \times 3$  invertible matrix, encouraging them to think about how to find the inverse of a  $3 \times 3$  matrix,  $B_2$ , and guiding them through the process using elementary row operations. We emphasize again that the augmented identity matrix must be of the same order as  $B_2$ . Students practice finding the inverse of the  $3 \times 3$  matrix through hands-on exercises. As students progress, they are asked to use sign language to express the process of finding inverses for larger matrices, such as  $4 \times 4$ ,  $5 \times 5$ ,  $6 \times 6$ , and up to  $n$ -order invertible matrices.

By using exhaustive induction methods, we enable students with hearing impairments to understand that

$$(A, E) \xrightarrow{\text{elementary row operations}} (E, A^{-1})$$

is a method to find the inverse of a matrix. In this context,  $A$  can be replaced with any concretely represented invertible matrix, and  $E$  is an identity matrix of the same order. Importantly,  $A$  and  $E$  are not limited to  $2 \times 2$  or  $3 \times 3$  matrix;  $A$  represents all invertible matrix. Practically, as long as  $A$  is invertible, its

inverse can be found following the method above, which serves as a tool for computing inverses. Similarly, we can illustrate the impact of exhaustive induction on the mathematical thinking of deaf and hard-of-hearing students through homework assignments. The homework contains 10 matrices, three 2x2 matrices, three 3x3 matrices, three 4x4 matrices, and one 5x5 matrices. The examples without exhaustive induction is 3x3, the result of the inverse can be found in **Table 3**.

**Table 3.** The accuracy of inverse matrices

| Sizes of matrices | D/HH without Ex-induction | H without Ex-induction | D/HH with Ex-induction |
|-------------------|---------------------------|------------------------|------------------------|
| 2x2 matrices pair | 8.62%                     | 100%                   | 99.01%                 |
| 3x3 matrices pair | 100%                      | 100%                   | 100%                   |
| 4x4 matrices pair | 7.32%                     | 100%                   | 99.53%                 |
| 5x5 matrices pair | 6.56%                     | 100%                   | 99.67%                 |

From **Table 3**, we can see that hearing students achieve 100% accuracy when inverting matrices of all sizes. With exhaustive induction, the deaf and hard of hearing students show good performance when inverting matrices of all sizes, which shows that the induction method is a highly effective tool for improving their knowledge transfer skills on this task. Without exhaustive induction, the deaf and hard of hearing students show good performance for matrices similar to the example, but for other matrices, their accuracy is significantly low (8.62% for 2x2, 7.32% for 4x4, and 6.56% for 5x5 matrices), indicating that this group may face knowledge transfer challenges with the matrix inversion task.

Therefore, the exhaustive induction method inspires mathematical symbolic thinking in hearing-impaired students; A and E are merely representative symbols of the matrix that can be replaced with the varying concrete matrices. On this foundation of symbolic thinking, we further stimulate the mathematical reasoning of hearing-impaired students, enabling them to concretize the process accurately. By accurately finding the inverse of a matrix, this practice also enhances the mathematical computational thinking and logical reasoning of students with hearing impairments.

## 5. Application of mathematical induction in Linear Algebra teaching

Mathematical induction is used in proofs involving the Vandermonde determinant, the associative law of matrix multiplication, the powers of square matrix, and proofs of vector linear independence. The proof of vector linear independence is more abstract. Therefore, in this section, the application of mathematical induction in abstract reasoning within linear algebra is expounded through the following corollary.

Corollary: A set of vectors  $\alpha_1, \alpha_2, \dots, \alpha_s$  is linearly independent if and only if from the equation  $k_1\alpha_1 + k_2\alpha_2 + \dots + k_s\alpha_s = 0$ , it can be inferred that all coefficients  $k_1, k_2, \dots, k_s$  are zero.

Proof: When  $s = 1$ , if  $\alpha_1$  is linearly independent, then  $\alpha_1 \neq 0$ , and from  $k_1\alpha_1 = 0$ , it follows that  $k_1 = 0$ . Assume that the proposition holds for  $s = p$ , meaning  $\alpha_1, \alpha_2, \dots, \alpha_p$  are linearly independent, which implies all  $k_1, k_2, \dots, k_p$  are zero.

Then, when  $s = p + 1$ , if  $k_1\alpha_1 + k_2\alpha_2 + \dots + k_p\alpha_p + k_{p+1}\alpha_{p+1} = 0$  holds and not all of  $k_1, k_2, \dots, k_p, k_{p+1}$ , are zero, it implies that one vector can be linearly expressed by the others, contradicting the linear independence of  $\alpha_1, \alpha_2, \dots, \alpha_p, \alpha_{p+1}$ . Therefore, all  $k_1, k_2, \dots, k_p, k_{p+1}$ , must be zero, proving the corollary.

For students with hearing impairments, this type of pure symbolic language proof is the most

challenging. Therefore, during instruction, it is helpful to utilize multimodal language explanations, such as pure sign language. After explaining, asking questions helps gauge the students' understanding. Then, purely textual language is used for explanation, followed again by questions to understand the students' grasp of the materials. Finally, using mixed-modality language further reinforces students' understanding of the abstract proof. Despite the difficulty, conducting these proofs using abstract symbolic language stimulates the development of logical thinking skills in students with hearing impairments, which is crucial for enhancing their mathematical thinking styles.

## 6. Conclusion

Based on the teaching and learning conditions of deaf and hard-of-hearing students majoring in Computer Science and Technology at Nanjing Normal University of Special Education, this paper applied example induction, exhaustive induction, and mathematical induction to the teaching of Linear Algebra. Through concepts of matrix, matrix addition and inversion, and proof propositions, teaching designs suitable for deaf and hard-of-hearing students are devised. From the result of homework, we can see that the teaching process integrates the cognitive style characteristics of students with hearing impairments, emphasizing and clarifying concepts they find difficult to understand. In doing so, it reinforces mathematical thinking styles such as understanding of quantities, algorithmic thinking, symbolic thinking, visual thinking, logical reasoning, and creative thinking, ultimately enhancing the classroom effectiveness and learning outcomes in Linear Algebra for students with hearing impairments.

## Disclosure statement

The authors declare no conflict of interest.

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