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The Past and Future of Aesthetics in Mathematics

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Abstract: This paper explores the connotations of mathematical aesthetics and its connections with art, facilitated by collaboration with Ester, an individual with an artistic professional background. It begins by tracing the historical evolution of aesthetics from the classical pursuit of authenticity to the modern shift toward self-expression in art. The discussion then highlights the similarities in the pursuit of truth between mathematics and art, despite their methodological differences. Through an analysis of aesthetic elements in mathematics, such as lines and function graphs, the article illustrates that the beauty of mathematics is not only manifested in cognitive processes but can also be intuitively expressed through visual arts. The paper further examines the influence of mathematics on the development of art, particularly how Leonardo da Vinci applied mathematical principles to his artworks. Additionally, the article addresses art students' perceptions of mathematics, proposes the customization of math courses for art students, and discusses future trends in the integration of mathematics and art, emphasizing the significance of art therapy and the altruistic direction of art. Lastly, the authors use a poster to visually convey the idea that the beauty of mathematics can be experienced through the senses.

Keywords: Mathematical aesthetics; Integration of mathematics and art; Applications of mathematics in art

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1. Introduction

For this assignment, the author collaborated with Ester, a high school classmate who graduated from the Art Institute of Chicago. She majored in AIADO (Department of Architecture and Design) and currently specializes in cosmetics design. Our recent focus has been exploring the aesthetics in mathematics, particularly the inherent beauty of the subject from a mathematical viewpoint. This prompted the author's curiosity about how art students perceive mathematics, which led to the invitation of Ester to join this learning journey.

During our discussions, we delved into the origins of aesthetics, the impact of mathematics on its development, and prospective directions for art among other topics. This collaboration has been profoundly meaningful. Previously, the author's observations of mathematical aesthetics were largely influenced by their background as a mathematics major, which inevitably led to a somewhat narrow interpretation of the subject. However, engaging with Ester provided a more holistic perspective on both mathematics and art. Mathematicians often celebrate the beauty and elegance of specific theorems, proofs, and definitions, valuing not just their truth but also their aesthetic qualities [1]. It seems that a true appreciation of art might be essential to fully grasp the concept of aesthetics in mathematics.

2. The history of aesthetics

Before delving into the aesthetics of mathematics, it is essential to understand the relevant historical and artistic background of aesthetics. It is equally important to recognize the significance of mathematics to art by exploring the era in which the concept of aesthetics emerged. Thus, the author consulted Ester about the history of aesthetics. She explained that the concept of aesthetics is quite broad, and the focus of classical aesthetics differs significantly from that of modernist art. In the classical era, when art primarily served the nobility or the Vatican, the primary function of oil painting was not unlike that of today's camera, focusing on photorealistic techniques and exploring how to accurately portray people. Artists like Leonardo Da Vinci studied anatomy to enhance their artistic precision. Therefore, aesthetics during that period was essentially a quest for extreme authenticity. With the advent of the camera, the primary purpose of painting shifted. People gradually moved away from realism towards self-expression, and the emphasis of contemporary art shifted towards exposition. The term "aesthetics," used in something akin to its modern sense, is commonly attributed to Alexander Baumgarten in 1735. However, the groundwork for aesthetics as a distinct branch of philosophy was laid earlier in the 18th century by thinkers such as the third Earl of Shaftesbury (Anthony Ashley Cooper), Joseph Addison, Jean-Baptiste Du Bos, and Francis Hutcheson, who were among the first to systematically explore aesthetics [2].

Both mathematics and art originated from an absolute pursuit of "truth," albeit in slightly different manners. While both fields are concerned with uncovering the truth, their methods diverge. Mathematics relies on analysis and proof, whereas art draws on the senses and emotions to convey its truths [3]. Despite their shared goal, there are notable differences between the two. The author views mathematics as a pursuit of truths that exist in nature, whereas art seeks to capture more abstract concepts, such as emotional expression through physical objects. When we engage with a painting or attend a concert, we can often discern the mood and emotional intent of the artist within the artwork or the music.

3. Aesthetic elements in mathematics

Ester shared that she perceives the aesthetics of mathematics in many casual ways, particularly through the lens of balance, much like in oil painting. For instance, the initial step involves mixing the flaxseed oil with pigment in a precise ratio. Though this action might seem minor, there is a strict formula where the quantity of each ingredient controls the wetness, dryness, and lubrication of the paint. When beginning to paint, proportionality is used to maintain the balance of the composition. In a landscape painting that includes figures, the distant scenery is completed first. Then, as the focus shifts to a closer view, the placement of the figures is carefully considered. All these decisions require a kind of instinctive calculation. It seems there is a complex set of computations running through her head, albeit unrecorded.

Throughout the author's studies, one question persistently arises: why is aesthetics in mathematics often overlooked? When people think of mathematics, the immediate association is with its strong logical structure. Rarely is mathematics first appreciated for its beauty. There are several reasons for this. Appreciating beauty requires tranquility and contemplation. Today's fast-paced lifestyle, coupled with the pressures of work and education, leaves little room for individuals to engage with the aesthetic aspects of mathematics. The most significant factor is that, unlike art or music, which can be directly experienced through paintings or compositions, the beauty of mathematics is rooted in a process of thought. This process is abstract and not readily accessible through tangible objects. For non-specialists, recognizing the beauty of mathematics can be challenging. Beauty is indeed in the eye of the beholder, yet it is also difficult to discern when the artwork is shrouded in darkness, obscured by an impenetrable cloud of symbols and jargon. Attempting to appreciate mathematics without grasping its inner mechanics is akin to reading a description of Beethoven's Fifth Symphony rather than experiencing its auditory power [4].

4. The use of lines

Although the aesthetics of mathematics are largely reflected in the process of thinking, there are tangible elements that can directly convey the beauty of mathematics, such as lines. The most intuitive display of lines' beauty is through function graphs, and among these, the most famous is perhaps the Cartesian heart function (**Figure 1**).

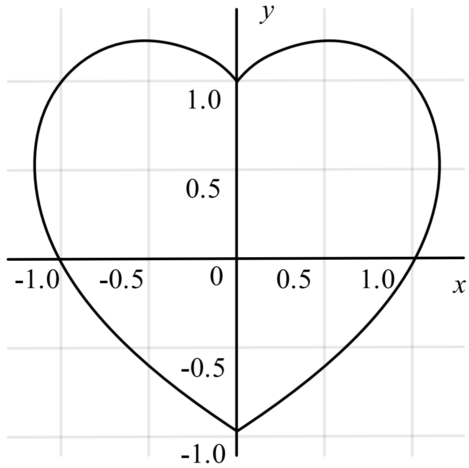

Figure 1. The most romantic function graph—Cartesian heart function

This heart is depicted by the equation $(x^2 + y^2 - 1)^3 = x^2y^3$. As we observe, the heart shape can be enlarged by increasing the constant, and similarly, it can be made smaller by decreasing the constant. However, this equation appears to be effective only when the constant is positive [5].

Ester concurs, noting that the lines and spaces within mathematics have an aesthetic appeal to her. She recounted her experience viewing the works of Julie Mehretu at a gallery in LA. Mehretu's art features several expansive paintings, each spanning tens of meters in width and height. Ester felt that these paintings conveyed a strong sense of three-dimensional modeling and architectural drawings, yet they also embodied an abstract randomness, reminiscent of the urban atmosphere during the bustle of the industrial age.

The author also found some paintings by Julie Mehretu for this purpose (**Figure 2**).

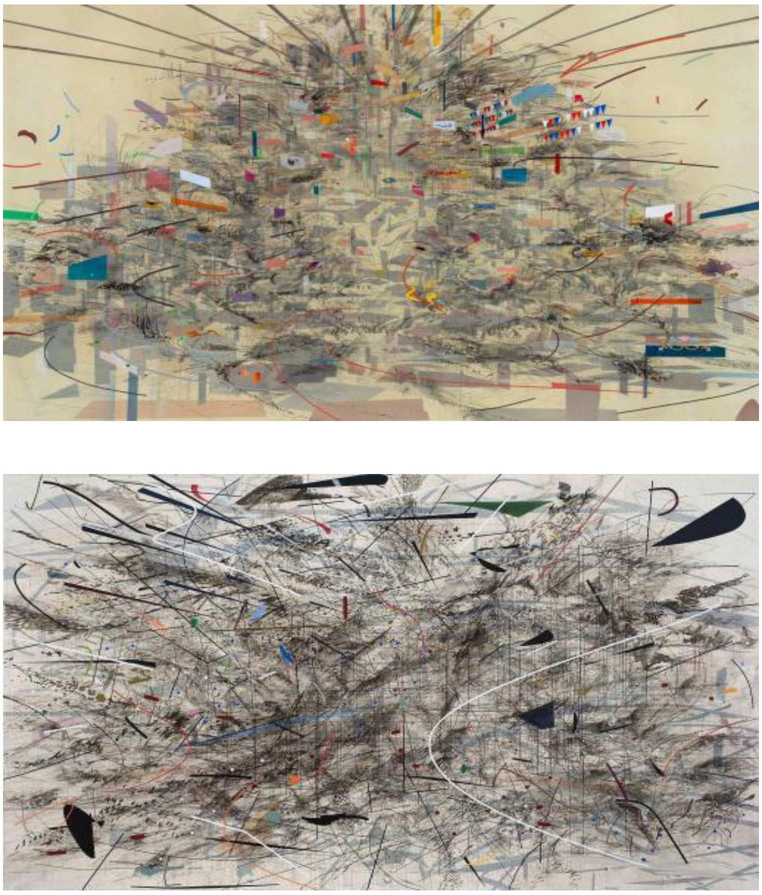

Figure 2. Julie Mehretu's artistic creation of lines and shapes [6-8]

Although not well-versed in artistic techniques, the author would like to discuss their impressions of these paintings from a mathematical perspective. Julie Mehretu's work is particularly striking. It evokes a type of problem commonly encountered in mathematics: functional graph analysis. This concept is a staple in college mathematics courses, where students are tasked with extracting necessary data from complex functional formulas, such as finding maximums, minimums, intersections, and variances. At first glance, Julie Mehretu's compositions might appear intricate and overwhelming. However, upon closer examination, they also convey a profound sense of spatial organization. Much like analyzing functional graphs, her art reveals an underlying order amidst the apparent complexity.

5. The contribution of mathematics to the development of art

Ester believes that the development of mathematics in art serves to digitize any seemingly complex structure. Leonardo da Vinci's study of animal anatomy is a prime example, which allowed him to understand the mechanics of animal bones and muscles. Similarly, his use of perspective in *The Last Supper* demonstrates his application of mathematical knowledge. Having mastered the concepts of proximity, where objects appear larger, and distance, where objects appear smaller, he achieved the "authenticity of painting" that was highly sought after in his era (**Figure 3**).

Figure 3. Da Vinci - II Cenacolo [9]

Although da Vinci is best known for his artistic works, he often considered himself more of a scientist than an artist. He utilized the mathematical principles of linear perspective, such as parallel lines, a horizon line, and a vanishing point, to create the illusion of depth on a flat surface [2]. The contribution of mathematics to art is not only reflected in the use of lines and the creation of spatial depth but also in the mathematical philosophy of seeking truth from facts. Historically, scientists have discovered mathematical patterns in nature [3]. Over the centuries, mathematical knowledge was not only applied as a natural science but also influenced artistic creation, giving rise to forms of artistic expression such as sketching. This intertwining of disciplines illustrates why the history of mathematics is also the history of art.

6. Art students' impressions of mathematics

Given the significant role that mathematics plays in the arts, the author was curious about how art students experience math classes differently from us. The author asked Ester whether she had taken math classes in college. She said that art students are not required to take math classes. She has not taken a math class since high school and is now relatively unfamiliar with math. In her daily life, she seldom uses specific mathematical knowledge. However, she noted that as a design student, she occasionally uses simple mathematical formulas when working with software or engaging in parametric modeling.

While this was somewhat surprising, it was not entirely incomprehensible. It sparked new insights about why most people do not perceive aesthetics in mathematics. As Ester pointed out, it is challenging for individuals to apply specific, high-level mathematical concepts in their everyday lives. When shopping, one cannot simply tell the cashier to wait while calculating the derivative of an item's price. However, the aesthetics of mathematics is most clearly demonstrated through the rigorous logic involved in proving theorems, which is why most people do not engage with mathematical knowledge post-graduation and are unable to appreciate its beauty.

Last week, the author came across an intriguing idea from John Sims. In an interview, he discussed his work on designing a math curriculum specifically for art students. This innovative approach underscores the substantial contributions that mathematics has made to the development of the arts. Consequently, the author believes there is a need to design math courses tailored for art students. Unlike their counterparts in mathematics, art students aim to enhance their artistic understanding through the study of mathematics. Advanced mathematical knowledge, such as theoretical proofs in abstract algebra, is not essential for artistic creation. In the author's view, art students could benefit from focusing on learning about functions in their math courses and experiencing the sense of balance that emerges from function graphs. Additionally, we could consider designing art courses suited for mathematics students. These courses should emphasize the connection between mathematics and aesthetics, rather than assessing the drawing skills and artistic creativity of mathematics students.

7. The future of mathematics and arts

Ester believes that the future of art will increasingly align with art therapy, suggesting that contemporary art, which has largely focused on issues of self-expression, will eventually transition toward altruism. Art therapy utilizes the art-making process to explore and address people's thoughts and feelings, linking it closely with psychology.

Similarly, the author holds parallel views about mathematics. The author once discussed with their teacher that what attracts the author most about mathematics is a specific branch: game theory. Their teacher also agreed that the process of discovering strategic interactions is both intellectually stimulating and aesthetically appealing. Much of this allure stems from the elements of the unknown and uncertainty, akin to how an artist may not envision the final appearance of a painting until it is complete. People are naturally drawn to the unknown, a phenomenon underscored by the recent popularity of the "blind box" challenge, where the contents remain a mystery until they are revealed. While mathematics is a rigorously exact science, where solutions are typically right or wrong, it also grapples with concepts that defy straightforward mathematical computation, such as luck. The recent surge in artificial intelligence has sparked concerns about machines replacing humans, a prospect that the author views with some reservation. Despite the unmatched computational and learning capabilities of artificial intelligence, it remains rudimentary in its ability to handle elements beyond the reach of current scientific understanding, such as luck. For instance, consider an extreme scenario where artificial intelligence competes against a human in Texas Hold'Em. While artificial intelligence might make the most strategically optimal decisions based on available data, it stands little chance against a human who is extraordinarily lucky, consistently drawing royal flushes. In this case, the formidable computing power of artificial intelligence falls short. Thus, the author believes the ultimate challenge for mathematics is to extend its reach to quantify and understand phenomena that currently elude scientific explanation. This exploration not only broadens the scope of mathematical inquiry but also deepens our understanding of the complex interplay between determinism and chance.

8. Conclusion

The author gained invaluable insights from this experience and is profoundly grateful to Ester for that. Our discussions enhanced the author's understanding of the interplay between mathematics and aesthetics. Why does mathematics relate to disciplines such as art and aesthetics? The author believes the primary reason is that the theories and concepts elucidated by mathematics have always existed in nature. The essence of mathematics is not merely abstract symbols and formulas, but a profound revelation and description of the fundamental laws of nature. It is like the wild animals on the African savannah; even if they do nothing, just their existence in the world makes them appear beautiful, which makes the author propose the concept that mathematics is a truth inherent in nature, existing in the universe irrespective of human discovery. Just as we need our eyes to appreciate the beauty of nature, our ears to enjoy music, and our taste buds to relish food. It is human nature to perceive beauty through these senses, and similarly, we can also experience the beauty of mathematics through them. One might wonder, how can mathematics be heard or tasted? In practice, when music students encounter a music sheet, instructors often simplify it into notations composed of Arabic numerals. In cooking, the precise measurement of ingredients to the gram exemplifies the aesthetic of mathematics. To truly appreciate the aesthetics in mathematics, we should look beyond textbook knowledge and observe its presence in everyday life. By doing so, we can appreciate the diverse beauty that mathematics offers.

Disclosure statement

The author declares no conflict of interest.

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