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A Study on High School Mathematics Teaching Design Based on Teaching for Robust Understanding: Taking the Cosine Theorem as an Example

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Abstract: Teachers' teaching behavior plays a crucial role in students' development, and there are problems in the current teaching behavior of mathematics teachers such as ignoring students' cognitive needs, lack of equal opportunities for students' classroom performance as well as lack of formative evaluation of students. In order to solve the phenomenon, this paper analyzes and explains how to promote teaching based on the Teaching for Robust Understanding (TRU) evaluation framework with the goal of focusing on the development of all students, taking the teaching design of The Cosine Theorem as an example, and provides ideas and methods for first-line high school mathematics teachers.

Keywords: Mathematics education; Teaching for Robust Understanding framework; Instructional design; Cosine Theorem

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1. Presentation of the problem

The new era has put forward higher requirements for the cultivation of talent, and the General Secretary has emphasized the need to insist on high-quality education as the lifeline of all kinds of education. The high-quality development of education cannot be separated from teachers' teaching. "What is effective teaching?" is a topic that scholars have long been concerned about. The classroom teaching behavior of mathematics teachers has a profound impact on the effectiveness of teaching.

In the current educational and teaching activities, many teachers do not delve deeply enough into mathematical research; they lack a thorough understanding of their students' situations; and their teaching methods are not sufficiently innovative [1]. These phenomena lead to mathematics teachers facing many difficulties in teaching. Therefore, in teaching, teachers should pay attention to the content of mathematics, cognitive needs, and students' participation and performance, and make formative assessments of students' performance in class [2]. This paper will take the teaching design of the Cosine Theorem as an example to further analyze how to reflect the students' subjective position in the classroom, pay attention to the overall development of students, and embody the nurturing value of the subject of mathematics.

2. TRU evaluation framework

The TRU framework is an evaluation framework developed by the University of California, Berkeley [3], which takes a comprehensive approach from the perspectives of both teacher teaching and student learning, and proposes five dimensions for evaluating teachers' teaching behaviors, described as follows:

(1) Mathematical content: The teaching of mathematical content focuses on the connection between knowledge;

(2) Cognitive demand: Teacher promotes students' cognitive development;

(3) Equitable access to mathematics: Teacher ensures that all students can participate in the classroom;

(4) Student motivation and expression: Students have the opportunity to express themselves;

(5) Formative assessment: Teacher provides timely feedback on classroom exercises [4].

These five dimensions address the issues mentioned above. In the following, the Cosine Theorem in Chapter 6, Section 4: Plane Vectors and Their Applications in the compulsory second book of the general high school mathematics textbook [5], the Humanistic Education Edition, is used as an example to construct an instructional design based on the Teaching for Robust Understanding (TRU) framework, and it is hoped that through this framework, the teaching and learning of teachers can be facilitated and inspired.

3. Instructional design of the Cosine Theorem

3.1. Design concept

This section describes the application of plane vectors in Chapter 6. The teaching design is based on the TRU framework, which guides students to apply their existing knowledge to solve real-life problems according to the mathematical content and the cognitive foundation of students. Through the introduction of real-life situations, we create students' "indignation" and "sentimental" states, and give full play to the students' initiative to solve the problem through independent investigation, group cooperation, and communication.

3.2. Teaching objectives

(1) To understand and master the Cosine Theorem, and know the relationship between the Cosine Theorem and the collinear theorem.

(2) To derive the Cosine Theorem with the help of vector operations. To prove the Theorem of Cosines geometrically to enhance the sense of application and develop the ability of logical reasoning [6].

(3) By observing and analyzing the characteristics of the Cosine Theorem, to understand the symmetry in the formula, penetrate the aesthetic education in teaching, and stimulate the interest in learning mathematics.

3.3. Teaching points

(1) Teaching key points: to understand and master the Cosine Theorem and its corollary, and to be able to perform simple calculations of solving triangles.

(2) Teaching difficulties: Deriving the Cosine Theorem by the vector method and the simple application of the Cosine Theorem [7].

3.4. Teaching process

3.4.1. Situation introduction

Teacher activities:

(1) Creating context and posing questions: Begin by showing an aerial documentary of the Red Flag Canal. Through the video and its narration, help students appreciate the spirit of self-reliance and hard work demonstrated by the people of Linxian County.

(2) Introducing the problem: After viewing the documentary, guide the discussion to the following problem: How did the people of Linxian County measure the length of the base of the mountain when constructing the Youth Cave?

Engineers and technicians first selected a suitable location, C, on the ground. They measured the distances from C to the foot of the mountain at points A and B, which were 700 meters and 600 meters respectively. They then used a latitude and longitude meter to measure the angle at point C between A and B, which was 60°. How can the length between points A and B at the foot of the mountain be calculated? Abstract this real-world scenario into a mathematical problem: In ABC, it is known that sides CA = 700 m, CB = 600 m, and C = 60°, that is, two sides and their angles. Find the length of the third side AB.

Student activity: Students suggest that the problem can be solved by using the method of heights and making a vertical line from AC to AC at point D.

Teacher posing question: Is it true that every time we encounter a problem involving two sides and an angle, we have to solve it using the height method? Is this not a bit troublesome? Can we find a relationship between the sides and angles of a triangle that can be expressed in an equation to solve this problem more quickly?

[Design intent] Creating a situation based on the nature of the problem. Combined with the TRU evaluation model, we start from the dimension of "mathematical content" and introduce mathematical problems through contextualization. After the students have proposed the solution to the problem, we help students to think out of the box, and then introduce the topic of this lesson.

3.4.2. Analyzing the problem and investigating cooperatively

Activity 1: Deriving the Cosine Theorem by the Vector Method

Teacher activity: Transform the above problem into a class problem: in ABC, it is known that C, the side opposite to A is a, and the side opposite to B is b, denoting the third side c. How to solve this problem?

Student activity: Students realize through independent thinking that if this is a right triangle, it can be expressed directly by the Pythagorean Theorem [8].

Teacher's follow-up question: If this triangle is not a special triangle, how can we represent it?

[Design intent] In accordance with the TRU evaluation framework, according to the cognitive needs of students, the concrete mathematical problems are transformed into abstract mathematical problems to guide students to think and understand a special case through independent thinking, preparing them for the subsequent exploration of the relationship between the collinear theorem and the cosine theorem.

Teacher: Ask students to think about what they have learned in this chapter.

Student: Vectors.

Teacher activity: Guide students to think about which of the operations of vectors studied include two sides and their angles.

Student activity: Based on the teacher's guidance, discover that the product of quantities of vectors can be applied to prove the problem: represent the three sides of a triangle as vectors, the $\overrightarrow{CB} = \vec{a}$, $\overrightarrow{CA} = \vec{b}$, $\overrightarrow{AB} = \vec{c}$, Then we will use $\vec{a}$ and $\vec{b}$ to represent $\vec{c}$. That is to say $\vec{c} = \vec{a} - \vec{b}$. At this point, it follows from the properties of the product of plane vector quantities that $\vec{c} \cdot \vec{c} = |\vec{c}|^2$. Through the above analysis, students are guided to do their own calculations and the final calculations to the results: $c^2 = a^2 + b^2 - 2ab\cos C$.

[Design intent] The "mathematical content" dimension of the TRU assessment framework states that the structure of classroom activities provides students with opportunities to experience thinking as mathematicians, and that coherent and focused discussions are conducive to the development of mathematical habits of mind. Therefore, guiding students to use their knowledge of vectors to prove the Cosine Theorem in relation to their previous knowledge is both a presentation and a demonstration of the application of vectors to plane geometry [9]. Emphasizing the holistic nature of knowledge and highlighting the fact that vectors are an important tool in the study of plane geometry problems.

Teacher activity: Please recall that the equation just derived is in the context of knowing C, the side a opposite to A, and the side b opposite to B to represent the third side, C, opposite to c. Think about it differently, how do you solve for the third side if you know the other two sides and the angles to which they are pinched? Invite students to do the derivation by themselves by analogizing the steps they just performed. Teacher walk-throughs and displays students' calculations.

Student activity: Students will analogize the above steps by calculating the third side if the other two sides and their angles are known, resulting in three equations: $a^2 = b^2 + c^2 - 2ab\cos A$, $b^2 = c^2 + a^2 - 2ab\cos B$, $c^2 = a^2 + b^2 - 2ab\cos C$.

Teacher-student activities: the students observe the three equations obtained, guided by the discovery that the formula has its own characteristics, and feel the beauty of symmetry in mathematics.

[Design intent] In teaching students to observe the specific form of the formula, in the concept of teaching penetration of aesthetic education, reflecting the value of mathematical education, so that students take the initiative to experience the beauty of mathematical knowledge.

Activity 2: Generating the Cosine Theorem

Teacher activity: The symbolic language of the cosine theorem has been obtained through derivation, can we get the textual language of the cosine theorem based on these three equations?

Student activity: Discuss with each other at the same table and share their results, other students add, and get the concept of the cosine theorem.

[Design intent] In the TRU evaluation framework, the dimension of "equitable access to learning content" states that classroom organization should ensure that all students have the opportunity to learn. Therefore, this design helps to deepen students' conceptual understanding of the cosine theorem through student-led discussions to gain concepts.

Activity 3: Proving the Cosine Theorem in Other Ways

Teacher activity: The teacher guides students to prove the cosine theorem in other ways [10].

Student activity: Groups share and discuss their ideas, while the teacher walks around and finds group representatives to share their ideas. Students suggest that the cosine theorem can be proved by constructing a right triangle.

[Design intent] In the TRU assessment framework, the dimension of "student motivation and performance" states that students should be given ample opportunities to express themselves in the classroom. Therefore, students are guided to prove the cosine theorem in different ways to enhance their mathematical thinking and abilities.

Activity 4: The Development of the Proof of the Cosine Theorem

Teacher activity: As early as before the third century A.D., Euclid first mentioned the problem of proving the cosine theorem in the Principia Geometrica; in the second half of the twelfth century, Ptolemy put forward the Ptolemaic Theorem in the Dacia Astronomica, which gave a method for proving the cosine theorem. In 1595, Pettistus proved the cosine theorem by geometric methods in Trigonometry: A Concise Treatment of the Solution of Triangles, and the cosine theorem was proved in the seventeenth century by De Morgan in the Fundamentals of Triangles. In the middle of the 17th century, De Morgan, in his Fundamentals of Triangles, ingeniously used the sum-angle formula and the sine theorem to derive the cosine theorem. It was not until the 1950s that the cosine theorem was proved using analytic geometry.

Activity 5: Analyzing the Cosine Theorem and the Pythagorean Theorem

Teacher activity: Guide students through the process of deriving the cosine theorem and think about how the collinear theorem is related to the cosine theorem.

Student activity: Students discuss the results together: the Pythagorean Theorem is a special case of the Cosine Theorem, and the Cosine Theorem is a generalization of the Pythagorean Theorem.

[Design intention] Combining the "cognitive demand" dimension of the TRU framework, starting from the hook and strand theorem that students are familiar with in junior high school, and letting students discover the relationship between the hook and strand theorem and the cosine theorem on their own, which is in line with the law of cognitive development of students.

3.4.3. Inspiring thought and extension

Teacher activity: The cosine theorem points out the relationship between three sides of a triangle and one of its angles. Applying the cosine theorem, you can solve the problem of determining the angle of a triangle by knowing the three sides of the triangle, how to determine it?

Student activity: Student exploration culminates in a corollary to the cosine theorem:

$$cosA = \frac{b^2 + c^2 - a^2}{2bc},\ cosB = \frac{c^2 + a^2 - b^2}{2ca},\ cosC = \frac{a^2 + b^2 - c^2}{2ab}$$

[Design intent] Incorporate the "learning opportunities" dimension of the TRU framework to help students engage in classroom activities that lead to inferences about the cosine theorem.

3.4.4. Application of new knowledge and consolidation of knowledge

Question 1: In ΔABC, if $C = \frac{\pi}{3}$, $a = 2$, $b = 3$. Then how long is c?

Question 2: In ΔABC, if a = $\sqrt{6}$, $b = 2$, $c = \sqrt{3}+1$. Find the size of A.

Teacher activity: The teacher evaluates the students' results and guides them to observe the similarities and differences between the two problems.

Student activity: Students conclude that the Cosine Theorem can be used to solve the problems of finding the third side by knowing two sides and their angles, and finding any angle by knowing three sides.

[Design intent] In the TRU framework, the dimension of "feedback on classroom practice" states that teachers should give students timely feedback based on the results of their classroom practice. Therefore, students are given the opportunity to deepen their understanding of what they have learned in this lesson through powerful instruction. Students can test themselves on the content of the lesson and discover real-life applications of the cosine theorem and its corollaries.

Question 3: Solve the problem of "building a youth cave" encountered before the start of the lesson.

Student activity: Students use the cosine theorem, bring in the relevant data, and find the result that the length of the foot of the hill AB is approximately 655.74 meters.

[Design intent] Finally solve the problems encountered before the beginning of the lesson to form a closed loop. Let students realize that mathematics comes from life and is applied to life.

3.4.5. Knowledge reviewing, summarizing, and improving

Teacher activity: Teachers review the content of this lesson from three aspects, namely, knowledge pathways [11], methods of thinking, and literacy enhancement.

[Design Intent] According to the requirements of the TRU framework for the dimension of "mathematical content," students can summarize, reflect on, and sublimate their knowledge, methods, and skills, and promote their understanding and awareness of the content and methods learned in this lesson.

3.4.6. Assignment for knowledge consolidation

Teacher activity: Separate tasks for consolidating knowledge and expanding thinking.

[Design Intent] Setting up after-school homework at different levels ensures that students develop their horizons while mastering the basics of mathematics.

4. Teaching strategies for high school mathematics based on TRU

Based on the five dimensions of the TRU evaluation framework, the following teaching strategies are derived from the instructional design of the Cosine Theorem in the hope that they will be helpful to teachers in their teaching.

4.1. Developing professionalism and focusing on math content

Professor Jianyue Zhang mentioned that the prerequisite for teaching good mathematics is to understand the content of mathematics by oneself [12]. It is necessary to understand its essence, but also to dig out its nurturing value. Teachers should pay attention to the wholeness of mathematics, the coherence of logic, and the universality of methods in the process of teaching [13]. Therefore, it requires teachers to consciously make connections between concepts and procedures when designing instruction to operationalize the goals of the unit's instructional design.

4.2. Focusing on the math classroom to promote students' development

Teachers should pay attention to the cognitive development level of students in the process of teaching by setting challenging questions to analyze and dig deeply into the teaching content. In this process, teachers should provide students with strong guidance and assistance and give full play to the leading role of students to build a solid foundation for math learning, so that students can independently solve the problems encountered.

4.3. Taking students as the main body and giving full play to their mobility

Teachers should pay full attention to every student in the classroom, give different learning opportunities to students at different levels, create an atmosphere in which all students actively participate in the classroom, actively express their ideas, and ultimately draw conclusions through independent exchange. Through this, we can realize "everyone can get a good mathematics education, and different people develop differently in mathematics."

4.4. Providing timely feedback on the learning process

The General High School Mathematics Curriculum Standards (2017 Edition Revised in 2020) [14] states that it is necessary to emphasize process evaluation, focus on literacy and improve quality. In the process of teaching, students should be guided to analyze the reasons on their own: "How did you get this question right?". It can not only help students to deeply understand the mathematical knowledge, but also help students to correctly understand themselves and enhance their self-confidence in learning mathematics [15].

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