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Difficulties in Learning and Teaching Numbers: A Literature Review on the Obstacles and Misconceptions of Learners and Instructors

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Abstract: This paper attempts to summarize a number of research studies on numbers. The purpose of this study was to investigate and identify the obstacles encountered by students when they are dealing with number reasoning (whole numbers, integers, and rational numbers) and the difficulties faced by pre-service teachers in teaching arithmetic, including their misconceptions and weaknesses when they teach arithmetic and operations. There are two main sections in this paper: students' cognitive obstacles for number reasoning, and pre-service teachers' misconceptions of arithmetic. With the summarized misconceptions and obstacles of both, students and teachers, this paper provides efficient and effective thinking strategies that may help both, learners and instructors overcome obstacles, revise misconceptions, and strengthen their understanding, in order to develop proficiency in number reasoning and arithmetic operations.

Keywords: Cognitive obstacles; Misconceptions of arithmetic; Number reasoning; Whole numbers; Integers; Rational numbers

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1. Introduction

Number reasoning and arithmetic operations are the core of students, teachers, and the curriculum from pre-kindergarten (pre-k) to eighth grade, including understanding numbers, representing numbers in multiple ways, and computing fluently with four operations according to the National Council of Teachers of Mathematics [1]. Whole numbers as the basic component in the number system provide a strong foundation for integers, rational numbers, and other numbers. The number systems are bounded in an interdependent relationship. From learning number names to mastering number operations and representations, students will come across a great deal of obstacles and difficulties, in order to develop proficiency in numbers, such as when they operate with single-digit and multi-digit numbers, learn numerical algorithms, as well as use estimation and mental arithmetic [2]. On the other hands, teachers who are responsible for helping students with proportional reasoning, the place-value system, the application of number lines, and algebraic thinking are also facing tremendous challenges. This paper includes a wide range of research studies investigating the misconceptions of both, students and teachers when they encounter numbers.

2. Students' cognitive obstacles for number reasoning

From empirical studies and historical data, many learners have difficulties when they deal with numbers, especially negative numbers, fractions, and rational numbers [3-6]. Gallardo concluded that less than 40% of the participants in the study were able to subtract integers [7]. Bishop and other researchers define negative numbers as “theoretically plausible, yet practically impossible” [4]. On the other hand, some researchers believe that when students are proficient in both, content knowledge and procedure knowledge of whole numbers, they are able to master other numbers. Mack views whole numbers as the basis for learning fraction when he investigated third graders and fourth graders, finding that students’ prior knowledge of whole numbers influences the meanings and representations they construct for fractions as they built on their informal knowledge of fractions [5]. As for rational numbers, Hunting and several other researchers assert that there is “an interdependence between development of rational-number knowledge and whole-number knowledge” [6]. Rational numbers can help stimulate and expand learners’ knowledge of whole numbers.

The aforementioned studies support the idea that whole numbers are the foundation of learning number reasoning since integers, whole numbers, and rational numbers are interwoven and interdependent. Therefore, in order to develop proficiency in numbers, it is vital to access and facilitate students’ prior knowledge of whole numbers at an early age [8].

Research has identified and documented cognitive obstacles and affordances within the domain of integers with both historical data and children’s thinking data, especially when children deal with the addition and subtraction of negative numbers. In a study [4], historical data was used based on five criteria of ways of reasoning: order, magnitude, logical necessity, computational, and limited. Among those five criteria, “limited” was the focus in the study. In particular, the authors generated four limitations in the historical data, which included “Addition Cannot Make Smaller,” “Subtraction Cannot Make Larger,” “Subtrahend < Minuend,” and “Negatives Rejected.” In their children’s thinking data, they selected 47 elementary school children, age ranging from six to ten, to participate in the study, who were approximately in grades one to four in the United States. The results indicated that negative numbers were the obstacles for learners’ whole number reasoning [9]. Historical data showed that for more than 1,000 years, mathematicians struggled with, operated on, and rejected negative numbers because of three reasons, but they eventually came to accept them. The first one was because of the lack of physical, concrete, or geometrically meaningful representation. Second, they struggled with negative numbers because they were the result of nonsensical operations. The third reason was related to the existence of negative numbers that made the notations of “addition cannot make smaller” and “subtraction cannot make larger” problematic.

As for children’s way of reasoning about negative numbers, Bishop and other researchers found that students face difficulties in viewing negative numbers as quantitates less than nothing, which falls under the first kind of obstacle [4]. Due to the lack of appropriate interpretations, children have always rejected negative numbers. The second kind of obstacle is removing something from nothing or removing more than one has. For example, one student in the study said, “Three minus five does not make sense because three is less than five”; yet another student answered the question in a similar manner, “Three minus five is zero because when you have three, you cannot take away five, so taking away three will leave you with zero.” The third obstacle is counter- intuitive situations involving routine interpretations of addition and subtraction. Students holding the wrong notations such as “addition cannot make smaller” and “subtraction cannot make larger” were not able to solve the following problem: “$4 + \square = 3$.” In addition and subtraction operations involving negative numbers, those misconceptions are identified as cognitive obstacles for learners. However, the same study also found cognitive affordances for students to overcome those difficulties, such as ordering relations, logical necessity and formalisms, as well as magnitude. From their research, students’ cognitive obstacles and affordances for number reasoning have been identified.

Another research conducted by Mack investigated students’ knowledge of fractions after giving explicit instruction on whole numbers [5]. After three weeks of treatment, the researchers found that the students were confused about the two different numbers. In particular, they overgeneralized the meanings of symbolic representations for whole numbers to fractions and those for fractions to whole numbers. In a one-to-one setting, each student received instruction on addition and subtraction of fraction. However, the students failed to relate the conceptual meaning of fractions to informal knowledge of whole numbers. For example, questions such as "If you have three eighths of a pizza and I give you two eighths more of a pizza, how much pizza do you have?" require students to access their informal knowledge of whole numbers. However, the instructors did not initially include verbal names for fractions, such as "three eighths" and "two eights." The students' informal knowledge was drawn upon by referring to fractional quantities in a manner such as "three of eight pieces" and "two of eight pieces"; therefore, their answers were "five of the eight pieces." In the study, Mack commented that "although students can draw on their informal knowledge to give meaning to mathematical symbols, they may not readily relate symbolic representations to informal knowledge on their own, even when problems presented in different contexts are closely matched." This study illustrates students' cognitive obstacle for number reasoning.

Hunting and several other researchers used the data of younger age children, in order to determine the effect of accessing whole number as prior knowledge when learning rational numbers and the role their whole-number knowledge might play in it [6]. The special setting of this study was the computer-based tool called Copycat. This application allowed students to explore relationships between whole numbers and rational numbers by manipulating virtual items in this computer-learning platform. For example, the children were encouraged to select an appropriate combination of denominator and numerator, in order to form fractions according to the specific input and output instructions. In the comparison task, the instruction required the children to compare $\frac{1}{2}$ and $\frac{1}{3}$ by asking which fraction is larger. By using the Copycat, the children set the fraction as $\frac{1}{2}$ and the input as 6; the output showed 3. The children repeated the procedure by setting the fraction as $\frac{1}{3}$ and the input as 6, in which the output showed 2. In that way, they claimed that $\frac{1}{2}$ is larger than $\frac{1}{3}$. The operator-like setting in Copycat provided a concrete and challenging environment to develop reasoning skills and strategic competence. The design and problem settings of this platform were based on the knowledge of whole numbers and number reasoning. In this study, students were able to solve difficult tasks in the computer program with the help of whole numbers. Based on the conclusion of the study, the researchers asserted that "this study underscores the importance of children having a firm foundation in whole-number strategies and relationships established in the early years of schooling." It clearly indicates that when instructions are given to facilitate the relationship between whole numbers and rational numbers, students are able to solve fraction comparison tasks successfully.

In summary of students' cognitive obstacles for number reasoning, those three studies above illustrate learners' cognitive obstacles and difficulties when they deal with numbers. Educators have attempted to access learners' prior knowledge of whole numbers when they teach negative numbers and its operations [4]. The purpose of activating students' prior knowledge of whole numbers in order to develop proficiency in rational numbers is to overcome cognitive obstacles for number reasoning [6]. Overall, strengthening the conceptual knowledge and procedural knowledge of whole numbers is an efficient and effective way to develop proficiency in other numbers, such as integers, fractions, and rational numbers.

3. Pre-service teachers' misconceptions of arithmetic

According to previous research, many pre-service teachers or prospective teachers do not have sufficient content knowledge and fluent procedural knowledge of teaching whole numbers and arithmetic in the United States [10]. For example, several researchers have found that pre-service teachers fail to justify multiplication in challenging tasks [11]. In another study, Thanheiser and other researchers found that the subjects were able to process the algorithms fluently but failed to present the conceptual meaning of the algorithms [12]. Harkness and Thomas pointed out that pre-service teachers with insufficient knowledge are unable to make correct judgments on students' work [13].

Such studies indicate that pre-service teachers' arithmetic knowledge must be strengthened, and their arithmetic misconceptions must be effectively and efficiently reversed, in order to meet the education standard as full-time mathematics teachers. It is very important to identity those misconceptions in the domain of whole numbers and arithmetic operations, including addition, subtraction, multiplication, and division [14].

In the research conducted by Chapman [15], he assigned three groups of tasks to pre-service elementary teachers, in order to develop their proficiency in using and teaching arithmetic operations. Those three groups of tasks were based on "reflection – inquiry – reflection" with peers involved. In particular, the first group of tasks aimed to access pre-service teachers' understanding of the nature and structure of arithmetic word problems and the conceptual meanings of arithmetic operations. The second group of tasks included "problem situations" relating to addition, subtraction, multiplication, and division. For example, "I have three marbles. My friend gives me five marbles. How many marbles do I have?". The third group of tasks required higher-level thinking. In addition to solving the problems, the pre-service teachers were required to change the questions, combine the four operations, compare, and then analyze those "problem situations." Those tasks facilitated new understandings of familiar concepts and promoted reflections and discourses about self, learners, instruction, and the curriculum. Additionally, they allowed pre-service teachers to simulate for real-world situation. By solving the tasks, pre-service teachers were provided with relevant, practical, and meaningful examples of arithmetic concepts. More importantly, the pre-service teachers were able to develop a deeper understanding of the conceptual knowledge of arithmetic operations and interactive mathematical concepts with respect to pedagogical strategies. The study reveals the importance of facilitating pre-service teachers' mathematical knowledge, especially in the domain of arithmetic operations.

According to Thanheiser's study in 2009 [16], she found that 80% of the participants failed in the task of interpreting multi-digits in the base-10 number system. Half of the participants wrongly read the number 389 as 300 ones, 8 ones, and 9 ones, which should be read as 3 hundreds or 30 tens or 300 ones, 8 tens or 80 ones, and 9 ones. In this study, a higher-level thinking was required of the participants when they were required to perform flexible grouping and regrouping. Thanheiser asserted that "with only procedural knowledge (knowing how to perform the steps of the algorithm), a teacher can solve a problem and demonstrate the procedure but cannot explain why the procedure is mathematically correct or why it is applicable in some situations and not in others." She pointed out the weakness of current pre-service teachers who have strong procedural knowledge but lack conceptual knowledge. This research result indicates the need of improving content knowledge among pre-service teachers in the United States.

Yackel, Underwood, and Elias designed a similar study as Chapman's. In the study [17], the researchers used mathematical tasks to foster pre-service teachers' concepts of early arithmetic operations. Specifically, they included the basic concepts of number, number relationships and strategies, as well as the coordination units of different rank. The approach of the study was to stimulate the participants in a base-8 world for six weeks. According to the researchers, they created eight analogs of instructional sequences in classroom teaching experiments in the elementary grades that have been proven successful in promoting deep conceptual understandings of basic arithmetic and place-value numeration. The purpose of the study was to "provide details about the instructional design" and "make it possible for others to make informed decisions about individual adaptions." For the majority of pre-service teachers, it was a challenging task to transfer from a base-10 world to a base-8 world without sufficient content knowledge. For example, the participants were given the task of interpreting the number 56 in a base-8 system and adding two numbers in a base-8 number system. Through these learning experiences, the participants were able to re-conceptualize their basic arithmetic concepts and learn how they can be advanced. In addition, justification and number reasoning could be improved and developed effectively and efficiently when pre-service teachers attempt to make sense of different number reasonings (a base-8 world). The study suggests a need for fostering pre-service teachers' conceptual knowledge of number reasoning and operation.

In another study [18], the researchers attempted to use manipulatives to revise the arithmetic misconceptions of pre-service teachers. There were two experiments based on two independent groups of pre-service teachers. Fifty participants engaged in the first experiment, while 39 participants were in the second one. The treatment of the study was to engage participants to solve operational problems involving whole numbers and fractions by presenting concrete representations or manipulatives in five classes. The two experiments were under the same procedure but with different groups of participants. The results were based on the performance of the pretest and posttest accessing division by fractions tasks, which indicated the improvement of arithmetic knowledge. In the study, the misconceptions of arithmetic were mainly reflected as the failure of pre-service teachers to explain the underlying meaning of how algorithms work. However, they showed fluent computation in the study. In other words, they lacked conceptual knowledge of arithmetic operations. The researchers encouraged pre-service teachers to use appropriate manipulatives, in order to deepen their understanding of operations. For example, base-10 blocks were found helpful in improving learners' understanding of place-value and operations. An effective approach known as "guided constructivism" was used in the study. Specially, learners were instructed on how to choose an appropriate model and embed it in actual problem situations.

This "guided constructivism" was raised because some learners were not fully stimulated in the symbolic environment, meaning that they do not understand "+," "-," "×," and "÷" in the problem settings. However, the manipulatives allow them to touch and control in order to understand the meaning of "joined with" in addition, "take away" in subtraction, "sets of" in multiplication, and "grouped into" in division. Both experiments showed no significant difference in the performance of the participants with "guide constructivism" in the treatment. With the help of manipulatives, the arithmetic misconceptions were reversed, and their arithmetic knowledge improved in accordance with the test results. The study indicates the importance of manipulatives and emphasizes the lack of content knowledge among pre-service teachers when they operate and compute numbers.

Lo, Grant, and Flowers also conducted a study and found that prospective teachers in elementary school have less conceptual knowledge of multiplication and fail to justify reasoning strategies [11]. In the study, the instructor accessed a "reasoning-based curriculum" to field-test prospective teachers in the topics of number and operation. All of the designed classes were with the focus of the understanding of place-value, flexibility of composing and decomposing numbers, both, additively and multiplicatively, as well as commutative and distributive properties for addition and multiplication. Specifically, Lesson 1 was to develop reasoning strategies; Lesson 2 was aiming at justifying 197 × 50 by using either area or equal groups representation; Lesson 3 was to compare different reasoning strategies; and Lesson 4 was on equivalent transformation. During the data analysis, there were six challenges identified by the researchers. The first two were associated with reasoning development and justification. The reason might be the lack of conceptual knowledge of the prospective teachers. The other four challenges were labeled as "describing versus explaining," "proof by picture," "coordinating area interpretations with strategies," and "coordinating equal groups interpretation." The prospective teachers could not tell the difference between describing the solving procedures and explaining them mathematically. Also, they failed to identify the role of pictures or diagrams, especially when they attempted to justify the commutative property for multiplication by using area or array interpretations. Some teachers attempted to utilize equal groups interpretation in class, but they made common errors, such as adding or subtracting groups of different size or switching the roles of the numbers in the middle of the justification. The researchers suggested that prospective teachers need to distinguish between addition and multiplication in order to develop a reasoning strategy for multiplication and addition. The study shows the weak justification strategies of pre-service teachers and suggests the need for strengthening their conceptual knowledge of number reasoning and arithmetic operations.

In another research, Harkness and Thomas conducted a study based on “Multiplication as Original Sin,” written by Corwin in 1989 [19]. In the study, pre-service teachers were required to reflect on how Corwin’s invented algorithm worked by providing proof and justification. The purpose of this research was to examine the participants’ understanding of the validity of Corwin’s “Upwards Method” and justify the provoking issues raised by them. In Corwin’s work, she invented a new algorithm called “Upwards Method,” which contradicted with her teacher’s “Downwards Method.” She applied the metaphor “original sin” to emphasize how her teacher criticized her invented algorithm to solve multiplication. The researchers believe that this method is based on commutative and distributive laws. For example, the question was to solve 34×23. Corwin’s teacher used the “Downwards Method,” which worked as follows: $34 \times 23 = 34 \times (3 + 20) = 34 \times 3 + 34 \times 20 = 102 + 680$. When this is written in the multiplication algorithm, it is solved from bottom to top. However, in Corwin’s method, it is solved from top to bottom, implying that $23 \times 34 = 23 \times 4 + 23 \times 30 = 92 + 690$. Both methods were worked mathematically. According to the result of this study, only 31% of the pre-service teachers showed insufficient understanding of those mathematical properties. However, most of them explained the procedural rules by memorizing them rather than illustrating the conceptual meaning of Corwin’s invented algorithm. The researchers pointed out that “one semester-long training course, for pre-service elementary teachers, which focuses on number sense and operations, is probably not enough to erase at least 12 years of mathematics as memorization of rules, procedures, and definitions.” If teachers lack understanding of algorithms and number reasoning, they will end up making wrong judgments of students’ creative work or invented methods just like Corwin’s teacher. It is a very dangerous and toxic situation in education. The study emphasizes that the need for improving pre-service teachers’ understanding of arithmetic and number reasoning is becoming more and more demanding.

Thanheiser, Whitacre, and Roy documented the prospective elementary teachers’ mathematics content knowledge in the United States from “a historical perspective” and “a current perspective” [12]. The study included an extensive literature review. From their work, “a historical perspective” referred to a summary of research studies prior to 1998, while “a current perspective” referred to research studies between 1998 and 2011. The researchers claimed that the “PTs in the United States and Australia continue to lack a conceptual understanding in this important area,” reflecting their weak content knowledge of arithmetic operations and number reasoning. From the studies prior to 1998, they discovered that prospective elementary teachers lack content knowledge because they failed to claim that the divisor must be less than the dividend, had difficulty relating the long-division algorithm to real-world situations, and had limited conceptions of zero, as evidenced by the fact that some of them answered incorrectly for questions involving division by zero. From the research studies between 1998 and 2011, they discovered that prospective elementary teachers relied on memorized procedures involving whole numbers and operations and had difficulty understanding the properties of zero. Therefore, the authors commented on the situation in the United States by stating, “There is still much work to do for the mathematics education community to better understand how PT’s conceptions develop and how this development can be facilitated.” The study provides horizontal and vertical perspective of viewing the content knowledge of prospective elementary teachers based on an extensive literature review, which highlights the fact that prospective elementary teachers have insufficient content knowledge in the States.

In summary of pre-service teachers’ misconceptions of arithmetic, the aforementioned studies reveal that pre-service teachers with procedural knowledge are unable to access their conceptual knowledge of arithmetic and operations. In other words, the weakness of pre-service teachers is reflected in the lack of understanding of number reasoning and arithmetic operations, thereby failing to justify why certain algorithms work and how they work [16]. The memorization of rules and fluent procedures cannot help to develop proficiency in numbers [13]. In addition, they fail to provide conceptual explanations of algorithms [11]. According to "Adding It Up: Helping Children Learn Mathematics," all five "strands" of mathematical proficiency are interwoven [20]. Other than the first two strands (conceptual understanding and procedural fluency), the other three, which are strategic competence, adaptive reasoning, and productive disposition, are also essential in both, teaching and learning numbers [20]. A lot of work is required to develop the proficiency of prospective teachers in terms of content knowledge [12].

4. Conclusion

Numerous studies focusing on the concerns of students' cognitive obstacles and pre-service teachers' misconceptions in the domain of numbers were summarized and included in this paper. The first section discussed the obstacles encountered by students when dealing with negative numbers, rational numbers, and number reasoning. The second section discussed the misconceptions of pre-service teachers, specifically, their weak conceptual knowledge of arithmetic operations. Therefore, in order to develop proficiency in number reasoning and arithmetic operations, both, students and pre-service teachers should balance their emphasis on conceptual knowledge and procedural knowledge. Furthermore, the other three "strands," strategic competence, adaptive reasoning, and productive disposition, also need to be strengthened [20].

Disclosure statement

The author declares no conflict of interest.

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