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Literature Review on the Impact of Symbol Sense on Algebra Learning in Middle School Students

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Abstract: This paper conducts a literature review to examine domestic and international research on symbolic literacy, defining a four-tiered structure of symbol awareness: perception, operation, reasoning, and representation. It analyzes the differentiated impact of each tier on algebra learning in junior and senior high schools, and constructs a tiered algebra teaching system that integrates secondary education. The study points out that existing research suffers from incomplete coverage of educational stages, a lack of assessment tools, and weak empirical evidence in teaching. It proposes future research directions such as longitudinal tracking, tiered teaching experiments, and digital integration, providing a reference for cultivating symbolic literacy in secondary schools.

Keywords: Symbol sense; Symbol awareness; Middle school students; Algebra learning; Hierarchical teaching

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1. Introduction

Algebra, as a core area of mathematics education, is not only a bridge connecting arithmetic and higher mathematics, but also a key vehicle for cultivating students' abstract thinking and logical reasoning abilities. However, for a long time, difficulties in learning algebra have been a common problem troubling both teachers and students. Many students, when faced with formulas like "" or "," often exhibit fear and resistance towards abstract letter symbols. This phenomenon reflects not merely a lack of calculation ability, but a deep-seated rupture in understanding the meaning of symbols. With the promulgation and implementation of the *Curriculum Standards for Compulsory Education: Mathematics (2022)*, the core competency-oriented mathematics education reform has placed higher demands on "symbol awareness" (formerly "symbol sense"). This signifies that algebra teaching must shift from simply focusing on the "basic skills" training of computational skills to cultivating comprehensive competencies in symbol understanding, symbolic representation, and symbolic application. Against this backdrop, re-examining the cognitive roots of difficulties in learning algebra and deeply analyzing the core role of symbol sense in the construction of algebraic thinking has significant theoretical and practical value for solving the dilemma of algebra teaching and implementing the new curriculum standards.

2. Research questions and methods

This paper takes middle school students as the research object and focuses on three questions: First, it sorts out the four levels of ability of symbol sense, operation, reasoning, and representation, and clarifies their differentiated impact on algebraic content such as polynomials, functions and sequences in middle school; second, based on the progressive law of symbol awareness, it constructs a coherent and hierarchical algebraic teaching path between junior and senior high school; third, it identifies the shortcomings of existing research and proposes directions for future research.

This study primarily utilizes literature review, combined with comparative and inductive analysis methods. It retrieves core Chinese and English literature from 2001 to 2025, categorizes and compares domestic and international theories and empirical findings related to symbolic perception, and integrates theoretical models and teaching cases to form a review research framework.

3. Literature review

Fey first mentioned symbol sense in 1990 and described it as an informal skill required to effectively handle symbolic representations and algebraic operations [1]. He believed that in order to cultivate symbol sense, the teaching objectives should include at least the following basic themes: First, being able to read algebraic representations to understand their possible forms or graphical representations. Second, being able to make informed comparisons of the order of functions of the form, which is a bridge between numerical and symbol sense. Third, being able to read function value tables or graphs, or interpret verbally stated conditions, to identify algebraic rules that express appropriate forms. Fourth, being able to check algebraic operations and predict the form of the results, or to make arithmetic estimations to determine whether the operations are correct. Fifth, being able to determine which of several equivalent forms is most likely to be the best for answering a particular question.

Analogous to “number sense,” Arcavi further elaborated on the concept of “symbol sense” in 1994, which is the intuitive ability of an individual to construct informal meaning in formal mathematical operations [2]. Arcavi pointed out that with the advent of Computer Algebra Systems, simply emphasizing mechanical symbol operations is no longer the core of algebra teaching. He believes that individuals with symbol sense should not only be able to manipulate symbols flexibly, but also have a metacognitive sense of “control” and aesthetic sense, such as being able to judge when to introduce symbols to assist in solving problems, and when to abandon tedious algebraic deductions and seek alternative paths such as graphics or numerical values. In addition, being able to “read” the mathematical structure hidden behind the symbols like reading text, rather than just performing mechanical logical deductions, is also an important part of this concept. This theoretical framework laid an important foundation for subsequent research on algebraic understanding, multiple representations, and technology integration theory.

Related to the concept of symbol sense, some authors focus on aspects of symbol sense in the use of technology, particularly in Computer Algebra Systems (CAS) [3]. CAS cannot pose mathematical problems, establish equations, or interpret the meaning of solutions; it can only provide algorithmic assistance in the process of solving problems. Therefore, Pierce and Stacey define “algebraic insight” as: “the algebraic knowledge and understanding required for students to correctly input representations into CAS, to efficiently check the entire calculation process and results for possible errors, and to interpret the output results as conventional mathematical representations" [4]. It includes two main aspects: one is algebraic expectation, which involves the identification and understanding of structure, objects, key features, principal terms, and symbolic meaning. The other is the ability to associate different representations, such as symbols with graphics and symbols with numerical values. In other words, this knowledge refers to the ability of students to use the tools and language of a calculator to process a mathematical problem and ultimately interpret and apply the results using conventional mathematical symbols and forms. Kenney adopted Pierce and Stacey's framework in her study and added "knowing how and when to use symbols" and "knowing when to abandon a certain representation" [5].

Similarly, Linchevski and Linveh use the term "structure sense" to describe the ability to "flexibly and creatively apply equivalent structures of representations" [6]. Through a study, they found that students lacked structure sense in both numerical and algebraic contexts, but also determined that students' errors were inconsistent. Some students answered certain questions correctly, but made mistakes on other questions with the same symbolic structure. Hoch and Dreyfus obtained similar results when studying how structure sense was affected by the number of brackets and the position of variables in a question [7]. They changed the structure of a set of questions and observed students' responses. The study also found that students lacked structure sense and were inconsistent in their handling of specific structures. Based on their findings, Hoch and Dreyfus elaborated on the definition of structure sense, calling it "a set of abilities that, independent of operational abilities, enable students to better apply previously learned algebraic skills."

Following the discussion on symbol sense above, Wang Xiong, a domestic scholar, deeply analyzed the internal mechanism of symbol sense development from the perspective of "representational schema" in cognitive psychology [8]. This study believes that symbol sense is not an isolated symbol recognition ability, but a cognitive structure in which individuals combine non-propositional mental graphics with conceptual properties based on physical environment and activity experience. Specifically, the spatial characteristics of mathematical formulas and formal operations (such as merging, decomposition, and position transformation) are often rooted in human-specific activity experience (such as counting and sorting). When learners can construct representational schemas that are isomorphic to mathematical forms, they can establish a cognitive link between the "form" of symbols and the "content" of mathematics. This view emphasizes that the cultivation of symbol sense is essentially to help students internalize external symbol operations into psychological schema structures, thereby avoiding meaningless mechanical imitation in algebra learning, which provides an important psychological explanatory framework for understanding symbol sense.

Xu Wenbin further clarified the internal logical connection between "number sense" and "symbol sense" from the perspective of cognitive development, providing an important theoretical supplement for understanding the formation mechanism of symbol sense [9]. The study pointed out that "number sense" is not a mysterious intuition, but an individual's awareness of numerical relationships and numerical patterns, as well as the ability to flexibly use this awareness to solve numerical problems. The literature emphasizes that the cultivation of symbol sense is rooted in a solid foundation of number sense. If students are only familiar with mutually independent standard calculation procedures but cannot distinguish numerical patterns and relationships, it is difficult to establish a connection between symbols and meanings. Therefore, the key to moving from "mechanical imitation" to "meaning construction" lies in guiding students to establish a connection between standard procedures and numerical structures. This view suggests that the formation and development of symbol sense is essentially a process of students gradually transitioning from flexible control of specific numerical operations to reasoning about abstract algebraic structures.

After clarifying the cognitive mechanism of symbol sense, domestic scholars further deconstruct it into specific teaching objectives and ability levels. Zhou Dongming pointed out that the cultivation of symbol sense is a gradual process from "recognition" to "understanding" and finally to "conscious application" [10]. This view emphasizes that the establishment of symbol sense first depends on the intuitive cognition and internalization of mathematical symbols (such as operators and relational symbols). Students must overcome the "fear" of abstract letters, which is the cornerstone of the budding of algebraic thinking. On this basis, the core manifestation of symbol sense is defined as a kind of "transformation" ability—that is, whether students can actively strip away non-essential attributes when facing specific situations (such as "the weight relationship between sheep and pigs"), use concise symbols (letters or custom symbols) to describe complex quantitative relationships, and realize the free conversion between textual language, graphic language and symbolic language. This view pushes symbol sense from static knowledge memory to the dynamic problem-solving field, emphasizing the important role of symbols as a "tool" in simplifying the thinking process.

In response, Peng Yongsong explored the intrinsic connection between symbol sense and "language sense" and "number sense" from the macro perspective of curriculum theory, providing a broader background for the cultivation of symbol sense [11]. He believes that mathematical symbols, as an international "language" that transcends time and space, cannot be learned without a specific language environment and a mother tongue foundation. This argument profoundly reveals the teaching implications of cultivating symbol sense: that is, symbol sense is not an isolated logical deduction ability, but a comprehensive quality built on a solid foundation of natural language comprehension (language sense) and mathematical cognition (number sense). Teachers need to play the role of "organizer" and "guide" in teaching, and help students build a bridge between formal symbol systems and quantitative relationships in the real world by designing rich mathematical activities.

4. A dimensional analysis of the influence of symbol sense on algebraic learning

This study is grounded in Zhu Liming's four-level model of symbol awareness and integrates Wang Xiong's image schema theory with Xu Wenbin's developmental framework from number sense to symbol sense. Based on these theoretical perspectives, the differentiated roles of the four levels of symbolic competence in junior secondary basic algebra and senior secondary abstract algebra learning are systematically examined [8,9,12,13].

Perception and recognition constitute the foundational level of symbol awareness. At this stage, students are expected to distinguish the forms, pronunciations, multiple meanings, and categories of mathematical symbols. In junior secondary education, insufficient development of this competence often leads students to confuse negative signs, subtraction symbols, and opposite-number notation, fail to distinguish operational symbols from relational symbols, and develop resistance toward letters and algebraic formulas. Consequently, they frequently make basic computational errors in polynomial operations and linear equations. In senior secondary education, these difficulties extend to confusion over function notation, sequence symbols, and differential notation, preventing students from correctly interpreting composite functions, general terms of sequences, and differential representations, thereby hindering their entry into advanced algebra. Students with well-developed symbolic recognition are able to construct clear mental representations of mathematical symbols, reducing cognitive load during algebraic operations, reasoning, and mathematical modeling throughout secondary education.

Understanding and operation build upon symbolic recognition and emphasize comprehension of the mathematical principles underlying symbolic transformations rather than mechanical manipulation. At the junior secondary level, students with weak competence in this dimension commonly make errors in applying the perfect square formula and factorization and struggle to manipulate algebraic representations containing variables independently of numerical values, resulting in early divergence in algebra achievement. At the senior secondary level, these deficiencies become evident when students encounter systems of equations, quadratic curves, trigonometric transformations, and differential calculus. They often memorize solution procedures without understanding the logic of equivalent transformations and therefore cannot flexibly simplify complex algebraic structures. By contrast, students who achieve a higher level of symbolic understanding can anticipate the general outcomes of symbolic operations. This finding is consistent with Fey's view that symbolic estimation and verification enable learners to perform equivalent transformations of algebraic representations independently across different stages of secondary education [1].

Association and reasoning represent the intermediate-to-advanced level of symbolic competence, relying on mathematical symbols to formulate conjectures and construct rigorous deductive arguments. In junior secondary education, students are mainly required to use simple symbolic representations to organize quantitative relationships and identify numerical patterns. Students with weak symbolic reasoning often depend excessively on lengthy verbal descriptions, resulting in fragmented logical structures. In senior secondary education, deriving general terms of sequences, proving inequalities, constructing algebraic proofs in solid geometry, and solving advanced calculus problems all require complete symbolic reasoning chains. Students lacking this competence experience difficulties in coordinating multiple conditions, conducting case analyses, and performing reverse verification, making it difficult to develop coherent algebraic reasoning. Xu Wenbin's research further demonstrates that a solid sense of numerical patterns provides the cognitive foundation for symbolic reasoning, whereas insufficient number sense continuously constrains the development of symbolic logic from junior to senior secondary education.

Abstraction and representation constitute the highest level of symbol awareness. This competence involves extracting essential quantitative relationships and patterns of change from real-world or mathematical contexts while flexibly switching among multiple forms of representation, including algebraic representations, graphs, tables, and coordinate systems. In junior secondary education, this ability is primarily reflected in equation construction and modeling with linear functions. Students with limited competence often fail to identify quantitative relationships correctly and are unable to interpret mathematical results within real-life contexts. At the senior secondary level, abstraction and representation expand to functional modeling, sequence applications, optimization problems involving inequalities, and transformations between algebraic and geometric representations in analytic geometry. According to Arcavi's theory of symbol sense, mature symbolic representation includes the metacognitive ability to select and appropriately shift among different representational forms, which distinguishes students with advanced algebraic thinking from ordinary learners [2].

The four dimensions of symbol awareness are not independent but interact closely with one another. Zhu Liming's empirical findings indicate significant positive correlations among all four dimensions [13]. Symbol

Sense and recognition support symbolic understanding and operation; symbolic operation provides the foundation for reasoning; and these three competencies together underpin abstraction and representation. An imbalance in any of the four dimensions during junior secondary education is likely to create discontinuities in subsequent senior secondary algebra learning, as weaknesses in one dimension can trigger cascading negative effects across the others. Only through the coordinated development of all four competencies can students successfully progress from basic symbol recognition and algebraic manipulation to logical reasoning and multi-representational mathematical modeling throughout secondary algebra learning.

5. Algebra teaching strategies based on cultivating symbol awareness

This section is structured around the four progressive dimensions of symbol awareness—perception and recognition, understanding and operation, association and reasoning, and abstraction and representation. Integrating the developmental progression of algebra from junior secondary basic algebra to senior secondary advanced algebra, it proposes a continuous and hierarchical instructional framework spanning Grades 7–12 based on the cognitive deficiencies exhibited by learners at different stages of symbolic development.

Perception and recognition constitute the foundation of symbol awareness and should be emphasized throughout the introductory stage of mathematics learning in secondary education. At the junior secondary level, instruction should employ concrete cognitive representations to establish students' mental schemas of mathematical symbols. Activities involving the classification and interpretation of symbols can help students distinguish the different meanings of the same letter in various mathematical contexts. Visual models, such as balance scales and line segments, can be used to differentiate operational symbols from relational symbols, while a gradual transition from concrete representations to symbolic notation helps reduce students' resistance to algebraic symbols. At the senior secondary level, explicit instruction should be provided on newly introduced mathematical notations, including composite functions, sequence notation, and differential symbols. By systematically classifying symbol types and clarifying their writing conventions and semantic boundaries, students can further refine their symbolic recognition system and establish a solid cognitive foundation for advanced algebra learning.

For learners who can recognize mathematical symbols but lack a conceptual understanding of symbolic operations, visualizing the logic underlying symbolic transformations should become the central instructional strategy. At the junior secondary level, algebraic rules such as factorization and perfect-square identities can be explained through geometric area models and the principle of equation balance. Discussions of representative computational errors can help students identify the cognitive causes of symbolic misconceptions, while introducing algebraic representations containing parameters enables learners to appreciate the generality of symbolic manipulation. At the senior secondary level, instruction should extend to trigonometric identities, systems of equations, and equivalent transformations in differential calculus. Rather than relying on memorized solution templates, teaching should emphasize the mathematical logic underlying symbolic transformations and cultivate students' ability to manipulate complex algebraic structures flexibly.

Association and reasoning, as intermediate-to-advanced dimensions of symbolic competence, require differentiated instructional tasks that align with students' cognitive development across secondary education. At the junior secondary level, emphasis should be placed on transforming verbal quantitative relationships into symbolic representations and establishing a standardized reasoning process consisting of hypothesis generation, symbolic generalization, and numerical verification. This process helps develop students' foundational inductive reasoning abilities. At the senior secondary level, instruction should incorporate more sophisticated reasoning tasks, including sequence proofs, inequality transformations, and algebraic proofs in analytic geometry. Multi-condition reasoning, case analysis, and integrated symbolic reasoning tasks should be strengthened to connect symbolic reasoning across equations, functions, and sequences, thereby bridging the discontinuity in algebraic reasoning between junior and senior secondary education.

Abstraction and representation represent the highest level of symbol awareness and require differentiated cultivation of mathematical modeling competencies across secondary education. At the junior secondary level, authentic real-life contexts should be incorporated into modeling activities, enabling students to translate among natural language, algebraic representations, and tabular representations and complete the basic modeling cycle. At the senior secondary level, open-ended modeling tasks involving function optimization, sequence applications, and the integration of algebraic and geometric representations in analytic geometry should be introduced. Students should be encouraged to define variables independently, construct symbolic models, and compare the applicability of different representational forms, including graphs, coordinate systems, and algebraic representations. Such instructional practices foster the metacognitive ability to select and flexibly switch among symbolic representations according to problem-solving needs.

Throughout secondary education, the cultivation of symbol awareness should follow a spiral progression. During the junior secondary stage, instruction should focus primarily on developing symbolic perception and basic operational competence while gradually introducing elementary reasoning and mathematical modeling. Grade 10 serves as a transitional stage, where instructional emphasis should be placed on addressing the gap in symbolic thinking between junior and senior secondary education and strengthening symbolic reasoning. In Grades 11 and 12, advanced algebraic topics such as differential calculus and comprehensive analytic geometry should be employed to integrate the four dimensions of symbol awareness, thereby promoting students' comprehensive development of algebraic thinking and symbolic competence.

6. Existing research shortcomings and future research prospects

6.1. Insufficient existing research

Existing research on symbol awareness has significant limitations. First, there is an imbalance in research across educational stages, with most empirical studies limited to junior high school. There is a lack of exploration into higher-order symbolic content such as sequences, functions, and derivatives in senior high school, and a lack of continuous longitudinal research from grades 7 to 12, making it difficult to fully present the continuous development of symbol awareness throughout secondary school and insufficient analysis of the barriers to the transition in symbolic learning between junior and senior high school. Second, the assessment system is incomplete. Existing evaluation scales are mostly adapted to compulsory education, lacking standardized assessment tools that cover complex mathematical notation in senior high school, and the stratified indicators cannot be adapted to the symbolic performance of all secondary school students. Third, teaching research leans towards theoretical concepts, mostly proposing differentiated instruction ideas without long-term classroom controlled experiments. The differentiated intervention effects on students with different symbolic levels lack quantitative data support, making it difficult to verify the practicality of strategies. Fourth, the research perspective is narrow, mostly confined to the algebra classroom, with very few studies combining information technology tools such as CAS and Geometer's Sketchpad, while neglecting the emotional and psychological analysis of students' fear of symbols.

6.2. Future research prospects

Further research can be deepened in four aspects: First, conduct longitudinal empirical studies across all educational stages, constructing an integrated assessment tool for symbol awareness from junior high to senior high school. Tracking surveys should be conducted at key junctures in grades 7, 9, and 12 to analyze the dynamic development characteristics of symbolic cognition and address the gap in algebra learning between junior and senior high school. Second, implement teaching intervention experiments, classifying students into different levels based on a four-layer framework of perception, operation, reasoning, and representation. Long-term teaching experiments should be conducted with control classes to quantify the actual effectiveness of the tiered approach and develop a coherent teaching plan that can be widely adopted. Third, broaden diverse research perspectives, exploring pathways for cultivating symbol awareness in a digital environment using computer algebra tools, and introducing cognitive psychology to analyze students' fear of symbols, considering both ability and emotional dimensions. Fourth, expand application scenarios, combining advanced high school algebra content such as sequences, analytic geometry, and derivatives to improve the design of teaching activities corresponding to advanced symbols, filling gaps in high school symbolic teaching research and perfecting a complete research system for symbol awareness serving algebra learning across all secondary schools.

Disclosure statement

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