Journal of Contemporary Educational Research, 2025, Volume 9, Issue 7
http://ojs.bbwpublisher.com/index.php/JCER
ISSN Online: 2208-8474
ISSN Print: 2208-8466

A Practical Study of Teaching Propositions in High School Mathematics Based on the TPACK Framework

Jianping Luo

School of Mathematics and Statistics, Henan Normal University, Henan, China

Abstract: Under the background of information technology in education, there is insufficient integration of technological knowledge, pedagogical knowledge, and subject content knowledge in the teaching of propositions in high school mathematics. Teachers mostly equate information technology with multimedia presentations, and students often memorize formulas mechanically, which leads to difficulties in the application of complex propositions. In this study, we take "the cosine formula of the difference between two angles" as an example. Based on the TPACK framework, we use contextual teaching and geometric drawing board demonstration to integrate subject content, pedagogical knowledge, and technological knowledge in teaching design and practice. It is found that by dynamically displaying the derivation process of the formula and guiding students to explore independently, it can help them understand the logic of the formula and improve their application ability. This study provides a paradigm for teaching propositions in high school mathematics and suggests that the TPACK framework can facilitate knowledge integration and cultivate students' mathematical literacy such as problem posing and creative inquiry, which is of great significance for teaching practice.

Keywords: TPACK; High school mathematics; Propositional teaching; Teaching practice

Online publication: July 25, 2025

1. Introduction

TPACK, i.e., Subject Pedagogical Knowledge for Integrating Technology, emphasizes that teachers should effectively integrate technological knowledge (TK), pedagogical knowledge (PK), and subject content knowledge (CK) [1]. In the era of information technology in education, information technology, as the core element of TPACK, has long had a place in most classroom teaching. Many educationalists and people working in education have affirmed the application of IT in classroom teaching. At the same time, the introduction of relevant national policies has also made modern information technology gradually become an indispensable tool for educators in classroom teaching. However, there are significant shortcomings in the application of the TPACK framework in the teaching of high school mathematics propositions: at the level of TK, teachers limit the tools such as the Geometry Board to static graphical displays, failing to dynamically present the coordinate transformations in the derivation of formulas; at the level of PK, the traditional mode of teaching lacks the students' independent investigation of propositional logic; at the level of CK, the connection between the derivation of formulas and the knowledge system of trigonometric functions was severed. This lack of knowledge integration has led to students' difficulties in realizing mathematical transformations when facing complex propositions, reflecting the fact that TPACK has not yet formed an effective application paradigm in the teaching of mathematical propositions. In recent years, there have been more studies on the integration of TPACK theory and teachers' skills in China, such as research by Xu and Li on the real-life dilemma of cultivating teachers' digital literacy, and their proposal of competence enhancement strategies based on the TPACK framework [2]; and Guo *et al.* focused on the group of teacher trainees and analyzed the influencing factors of numerical competence and its enhancement mechanism under the TPACK theory [3]. These studies provide multiple perspectives on the integration of TPACK theory and teachers' skills from different educational subjects, but there are relatively few studies related to high school mathematics instructional design. While propositional teaching is a very important part of high school mathematics teaching, whether or not information technology can be used appropriately in this part of the classroom is related to the actual effectiveness of teachers' teaching. Therefore, based on the previous research, this study aims to study the teaching design of propositional problems in high school mathematics based on the TPACK perspective by taking "the cosine formula of the difference of two angles" as an example.

2. Teaching mathematical propositions in the TPACK framework

Mathematical propositions, including laws, formulas, theorems, properties, axioms, etc. [4], are important learning vehicles for students' cognitive development. The teaching of mathematical propositions has a very important position and role in high school mathematics teaching, the key to teaching is not to let students memorize axioms, theorems, formulas, laws, but to let them experience the discovery and occurrence of mathematical propositions, deduction and proof, variation and derivation, understanding and application of the whole process.

TPACK is a knowledge framework that integrates CK, PK, and TK. In terms of subject knowledge, teachers use TPACK to accurately grasp the mathematical knowledge system related to propositions in order to teach them effectively; in terms of pedagogy, TPACK facilitates the flexible use of various pedagogical methods, such as inquiry and problem-driven teaching in propositional teaching to stimulate students' thinking; and in terms of technology, the use of Geometry Drawing Board and other technologies to present graphical relationships and data changes in propositions intuitively assists students' understanding of abstract propositions to enhance the effectiveness of teaching and learning.

2.1. The core elements of TPACK

Content knowledge (CK): Expertise about a specific subject area, such as theorems, formulas, conceptual systems, etc. in mathematics; it constitutes the basic material for teaching and learning, and is the core of the subject matter that the teacher imparts to the students.

Pedagogical knowledge (PK): This refers to the knowledge and skills needed by teachers to design curricula and develop lesson plans, including how to develop appropriate teaching objectives and lesson plans, and select teaching strategies and assessment methods that are responsive to the needs of different students and pedagogical contexts [5].

Technical knowledge (TK): This refers to teachers' knowledge of various IT tools (e.g., multimedia courseware production software, online teaching platforms, educational apps, electronic whiteboards, etc.), including their functions, methods of operation, scope of application, and the role they can play in teaching situations.

The three elements are not independent of each other, but are closely related and intertwined. From their cross-fertilization, four new composite elements are created: subject pedagogical knowledge (PCK), subject content knowledge (TCK) of the integrated technology, pedagogical knowledge (TPK) of the integrated technology, and subject pedagogical knowledge (TPACK) of the integrated technology [1], as shown in **Figure 1**.

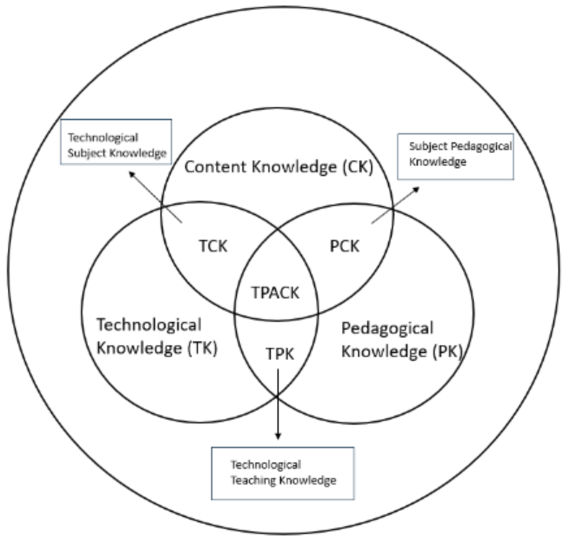

Figure 1. Schematic diagram of the TPACK model

Teaching mathematical propositions under the TPACK framework requires teachers to use the Technology-Packed Mathematical Knowledge for Teaching and Learning of Mathematical Propositions (TPACK) based on the three core elements of CK, PK, and TK. Before designing the teaching process, the TPACK framework should be used as the basis, and the elements in the framework should be analyzed in relation to the teaching content, so as to find the fit between information technology, mathematical pedagogies, and mathematical propositions, and to achieve the optimal efficiency of the classroom [1].

2.2. Case study

This section discusses a case study of TPACK "cosine formula of the difference between two angles" based on the practice of high school mathematics propositions.

2.2.1. Content analysis

The cosine formula of the difference between two angles is from the first lesson of Chapter 5.5 Triangular Constant Transformations of the Compulsory Mathematics of High School of the Renjiao Edition, which belongs to the formula study type class, introducing the cosine formula of the difference between two angles. This formula is the logical starting point of the other trigonometric constant transformations, unlike the

traditional special angle relationship, this formula is for any two angles, their vertices are the origin of coordinates, but the position of the terminal edges can be arbitrary. The textbook uses the rotational invariance of the circle and the formula for the distance between two points to establish a link between the coordinates, the derivation of the formula, this method is simple and clear, and the derivation of the induction formula of the same method, reflecting the link between knowledge, so this lesson in the trigonometric function teaching play a role in the beginning and end.

2.2.2. Component analysis

Using TPACK theory as a guide, we analyzed the TPACK components in the teaching of the cosine formula for the difference between two angles, as shown in **Table 1**.

Table 1. Component analysis of TPACK in the cosine formula for the difference of two angles

TPACK	Component
CK	Understand the derivation process and the main steps of the cosine formula of the difference between two angles, master the form and symbolic characteristics of the cosine formula of the difference between two angles, and be able to use the formula to find the value and calculation.
PK	Contextual, problem-driven, self-directed, co-operative, intuitive, lecture-practice combination.
TK	Multimedia presentation, Geometry board presentation.
PCK	Combined with the abstract nature of the formula for the cosine of the difference between two angles and the logical nature of the derivation, the use of contextual introduction and inquiry-based teaching strategies, first of all, to raise questions to stimulate students to think, and then let the students understand the derivation process of the formula through independent exploration, group discussion, and finally through the explanation to strengthen understanding. In view of the diversity of formulae, using the strategy of combining lecture and practice, through the example of students in the practice of formulae in different situations to master the application of methods.
TCK	Make use of the animation, pictures, and videos in the multimedia courseware as well as the dynamic operation function of the Geometry Drawing Board software to present the derivation process of the formula for the cosine of the difference between two angles and the related concepts in an intuitive and dynamic way. For example, the animation shows how the cosine of the difference of two angles is represented by the change of coordinates of the point on the unit circle, so that students can understand the source of the formula more clearly.
TPK	In the context of the introduction strategy, the use of multimedia to display relevant pictures and videos of the problem situation, to enhance the real sense of the situation and attraction, better to trigger students' thinking. In the inquiry-based teaching strategy, the dynamic operation of the Geometry Drawing Board provides students with a more convenient and intuitive tool to improve the efficiency and effectiveness of inquiry activities.
TPACK	Through the multimedia courseware to show the problem situation to lead to the formula investigation (TK and CK integration), the use of inquiry teaching to guide students to use the geometry board software to explore the formula derivation (PK and TK integration), according to the student's understanding of the knowledge of the use of online resources for the targeted expansion (TCK and PCK integration), etc., to achieve the subject of the content of the knowledge, pedagogical knowledge and technological knowledge of the integration of the full range of knowledge.

2.2.3. Classroom fact sheets

(1) Situation introduction, questioning to enlightenment

Teachers first use multimedia to present the news: low altitude "new energy" outlines a new picture of air traffic, the successful test flight of the flying taxi, not only means that the low altitude economy from science fiction to reality, but also marks a major innovation in China's automotive technology.

Teacher: Now suppose that the horizontal distance between the flying car and the obstacle in front of it is measured to be 20 km during the test flight, and the angle of elevation is 15°. As shown in **Figure 2**, how many kilometers should the flying car continue to travel in the direction of the angle of elevation in order to avoid the obstacle?

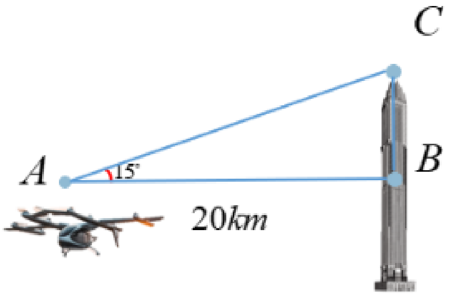

Figure 2. Schematic diagram of the situational model

Student constructs a triangle solution using previously learnt trigonometric functions, to get $AC = \frac{20}{\cos 15^\circ}$

Teacher follows up with: cos 15° equals what?

(Students cannot get the result with their existing knowledge, to stimulate the desire to know, and then introduce the topic)

Design intent: A good learning context is like a link between students' cognition and the real world [1]. This link is based on the major innovation of new kinetic vehicle technology, combined with the practical, through the integration of technical knowledge, create a mathematical learning context, establish a bridge between the reality of concrete problems and mathematical abstraction (TCK), in order to arouse the students' curiosity and activate their thinking.

(2) Hands-on, bold conjectures

Question 1: What does cos 15° equal?

Teacher: Can we use the trigonometric values of the special angles we have learnt to transform them?

Student: cos 15° can be converted to cos (60°–45°) or cos (45°–30°).

Teacher's follow-up question: cos (45°–30°) What is it equal to again?

Student: guessed cos (45°–30°) = cos 45° – cos 30°, but found that the two sides have different signs, so obviously the equation does not hold.

Teacher: If we generalize the data, this problem can be abstracted into the expansion of cos (α–β). First from the special angle intuition, using the previous learning of the induction formula, you can quickly say, cos (90°–30°) cos 90° cos 30° + sin 90° sin 30° results?

Student: both equal to 1/2.

Teacher: What can you find out from this? With this conjecture, please complete **Table 2**, and conjecture cos (α–β) the expansion of how, independent thinking and then group discussion.

Table 2. Form

cos (60°–30°)	cos 60°	cos 30°	sin 60°	sin 30°
cos (120°–60°)	**cos 120°**	**cos 60°**	**sin 120°**	**sin 60°**

Students fill out the form and find out: cos (60°–30°) = cos 60° cos 30° + sin 60° sin 30°

cos (120°–60°) = cos 120° cos 60° + sin 120° sin 60°

Conjecture after collaborative enquiry: cos (α–β) = cos α cos β + sin α sin β

Design intention: Through problem 1, students are guided to review special trigonometric values, and abstract from the concrete to find the cosine formula of the difference between any two angles, to develop students' thinking from the special to the general. The students' curiosity is aroused after experiencing the wrong conjecture. Through independent thinking, group work, and communication, improve the table, gradually formed a conjecture on the cosine formula of the difference between two angles (PCK).

(3) Rigorous derivation, rigorous argumentation

Question 2: How do you prove cos (α–β) = cos α cos β + sin α sin β?

Teacher: Please recall that the previous study of trigonometric functions was carried out in the unit circle, so we might as well analogize the idea and start to explore. As shown in **Figure 3**, the unit circle intersects the positive semi-axis of the *x* axis at the point *A*(1,0), and the non-negative semi-axis of the *x* axis is the starting side of the angles, α β, and their terminal edges intersect the unit circle at the points P_1, A_1 [6]. $\angle A_1OP_1$ How much is it equal to? How do you make it in the unit circle?

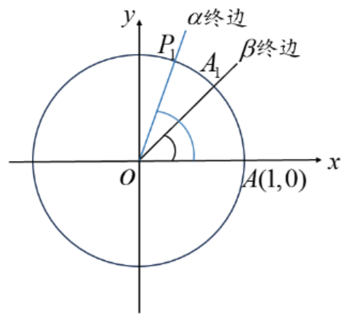

Figure 3. The unit circle intersects the positive semi-axis of the *x* axis at the point *A*(1,0)

Student: $\angle A_1OP_1 = \alpha - \beta$, you can make the starting side of this angle coincide with the non-negative half-axis of the *x* axis.

Teacher: How is the final side determined? As shown in **Figure 4**. Teacher shows students an animation to guide the observation of the characteristics of the terminal side of an angle.

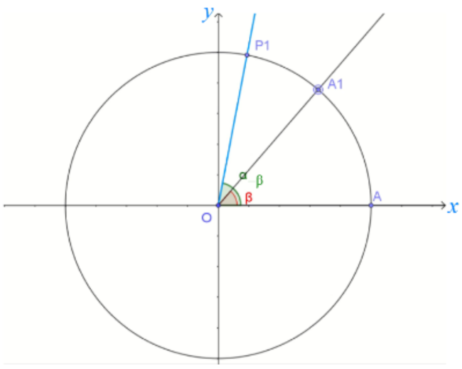

Figure 4. To determine the final side

Student: Find out that by rotating the sector A_1OP_1 around the point *O* clockwise by the angle β, A_1 coincides with the point *A*, where the point P_1 is located, which is the intersection of the terminal side of the angle α–β with the unit circle.

Teacher: According to the definition of trigonometric function, can you express the coordinates of these three intersection points? Further, connect *AP* A_1P_1, observe the graph, what equivalence relationship can you find?

Student: $P_1(\cos\alpha, \sin\alpha)$ $A_1(\cos\beta, \sin\beta)$ $P(\cos(\alpha-\beta), \sin(\alpha-\beta))$, because the sector *AOP* is rotated from the sector A_1OP_1 around the point *O*, according to the rotational symmetry of the circle, we can get the arc *AP* is equal to the arc A_1P_1, and then get $AP = A_1P_1$.

Teacher: Next, please think about it, can we use the coordinates of the point to express the equivalence?

Student: using the formula for the distance between two points, substituting gives,

$[\cos(\alpha-\beta) - 1]^2 + [\sin(\alpha-\beta)]^2 = (\cos\alpha - \cos\beta)^2 + (\sin\alpha - \sin\beta)^2$

Simplifying gives: $\cos(\alpha-\beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$

Teacher: In particular, when is substituted into the equation, it can be found that the equal sign also holds, so we have completed the proof of the equation.

Design intention: By guiding students to recall their past experience of studying trigonometric functions in the unit circle, and analogizing the investigation of cos (α–β) unfolding proofs, we also aim to strengthen the connection within the trigonometric function knowledge system (CK). In the process of determining the terminal side of the angle α–β, the teacher plays an animation to assist the teaching, to visualize the abstract geometric dynamic process of rotating the angle and determining the terminal side (TCK), to inspire the students to observe and think on their own, so that they can better participate in the subsequent derivation and argumentation, which is in line with the teaching principle of guiding in-depth thinking by intuitive perception. In addition, the problem-oriented approach guides students to find the equivalence relationship expressed in coordinates, gradually clarifies the ideas and methods of proof, and develops students' independent thinking and problem-solving ability (PK).

(4) Generalizing formulas, returning to contexts

The teacher guides the students to observe the cosine formula for the difference between two angles:

For any angles, α β, there are cos (α–β) = cos α cos β + sin α sin β.

This formula gives the relationship between the sine and cosine of an arbitrary angle α β, and the cosine of its difference angle α–β, known as the cosine of difference formula, abbreviated as $C_{(\alpha-\beta)}$.

Students are guided by the teacher to conclude that: the signs on both sides of the equation are positive and negative, and opposite, the right-hand side of the equation is a multiplication of trigonometric functions of the same name and then addition, the cosine is in the first place and the sine is in the second place, and the angles α β are arbitrary.

Teacher: Let's go back to the problem we encountered at the beginning, please do some calculations to find out how much cos 15° is equal to?

Student: you can convert it to cos (45°–30°), and then substitute it into the formula you learnt today to get the result.

Design intention: To guide students to observe the specific form and characteristics of the formula for the cosine of the difference of two angles, aiming at developing students' ability to summarize and think logically (PCK). This helps students to deeply understand the connotation of the formula for the cosine of an angle of difference, strengthens their memory of the structure and properties of the formula, enables them to accurately grasp the conditions for the application of the formula in different situations, and transforms the formula from an abstract mathematical expression into a comprehensible and usable knowledge tool. Finally, the teaching strategy of contextual regression (PK) was adopted. This enables students to apply the difference cosine formula they have just learnt to real problems immediately, consolidates their understanding and application of the formula in practice, makes students feel the practicality of the new knowledge, and at the same time checks whether students have really mastered the application of the formula to deepen their learning impression.

(5) Learning by applying, deepening new knowledge

Teacher: After learning the cosine formula of the difference between two angles, students think about it, what is the connection between this and the induction formula we learnt earlier? Please use the formulas learned today to prove the following two induced formulas.

Prove it using the formula $C_{(\alpha-\beta)}$:

$$(1)\ cos\left(\frac{3\pi}{2}-\alpha\right)=-\,sin\,\alpha\,;$$

$$(2)\ cos\,(-\alpha)=cos\,\alpha\,.$$

After the students have proved it, the teacher will point out that the induced formula is in fact a specialization of the cosine of difference formula, and the cosine of difference formula is also a generalization of the induced formula.

Design intention: To guide students to think about the connection between the cosine formula of the difference of two angles and the induction formula they learnt before, and to let students use the formula they learnt to prove the induction formula. This helps students to establish a close connection between the new knowledge (the cosine formula of the difference of two angles) and the old knowledge (the induction formula), and improve the construction of the trigonometric knowledge system in students' minds (CK). Finally, it is pointed out that the induction formula is a specialization of the difference angle cosine formula, and the difference angle cosine formula is a generalization of the induction formula. This summarizing link helps students to improve their ability to summarize and think logically by comparing and contrasting the relationship between the two formulas. Students can clearly understand the position and role of different formulas in the mathematical knowledge system, and learn to shift their thinking from the particular to the general and from the general to the particular, further optimizing their mathematical thinking (PCK).

(6) Traceability, cognitive expansion

Teacher: After feeling the charm and use of new knowledge, we cannot help but look back to the source of human wisdom, from the ancient Greek period to the modern period, countless mathematicians like Pythagoras Kepler and other explorations and attempts, so that we can stand on the shoulders of giants to continue to move forward, as shown in **Figure 5**.

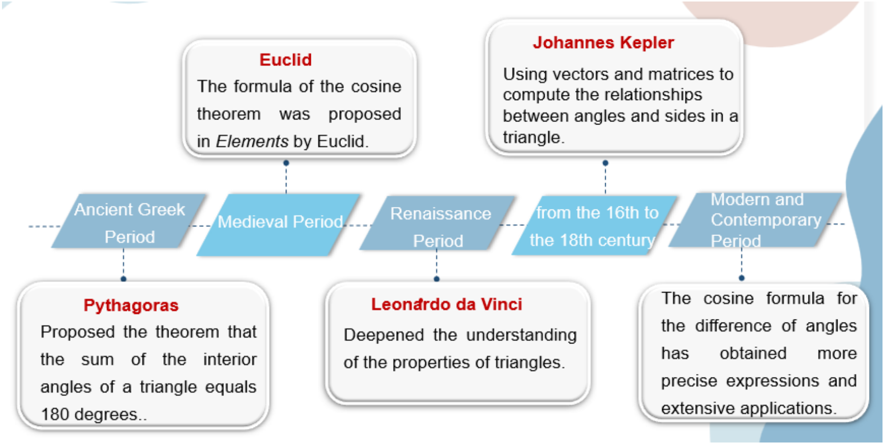

Figure 5. Development history

Design intention: Through the multimedia courseware to show the contribution of ancient mathematicians, make the teaching images more vivid (TCK). At the same time, students will understand that the knowledge they have learnt is not created out of thin air, but has a deep historical origin and the wisdom of many mathematicians, which enriches their knowledge of trigonometric functions and makes them understand that these formulae carry the long-term exploration of human beings behind them, thus expanding the cultural connotation of subject knowledge.

3. Conclusion

The study of propositional teaching in high school mathematics under the framework of TPACK can bring many insights into teaching practice. Teachers need to deeply understand the interconnectedness and logical structure of mathematical knowledge, which is the basis for effective propositional teaching. In the design process of teaching mathematical propositions, the teacher firstly creates an intuitive teaching situation by integrating the technical knowledge of mathematical propositions (TCK), and then triggers the students' desire for knowledge; secondly, the teacher uses the integrated knowledge of mathematical propositional teaching method (PCK) to pose a series of questions, and then enables the students to complete the discovery, demonstration, formation, and application of formulas and theories during problem-solving by means of independent investigation and cooperative group investigation. On this basis, teachers can use the Geometry Board or GGB software to demonstrate the dynamics of straight lines, ellipses, hyperbolas, and other geometric shapes, which also requires teachers to have pedagogical knowledge of integrated technologies (TPK). The TPACK framework requires that teachers have the ability to integrate technology with mathematical propositions and pedagogies in order to develop students' mathematical literacy in problem solving, creative inquiry, self-direction, information use, and systems thinking.

Disclosure statement

The author declares no conflict of interest.

References

[1] Gao Y, Zhou Y, 2020, Teaching Design of Mathematical Concepts in the Perspective of TPACK—Taking "Angle of Inclination and Slope of a Straight Line" as an Example. Research on Secondary Mathematics (South China Normal University), (18): 2–5.

[2] Xu M, Li H, 2025, Realistic Dilemmas and Realisation Strategies of Teachers' Digital Literacy Cultivation under TPACK. Teaching and Management, (18): 53–57.

[3] Guo F, Gao J, Zhu H, 2025, Research on the Influencing Factors and Enhancing Mechanism of Teacher Trainees' Digital Competence Based on TPACK Theory. Shanghai Educational Research, (02): 16–23 + 73.

[4] Nie L, 2021, Design of High School Mathematics Proposition Teaching under the Concept of STEAM Education. China Education Technology Preparation, (07): 78–81.

[5] Tang T, 2023, A Study on the Design of High School Mathematics Teaching Based on the TPACK Framework—Taking "Exponential Function" as an Example. Mathematics Learning and Research, (23): 140–142.

[6] Hu Z, 2023, Focusing on Formula Derivation for Effective Teaching—The Teaching Design of "the Cosine Formula of the Difference between Two Angles." Secondary Mathematics Teaching Reference, (06): 23–25.