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The Evaluation of Alternative Risk Control Schemes Based on Cumulative Prospect Theory

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Abstract: Based on the analysis of the evaluation problems associated with the risk control scheme for major engineering projects, the evaluation method of the risk control scheme considering the irrational behavior of evaluation members in fuzzy random environment is proposed. Firstly, a maximum entropy model corresponding to any evaluation member is established by using triangular fuzzy random variables and grey correlation coefficient in order to obtain the weight of each risk factor of the member. Secondly, a nonlinear programming model is established according to the principle of minimizing deviation to estimate the weight of different evaluation members on the evaluation of alternative risk control schemes. Lastly, the cumulative entropy model is used to calculate the weight of risk control schemes. Cumulative prospect theory obtains the comprehensive prospect utility value of each alternative to determine the optimal alternative.

Keywords: Major project; Risk assessment; Cumulative prospect theory

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1. Introduction

The implementation of major engineering projects is a typical high-risk operation, therefore, the safety management of major engineering projects is exceptionally challenging [1]. The implementation unit of major engineering projects should actively carry out the construction of risk investigation and control system in accordance with the relevant national and local requirements. The evaluation of the risk control scheme is an important link in the process of risk investigation and control, and the selection of a reasonable risk control scheme is of great significance for enterprise security management.

The expert scoring method was used to represent the preference information of evaluation objects [2]. This method is easier to use, but in the actual operation process, evaluation members often prefer to give higher scores in risk evaluation, resulting in low accuracy of conclusions obtained by this method [3]. The analytic hierarchy process (AHP) was introduced to require evaluation members to make pound-to-pair comparisons of evaluation objects, which improved the consistency of evaluation conclusions [4]. However, there are many uncertain and complex information in the decision-making process of the risk control scheme, which increases the difficulty of the popularization and application of AHP and network analysis [5]. Some scholars have introduced the fuzzy method and multi-attribute decision-making method to study the evaluation of risk control schemes [6]. The decision-making method based on fuzzy theory can better reflect the fuzziness of preference information, and the decision-making method based on multi-attribute is very consistent with the multi-factors of the evaluation of risk control schemes, and the conclusions obtained are more in line with the actual situation. These methods of evaluating risk control schemes are becoming new research topics.

It should be further considered that empirical expectations are often present in the evaluation of risk control schemes of major engineering projects by evaluators. In the actual evaluation process of risk control schemes, the importance and relevance of each key indicator are the key factors affecting the determination of the weight of the indicator. In addition, the re-evaluation process is complicated by varying circumstances of different evaluation members with correspondingly distinct levels of importance, which increases the difficulty in forming the evaluation conclusion. However, the traditional expected utility theory cannot explain the reference dependence existing in the evaluation process of evaluators. Therefore, the prospect theory is applied in this paper to solve the above problems [7]. In this paper, by using triangular fuzzy random variable, we can accurately describe the relationship of redundancy, complementarity, and preference among evaluation indicators under different alternative schemes, and use the cumulative prospect theory to describe the different risk preferences and reference dependencies of evaluation experts, so as to select the best risk control scheme for major engineering projects.

2. Proposed approach

2.1. Estimating marginal expectation and variance decision matrix of alternative risk control schemes for major engineering projects

Assuming that the evaluation value of evaluation indicators of alternative risk control schemes by evaluation members is triangular fuzzy random variable, a decision matrix corresponding to evaluation members is constructed:

$$\begin{cases} \chi^k = [\chi_{ij}^k(\beta_{ij}^k)]_{m\times n} \\ \chi_{ij}^k(\beta_{ij}^k) = [\gamma_{ij}^k(\beta_{ij}^k) - a_{ij}^k, \gamma_{ij}^k(\beta_{ij}^k), \gamma_{ij}^k(\beta_{ij}^k) + b_{ij}^k] \end{cases} \tag{1}$$

The expectation and the variance of the triangular fuzzy random variable are determined by (2) and (3), respectively.

$$E^k = [E(\chi_{ij}^k)]_{M\times N} \tag{2}$$

$$V^k = [V(\chi_{ij}^k)]_{M\times N} \tag{3}$$

According to the corresponding decision matrix, the expectation and the variance of the triangular fuzzy random variable are determined as $E(x_{ij}^k)$ and $V(x_{ij}^k)$, respectively. The expected mean value of alternative risk control schemes for major engineering projects can be obtained by (4):

$$\overline{E}_{ij} = \frac{E(\chi_{ij}^1) + E(\chi_{ij}^2) + L + E(\chi_{ij}^l)}{l} \tag{4}$$

2.2. Obtaining the evaluation member weights of alternative risk control schemes for major engineering projects

The maximum entropy model based on the objective weight of the grey correlation coefficient is used to solve the weight of the evaluation members on different evaluation indicators.

The grey correlation coefficient matrix corresponding to the weight of key indicators can be obtained by equation (5):

$$\xi^k = \begin{bmatrix} \xi_1^k(1) & \xi_1^k(2) & \ldots & \xi_1^k(N) \\ \xi_2^k(1) & \xi_2^k(2) & L & \xi_2^k(N) \\ M & M & O & M \\ \xi_M^k(1) & \xi_M^k(2) & L & \xi_M^k(N) \end{bmatrix} \quad (5)$$

According to the decision matrix of each evaluation members, the weight variance of evaluation members for alternatives is determined as:

$$D_2^k(i) = \frac{1}{N}\sum_{i=1}^{N}\left(\pi_j^k(i) - \frac{1}{N}\right)^2 \quad (6)$$

Assuming that the weight of the evaluation members for the alternative project is w_k. According to the principle of the deviation minimization between the comprehensive expected value of each alternative and the mean of the comprehensive expected value of the evaluation team, a deviation minimization model is established as follows to determine the weight corresponding to each evaluation member:

$$\min R = \sum_{i=1}^{M}\sum_{k=1}^{L}\left[w_k\left(M(E_i^k) - \overline{M}(E_i^k)\right)^2\right] \quad (7)$$

$$\begin{cases} M(E_i^k) = \sum_{j=1}^{N}\pi_j^k E[\chi_{ij}^k]; \\ \overline{M}(E_i^k) = \sum_{j=1}^{N}\pi_j^k \overline{E}_{ij}; \\ \sum_{k=1}^{L} w_k = 1; \end{cases}$$

Through solving the above model, the weight vector of the key indicators of the risk control scheme of major engineering projects corresponding to the evaluation members is determined as $\pi^k = \{\pi_1^k, \pi_2^k, \cdots, \pi_N^k\}$.

2.3. Determining the combined marginal utility of alternative risk control schemes

According to the value function formula in prospect theory, the positive prospect value function and negative prospect value function of triangular fuzzy random variables are defined as:

$$v^+\left(\xi_{ij}^k\right) = \left(E(\xi_{ij}^k) - \mu_j^k\right)^\alpha \quad (8)$$

$$v^-\left(\xi_{ij}^k\right) = -\lambda\left(\mu_j^k - E(\xi_{ij}^k)\right)^\beta \quad (9)$$

In (8) and (9), α and β represent the degree of perception of "gain" and "loss" of each index by members of the risk control scheme evaluation of major engineering projects, respectively.

According to the weight vector of key indicators, the marginal value of each alternative corresponding to the evaluation members is defined as:

$$u_i^k = \sum_{j=1}^{p} v^+\left(\xi_{ij}^k\right)\pi_j^k + \sum_{j=p+1}^{N} v^-\left(\xi_{ij}^k\right)\pi_j^k \quad (10)$$

According to the decision weight of members, the comprehensive marginal utility of each evaluation member for alternative solutions can be obtained as follows:

$$U_i = \sum_{k=1}^{L} w_k u_i^k \quad (11)$$

Lastly, each alternative scheme can be properly sorted to obtain the best alternative by the triangular fuzzy random variable order relation.

3. Steps of selecting the alternative risk control scheme

In order to make full use of water resources in a certain area of southwest China, a major project was approved by relevant government departments. An evaluation team including three safety management experts was set up to select the risk control scheme, which has preliminarily drawn up four alternative risk control schemes, and the evaluation team needs to select an optimal risk control scheme from the four schemes for risk management. The selection is according to four indicators: personnel allocation, personnel behavior habit, usage of equipment, and organization and management of project implementation.

Step 1: The evaluation members evaluated the performance of alternative risk control schemes on various evaluation indicators using triangular fuzzy random variables with real value random variables.

Step 2: By (2) and (3), the expectations and variances of the evaluation members for the four alternative risk control schemes are calculated respectively, then the expected mean for the evaluation team is available by (4) as follows.

$$E^1=\begin{bmatrix}0.7887 & 0.6685 & 0.7282 & 0.6398\\ 0.8197 & 0.7513 & 0.8062 & 0.5880\\ 0.7950 & 0.7346 & 0.7253 & 0.5899\\ 0.7478 & 0.7067 & 0.7093 & 0.5893\end{bmatrix},\ V^1=\begin{bmatrix}1.6598 & 2.4375 & 1.8249 & 2.6276\\ 1.4888 & 1.7977 & 1.5887 & 2.9941\\ 1.5646 & 1.9066 & 1.8523 & 2.7600\\ 1.8526 & 2.0174 & 1.9627 & 3.1705\end{bmatrix},\ E^2=\begin{bmatrix}0.8025 & 0.7267 & 0.8300 & 0.7044\\ 0.7526 & 0.6782 & 0.8514 & 0.6423\\ 0.8408 & 0.7446 & 0.7899 & 0.5789\\ 0.7296 & 0.6498 & 0.8075 & 0.6136\end{bmatrix},\ V^2=\begin{bmatrix}1.5639 & 2.0202 & 1.3955 & 2.0758\\ 1.7457 & 2.1908 & 1.4670 & 2.2829\\ 1.4410 & 1.9072 & 1.6153 & 2.9598\\ 1.9060 & 2.2808 & 1.5662 & 2.7285\end{bmatrix},$$

$$E^3=\begin{bmatrix}0.7694 & 0.6611 & 0.7937 & 0.6594\\ 0.7196 & 0.6869 & 0.7682 & 0.6943\\ 0.7652 & 0.6828 & 0.7726 & 0.6088\\ 0.7842 & 0.6548 & 0.7067 & 0.6289\end{bmatrix},\ V^3=\begin{bmatrix}1.7697 & 2.3419 & 1.5884 & 2.3728\\ 1.9056 & 2.0750 & 1.8279 & 2.0470\\ 1.6674 & 2.1653 & 1.7441 & 2.7945\\ 1.6922 & 2.2808 & 1.9883 & 2.5297\end{bmatrix},\ \overline{E}=\begin{bmatrix}0.7869 & 0.6954 & 0.7840 & 0.6679\\ 0.7639 & 0.7055 & 0.8086 & 0.6415\\ 0.8003 & 0.7113 & 0.7626 & 0.5925\\ 0.7539 & 0.6704 & 0.7412 & 0.6106\end{bmatrix}.$$

Step 3: In order to determine the subjective weights of the evaluation members on the four evaluation indicators of the project risk control scheme, the following nonlinear planning corresponding to the first evaluation member was established as follows by (5):

$$\max T^1=-\left(\pi_1^1\ln\pi_1^1+\pi_2^1\ln\pi_2^1+\pi_3^1\ln\pi_3^1+\pi_4^1\ln\pi_4^1\right)$$

$$s.t.\begin{cases}\pi_1^1+\pi_2^1+\pi_3^1+\pi_4^1=1;\\ 0.1891<\pi_1^1<0.2703;\\ 0.1442<\pi_2^1<0.3674;\\ 0.2149<\pi_3^1<0.2911;\\ 0.2387<\pi_4^1<0.2664;\\ 0.0001<\dfrac{1}{4}\sum_{j=1}^{4}\left(\pi_j^1-\dfrac{1}{4}\right)^2<0.0063.\end{cases}$$

Using the nonlinear programming software package of Matlab to solve the above model, the subjective weights of the evaluation members on the four evaluation indicators can be obtained as follows:

$$\pi^1=(0.2515\ \ 0.2500\ \ 0.2504\ \ 0.2481),\ \pi^2=(0.2506\ \ 0.2487\ \ 0.2551\ \ 0.2456),\ \pi^3=(0.2536\ \ 0.2485\ \ 0.2524\ \ 0.2455).$$

Step 4: Based on the expectations and subjective weights of the above three assessment members, the comprehensive marginal expectation and the average comprehensive marginal expectation of the assessment members, and the four alternative risk control schemes were obtained as shown in **Table 1**.

Table 1. The combined marginal expected value and the mean combined marginal expected value of the alternative

	$M(E_1^k)$	$M(E_2^k)$	$M(E_3^k)$	$M(E_4^k)$	$M(E_5^k)$
$k=1$	0.6993	0.7337	0.7043	0.6815	0.7047
$k=2$	0.7666	0.7322	0.7396	0.7011	0.7351
$k=3$	0.7216	0.7175	0.7082	0.6944	0.7105

Step 5: According to the principle of the deviation minimization between the comprehensive expected value of each alternative risk control scheme and the mean of the comprehensive expected value of the evaluation team, a deviation minimization model is established to determine the evaluation weight of the evaluation members to the alternative risk control scheme.

$$\min R = 0.0014w_1^2 + 0.0022w_2^2 + 0.0004w_3^2$$

$$\begin{cases} w_1 + w_2 + w_3 = 1 \\ w_1 > 0, w_2 > 0, w_3 > 0 \end{cases}$$

Using the nonlinear programming software package of Matlab to solve the above model, it can be obtained that the evaluation weights of each evaluation member on the alternative risk control schemes of water conservancy projects are $w_1 = 0.3833$, $w_2 = 0.3325$, $w_3 = 0.2843$.

Step 6: According to the experience and preference of evaluation members, the psychological expectation reference point matrix of evaluation indicators of alternative risk control schemes for the project is determined as follows:

$$\mu = \begin{bmatrix} 0.7878 & 0.7153 & 0.7423 & 0.6018 \\ 0.7814 & 0.6998 & 0.8197 & 0.6348 \\ 0.7596 & 0.6714 & 0.7603 & 0.6479 \end{bmatrix}.$$

By using the value functions (8) and (9) of the prospect theory, the prospect value function matrix of the triangular fuzzy random variable corresponding to the evaluation members' key indicators of alternative risk control schemes is estimated as follows:

$$V^1 = \begin{bmatrix} -0.0047 & 0.0676 & 0.0235 & -0.1266 \\ -0.1085 & -0.1207 & -0.2000 & 0.0231 \\ -0.0293 & -0.0697 & 0.0277 & 0.0203 \\ 0.0589 & 0.0152 & 0.0497 & 0.0211 \end{bmatrix}, V^2 = \begin{bmatrix} -0.0754 & -0.0934 & -0.0401 & -0.2156 \\ 0.0441 & 0.0342 & -0.1079 & -0.0304 \\ -0.1875 & -0.1463 & 0.0454 & 0.0790 \\ 0.0739 & 0.0716 & 0.0207 & 0.0337 \end{bmatrix}, V^3 = \begin{bmatrix} -0.0384 & 0.0178 & -0.1130 & -0.0442 \\ 0.0589 & -0.0575 & -0.0318 & -0.1509 \\ -0.0235 & -0.0439 & -0.0469 & 0.0577 \\ -0.0863 & 0.0271 & 0.0761 & 0.0306 \end{bmatrix}.$$

Step 7: The marginal utility decision matrix corresponding to each evaluation member regarding the four alternative risk control schemes is determined by (10).

$$U^1 = [-0.0100\ \ \text{-}0.0998\ \ \text{-}0.0131\ \ 0.0358], U^2 = [-0.1053\ \ \text{-}0.0154\ \ \text{-}0.0524\ \ 0.0499], U^3 = [-0.0447\ \ \text{-}0.0444\ \ \text{-}0.0145\ \ 0.0116].$$

Step 8: The comprehensive prospect utility vector of the evaluation team for the four alternative risk control schemes is estimated by (11) as follows: $U = [-0.0532\ \ \text{-}0.0533\ \ \text{-}0.0267\ \ 0.0324]$.

From the comprehensive prospect utility ranking of the four alternatives, it can be seen that the fourth alternative risk control scheme has the highest comprehensive prospect utility value, and the comprehensive prospect value is greater than 0, indicating that the scheme can meet the expectations of the evaluation members on the whole.

4. Conclusion

Aiming at the irrationality and fuzzy randomness of evaluation members, this paper proposes a risk control scheme selection method for major engineering projects based on cumulative prospect theory and triangular fuzzy random variables. Firstly, the triangle fuzzy random variables are used to describe the interrelation of each evaluation index, and the marginal expectation and variance decision matrix of alternative risk control schemes for major engineering projects are obtained. Secondly, a maximum entropy model corresponding to any evaluation member is established by using the grey correlation coefficient to obtain the evaluation index weight corresponding to the evaluation member. Thirdly, based on the principle of deviation minimization, a nonlinear programming model is established to estimate the weight of different evaluation members to the evaluation of alternative risk control schemes for major engineering projects. Lastly, the cumulative prospect theory is used to obtain the comprehensive prospect utility values corresponding to each alternative risk control scheme of the evaluation team to determine the optimal alternative.

Disclosure statement

The authors declare no conflict of interest.

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