International Education Forum, 2024, Volume 2, Issue 11
http://ojs.bbwpublisher.com/index.php/IEF
Online ISSN: 2981-8605

A Blended Teaching Approach to Linear Algebra

Haiyan Liu*

College of Jincheng, Nanjing University of Aeronautics and Astronautics, Nanjing 211156, China

***Corresponding author:** Haiyan Liu, ziluolan7812@sina.com

Abstract: Linear algebra is an important fundamental course for university students in technology and science. However, most students think linear algebra is very abstract. To give students a much better learning effect, we chose a blended teaching method that uses both offline and online resources. By analyzing the characteristics of linear algebra, the aim of teaching, and the characteristics of human understanding. Concrete teaching methods and some linear algebra examples are displayed in this paper.

Keywords: Linear algebra; Blended teaching; Online and offline teaching

Online publication: December 24, 2024

1. Introduction

Linear algebra is an important fundamental course for university students in technology and science. However, linear algebra textbooks lack concrete application examples, some of the definitions lack background context, and the lesson time is limited [1,2]. Therefore, the traditional teaching according to the textbook seems very abstract. The traditional teaching method is limited. It no longer adapts to the new requirements of new situations which need more talents who have much better research ability, innovative ability, and application ability. Additionally, the traditional method is not beneficial for the development of personal quantity. Much research has been done to improve the teaching method. One of them is blended teaching [3,4]. Blended teaching uses two kinds of resources: online and offline. With these abundant resources, we can adequately utilize the provided information and software. By analyzing the characteristics of linear algebra, the teaching goals we aim to achieve for students, and the nature of human understanding, we propose a blended teaching approach for linear algebra. Before proposing the specific blended teaching approach, we will first discuss the considerations involved.

2. Consideration of blended teaching for linear algebra

In this section, we describe the considerations that serve as guiding principles for blended teaching in linear

algebra. These considerations include the characteristics of linear algebra, the teaching goals we aim to achieve for students, and the nature of human understanding.

2.1. Characteristics of linear algebra

Linear algebra is derived from studying the linear system of equations. It primarily deals with linear relationship problems. The primary study objects involve vectors, vector space, linear transformation, and linear systems of equations with finite dimensions. Linear algebra can be widely used in computer science, physics, economy, biology, pattern recognition, data science, circuit analysis, and so on. Next, we will analyze the characteristics of linear algebra.

Firstly, from the application domain of linear algebra, it is clear that linear algebra has abundant practical applications. It can provide colorful examples.

Secondly, the linear algebra textbook lacks application examples, and some definitions in the textbook lack background and meaningful explanations. As a result, linear algebra appears very abstract to students. For example, consider the concept of matrix rank: why is the highest order of the algebraic minors of a matrix that is not equal to zero defined as the rank of the matrix? If the teacher simply presents the definition without explanation, it can seem very abstract. Students need to grasp the underlying meaning of the concept to fully understand it.

Thirdly, in practical applications, the matrix may have huge order and the linear system of equations will have many variables or many equations. Students should have the ability to choose the right software to program.

2.2. The teaching aim

The teaching aim acts as our guide. The primary teaching problem is what kind of person we hope to cultivate. For the aspect of individuals, we regard the students as vivid people, so we hope to stimulate and lead students to the route of self-development.

For the aspect of society, we are in the artificial intelligence age. Nowadays, there is a need for more innovation. We hope to cultivate within the students the capability of research, innovation, application, and cooperation. The aim is for individuals and society to supplement each other.

2.3. The characteristics of human understanding

As humans, we are willing to study the things that interlink with us. The formation of thinking is from concrete to abstraction and then from abstraction to concrete. According to these aims and characteristics, we design the strategy of blended teaching.

3. The blended teaching method for linear algebra

According to the aims and characteristics mentioned above, in our blended teaching, we will design some questions to lead students to think over, to discover the rules of the questions, and to verify the rules they are provided with. Then, we hope to reinforce their recognition through further exercises, reading, and studying. Finally, we hope that students can solve some practical problems or find more rules with the knowledge they have gained.

We divide the study process into three stages: the stage before class, the stage during the class, and the

stage after class. The ideas mentioned above will run through all of these study processes. Next, we will describe the concrete teaching method and examples used in every stage as follows.

3.1. The stage before class

The stage before class is designed to help students understand the relevant background knowledge, the origin of the problem, its historical development, and stories about the mathematicians involved. This stage encourages students to think critically and draw preliminary conclusions related to the concepts they will learn.

To reach this aim, we can give the students a self-study list. This can aid students to prepare more effectively. The list contains some tasks such as exercises, discussions, and some resources that need students to read.

Online classrooms such as Rain-Classroom can be used to give the students some resources to prepare lessons before class. The resources may contain PowerPoint presentations (PPT), videos, resources from the Internet, documents, and other exercises that can push students to think about the relevant knowledge in advance. For example, before starting class on the topic of determinants, some resources will be given in advance to the students.

Firstly, some practical examples and exercises that can be solved using a linear system of equations will be uploaded online. The examples are as follows.

(1) Example 1: We can use the linear system of equations and computed tomography X-ray to get the quality of some human brain organs.

(2) Example 2: We can use the linear system of equations to solve some chemical problems, such as balancing chemical equations.

Through these practical applications, students can appreciate the diverse and colorful applications of linear systems of equations in different domains. Next, we will provide students with two exercises designed to help them think critically about better methods for solving linear systems of equations.

(1) Exercise 1: Students will solve five linear systems of equations, each consisting of two equations with two variables, using methods they learned in high school. Then, we will pose a follow-up question: What should we do if there are 100 such systems to solve? This encourages students to consider whether there are faster and more efficient ways to handle large numbers of equations.

(2) Exercise 2: Students will solve a specific set of equations, as follows:

$$\begin{cases} a_{11}x_1 + a_{12}x_2 = b_1 \\ a_{21}x_1 + a_{22}x_2 = b_2 \end{cases} \quad (1)$$

We will then prompt students to think about whether the solutions from **Exercise 2** can help solve these 100 equations more efficiently. Furthermore, we will ask them to consider ways to remember or reuse the solutions to improve efficiency.

The purpose of **Exercise 2** is to guide students in understanding the necessity of using ready-made solutions to tackle large systems of equations and exploring strategies for retaining and applying these solutions. By preparing these examples and exercises online, students can better understand the concept of determinants and their role in solving linear systems of equations.

3.2. The stage during class

In this stage, the teacher's task is to guide students in forming concepts and conclusions based on the problems we study. Students will realize that definitions and theorems are not rigid facts to be memorized; rather, they are concepts to be discovered.

In this way, we can cultivate students' scientific literacy, the spirit of exploration, and the ability to cooperate by discussing in teams. For example, in the section on inverse-matrix, the concrete teaching process will be divided into three steps.

(1) Step 1: In this step, we will provide students with some background on the application. Specifically, we sometimes need to encrypt a matrix. A matrix can store various types of information, and to keep it confidential, we may want to encrypt the matrix. The teacher can have students take on the role of cryptographers and think about the best way to encrypt an information matrix.

The teacher can organize students into teams for discussion. Drawing on all the matrix calculations they have learned, students can discuss the problem and submit their answers online. By reviewing the answers posted online, everyone will see that there are different possible approaches. The class can then further discuss which method involves the most complex calculation, ensuring the highest level of security for the matrix through matrix operations.

Through the discussion, students will conclude that matrix multiplication is the most complex operation in matrix calculations, making it the safest method for encrypting a matrix. Let the information matrix be X (named as plaintext matrix), the encryption matrix be A, and the encrypted matrix be C (referred to as ciphertext matrix). Then, we can get the calculation $AX = C$ or $XA = C$.

(2) Step 2: The next question is how to decode the matrix. In this step, students can learn methods to solve an unknown problem by comparing it with the knowledge they are already familiar with.

The teacher can remind students that matrix multiplication is similar to number multiplication. If we want to decode, we can draw a parallel with number multiplication. This understanding will allow us to decode the matrix, $X = A^{-1}C$ or $X = CA^{-1}$.

(3) Step 3: To define the inverse matrix, in this step, the teacher helps students understand that when we create new concepts in research, we define these new concepts. Therefore, we need to define the inverse matrix A^{-1}.

In this stage, we see that we define the inverse matrix through the process of solving a practical problem. Through this training, students will understand that we can create definitions or conclusions based on our needs during problem-solving. We also observe that online classes facilitate more efficient interaction. Additionally, if students have questions or new ideas, they can promptly write them down in the online class, where they will be displayed in the PowerPoint presentations. The teacher can then make relevant adjustments based on these questions in real-time. Other students may gain new insights from these questions or ideas. Moreover, using the online class platform, the teacher can assign exercises online and quickly monitor the learning progress of all students.

3.3. The stage after class

The stage after class is crucial for students to digest, consolidate, and reinforce their understanding of the knowledge they gained in class. We will provide an example to demonstrate what is done after class. The example focuses on the section about eigenvalues and eigenvectors of matrices. We will give three tasks for

the students to complete.

(1) Task 1: We will give students some exercises to calculate the eigenvalue and eigenvector. It will give students a chance to familiarize themselves with the definition and the calculation.

(2) Task 2: We will let students calculate the eigenvalue and eigenvector using software such as Matlab. In this way, the students can use proper software to calculate.

(3) Task 3: Through online platforms, students will be given some documents on the application of eigenvalue and eigenvector. For example, documents on "Shallow Discussion of the Application of Eigenvalues and Eigenvectors," "Economics Perspective of Understanding Eigenvalues and Eigenvectors," and so on [5,6]. These documents can be found in academic journal databases such as CNKI, Wan Fang Data, Google Scholar, and so on. By reading these documents, students can further master the essence and applications of eigenvalues and eigenvectors.

4. Conclusion

In summary, we presented the concrete flow of the blended teaching method for linear algebra. Through the use of abundant online resources and proper teaching design, we can help students master the course content. By studying this course, we can also cultivate students' research abilities, innovative thinking, application skills, and teamwork. Our next task is to establish a comprehensive resource database and examples for each concept in the linear algebra course. **Figure 1** illustrates the overall concept of using the blended teaching method for linear algebra.

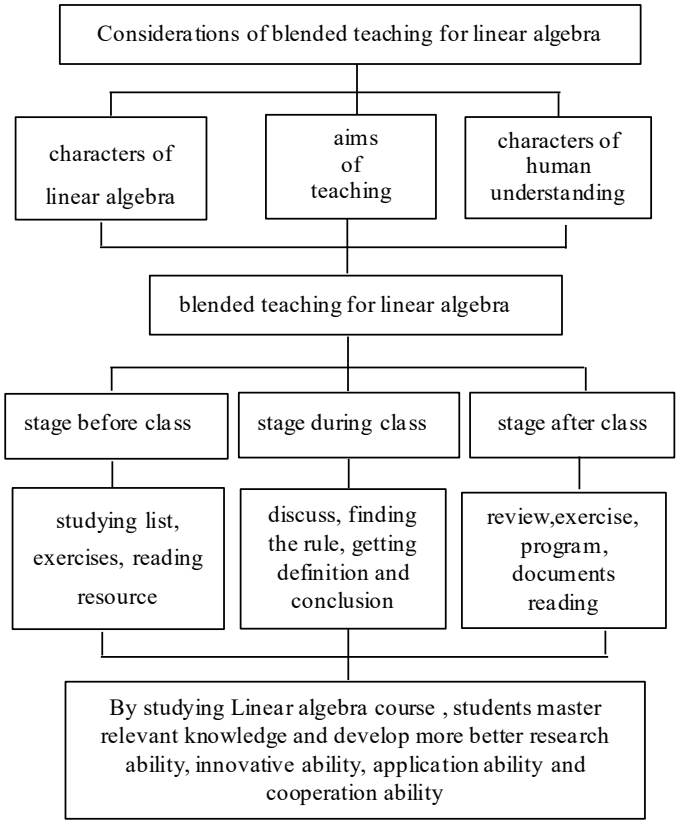

Figure 1. Blended teaching method for linear algebra

Disclosure statement

The author declares no conflict of interest.

References

[1] Xiao JP, Du XC, 2024, Phytophysiology Blended Teaching Design and Exploration based on Rain-Classroom. Journal of Higher Education, 10(1): 103–106.

[2] Zhu Q, 2022, The Application of Blended Teaching for Linear Algebra. Computer Knowledge and Technology, 18(6): 175–177.

[3] Tongji University School of Mathematical Sciences, 2023, Linear Algebra. Higher Education Press, Beijing, China.

[4] Lay DC, Lay SR, 2023, Linear Algebra and Its Application. China Machine Press, Beijing, China.

[5] Wang YX, Feng WS, 2020, Shallow Talk the Application of Eigenvalue and Eigenvector. Mathematics Learning and Studying, 10: 10–11.

[6] Wei X, Wang JQ, 2024, On Economics Perspective of Understanding Eigenvalues and Eigenvectors. Studies in College Mathematics, 27(3): 80–82.