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Mathematical Problems in the Case of a Curriculum: Analysis based on Context and Contextualization

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Abstract: The objective of this study is to analyze the context and contextualization of problems from the case of the Costa Rican curriculum developed in the 2012 reform. The mixed methodology consists of a qualitative content analysis and a subsequent quantitative account. A system of categories is built from a literature review leading to the study of types of context and types of contextualization. The 59.5% out of the total of 141 problems has a scientific/mathematics context, with contextualization of the active type in only a few of them. Moreover, contexts of the rural and indigenous types are absent. We conclude therefore that some disconnections exist between the theoretical curricular basis and the problems exemplified. Finally, we propose the further discussion of our categories as indicators for the design of mathematical problems with contextualization of the active and significant types.

Keywords: Context and contextualization of mathematical problems; Mathematical curriculum; Theoretical foundations of the curriculum; Active and meaningful contextualization

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1. Introduction

For more than six decades, authors such as Puig (1955) [1] have indicated the need to present school mathematics in relation to natural and social life. Moreover, since the 1990s, several studies have highlighted the importance of the historical-cultural context in the construction of mathematical knowledge and have conceived mathematics as a cultural product, the result of people's activities [2,3]. Despite this, we start from the assumption that the experience of educational curricula in different parts of the world is still far removed from the realities and cultures of learners.

Several theoretical approaches to mathematics education have been interested in the importance of the context of the problems presented in the classroom. The so-called Realistic Mathematics Education, for example, proposes the teaching of mathematics connected to reality, so that it is close to the learner and relevant to society [4]; thus, mathematics is conceived as a human activity of recognizing and solving problems in the environment. The so-called Culturally Relevant Teaching puts the focus on the learner's experience and culture; this implies starting from cultural and socially relevant situations for the development of critical thinking [5].

For the so-called Ethnomathematics Programme [6], multiple mathematical practices emerge from everyday activities that are permeated by context and culture. Thanks to this programme, instructional aids, materials and the possibility of linking content and creating curricula that take into account the context and cultural values of mathematics have emerged [7]. Fuentes (2013) indicates that ethnomathematics can be brought into the classroom and contribute to teachers in the planning, execution and evaluation of their teaching practice by considering, in mediation activities, that the social world and the culture in which the learner lives are crucial for their training [8]. Note that we indicate with Ethnomathematics, capital and singular, the research programme, and with ethnomathematics, lowercase and plural, the various cultural mathematics that are the object of study of the Programme. Along the same lines, Peña (2014) [9] considers that in formal education it is necessary to contextualize mathematics, since:

> ... mathematics disconnected from history, from other knowledge, and from the environment, has naturally led us to ignore students' mathematical knowledge, which has had pedagogical implications for the potential development of the mathematical thinking of students from culturally homogeneous or diverse classrooms. (p. 175)

Because of the importance we attach to the local context, we frame our research in Ethnomathematics, without forgetting contributions from other approaches to the notions of context and contextualization. This is ongoing research in the framework of the first author's doctoral thesis research.

Costa Rica, the country whose curriculum is the subject of our research, has been attentive to these educational changes. In 2012, it implemented a reform with the implementation of a new mathematics curriculum that materialized in the Mathematics Study Programmes document [10], which emphasizes problem solving linked to the learner's physical, social and cultural environment. For the director of this reform, the contextualization of mathematics plays a determining role [11]; in his analysis of the first years of implementation of the reform, he insists on the relevant role of real contexts in the teaching of mathematics, indicating that "experiences close to real, everyday life" should be encouraged (Ruiz, 2013, p. 53) [12]. Somewhat later, the international interest in this curriculum reform is demonstrated by Planas (2015), who characterizes it as a perspective of praxis in mathematics education that should be examined in depth [13].

The Mathematics Programmes of Study is a 518-page document, which is provided to every teacher, both primary and secondary, and covers grades one to eleven. The document contains information relating to each thematic unit (relations and algebra, statistics and probability, numbers, measurement and geometry), structured according to three elements: knowledge, specific skills and specific indications. The latter are intended to provide teachers with concrete examples of tasks and problems to present in the classroom and are a close reference material for teachers in the country.

In our work, we analyzed the context and contextualization of the mathematical problems present in the specific indications of the Costa Rican Study Programmes for the third and fourth cycles (from seventh to eleventh grade). In particular, we analyzed the connection between the theoretical foundations on active contextualization set out by the MEP (2012) [10] and the examples proposed in the specific indications. That is, we examine whether what is stated in the curriculum rationale is applied in the proposed examples. At the same time, we address the presence of problems with meaningful contextualization, based on the ethnomathematical perspective presented in Albanese *et al*. (2017) [14].

2. Mathematical problems, context and contextualization

Unlike exercises, mathematical problems require a high cognitive demand from the solver as they do not have an algorithmic procedure that leads directly to the solution [15]. The Mathematics Syllabus of the MEP (2012) [10] does not go into the differentiation between exercise and problem, so we focus on the analysis of problems assuming that they can indeed be considered as such.

The relationship between mathematics and the context is intrinsic to the historical development of mathematical knowledge that has been constructed in each society and culture in order to respond to emerging problem situations [6]. Niss (1995) [16] points out the importance of mathematics in society according to its applications to: (1) applied science, as mathematics is the foundation of scientific areas such as physics, engineering, biology and economics, each of them in turn with socially transcendent applications and implications; (2) specialized practices, through which decisions and predictions are made for social, economic and natural phenomena; and (3) the non-specialized practices of everyday life. In this respect, and considering that education is framed in a social context, mathematics is not detached from values, interests, nor from ideological, political, economic, and cultural circumstances. Regarding the notion of context in mathematics education, and in accordance with Niss (1995) [16], Planas and Alsina (2009) [17] suggest that the situation that generates questions or problems that require the activation of mathematical knowledge and practices should be taken into account. Thus, the context encompasses situations that make sense to the learner and fosters his or her critical mathematical thinking, in line with Gutstein *et al.* [5]

Another approach to the notion of context is that of Ramos and Font (2006), who analyzed it by differentiating between two types in mathematics [18]. On the one hand, the context "as a particular example of a mathematical object" (p. 532) and, on the other hand, the context that allows framing this object in the environment. Hence, the context can be 1) a purely mathematical entity, for example, the context of vector spaces, or 2) the situation of the environment where a particular content is developed. This second conception of context (which is the one we take in this study), is indicated as ecological use, since the mathematical content will have a different meaning or use, depending on the place, social group or time of reference.

With slightly different nuances, Nuñez and Font (1995) refer to contextualization in mathematics education as "working with concepts in different concrete contexts in order to achieve, on the one hand, their significance and functionality and, on the other, to facilitate the processes of abstraction and generalization" (p. 293) [19]. Thus, the contextualization of a mathematical problem is intrinsically related to the ecological use of the context in which it is framed.

2.1. Relevance of the context

There are several classifications of contexts in mathematics problems. We review the ones we took into account to generate the system of categories. Rico (2006) analyses the contextualization of mathematical problems in the so-called phenomenology analysis, as part of content analysis [20]. Based on the proposal of the OECD (2004) [21] and in relation to Niss (1995) [16], he classifies the contexts of problems according to the situation in which they are framed:

Personal. Referring to the learner's daily activities (non-specialized practices).

Educational and occupational. Relating to a school or work environment (specialized and non-specialized practices).

Public. Concerning the community and important events in public life (specialized practices).

Scientific. Concerning the understanding of a technological process, a theoretical interpretation or a mathematical problem (applied science).

In this research, in addition to the above types, we consider other categories for the contexts of mathematical problems. First, we take 'Costa Rican context' since the Mathematics Study Programmes of the MEP (2012) [10] emphasize that the curriculum should respond to the national reality. Referring to Costa Rica, it is made clear that "the use of problems as generators of the organization of lessons offers valuable opportunities to connect with the needs of our country" (p. 19). This encourages the presentation of problems that come from the Costa Rican environment and idiosyncrasies. Secondly, we take 'rural context' and 'urban context''. Aroca (2003) expresses the need to consider mathematics coming from both rural and urban areas of the countries [22]:

> 'Many of the rural and urban ethnomathematics are an essential part of the cultural reserve of a country because they are unique, because of that historical accumulation, because of the region where they develop, because of the type of values they preserve, because of the cultural systems that give them meaning, because of their traditional way of being transmitted that involves one or all the senses.' (p. 116).

This researcher stresses the importance of bringing to the classroom the contexts, both urban and rural, where mathematics is developed. We consider urban context to be that related to the city or town; rural context to be that which involves country life; and we add 'indigenous context', which refers to that which contemplates aspects of the life of some native group of the country, given that they usually present different characteristics with respect to non-indigenous rural contexts.

At the time of the study, according to the most recent report of the *Instituto Nacional de Estadística y Censo* (National Institute of Statistics and Census) (2012), the urban population in Costa Rica was 72.8% and the rural population 27.2%. The indigenous population corresponds to 2.4% of the inhabitants, i.e., 104,143 indigenous people, of which around 80% live in rural areas [23].

2.2. Relevance of contextualization

The Mathematics Programmes of Study of the MEP (2012) explain that contextualization strengthens the active role of the learner, engaging them in their learning through the use and design of mathematical models [10]. A difference is established between what was worked on in past curricula as contextualization, and what is now called active contextualization. In contrast to mathematical problems that are "decorated" with everyday elements that are dispensable, this new alternative proposes that mathematical problems present contexts that are necessary to be solved:

> Design problems taken from the press, from the school, from the community, from the classroom, from the Internet. The same traditional "problems" that appear in many textbooks (as an appendix) can be enriched if they are placed in the perspective of modelling and used to build higher cognitive skills (p. 96).

For Ruiz (2017), a problem is artificially contextualized when the context provided is not necessary for its resolution [24]. This author conceives these contexts and the contextualization to which they give rise as ornaments and proposes not to use them. Freudenthal (2002) has already explained in a similar way that focusing on context as "noise" disturbs the clarity of the mathematical message [4].

In MEP (2012), it is argued that active contextualization is possible with modelling and by manipulating real environments, according to each educational level [10]. At the same time, the use of history is suggested to make connections in a natural and realistic way, considering not only the contributions of the so-called Western cultures, but also those produced in America.

In Ruiz's (2017) analysis of the implementation of the Mathematics Programmes of Study and related to the contextualization of problems, it is written: "situations [...] must have meaning and not be artificial, [avoiding] abstract mathematical situations disguised by means of a real context" (pp. 72-73) [24]. In order to assess whether a problem is well contextualized (which is interpreted as equivalent to the so-called active contextualization), Ruiz states that:

> [T]he real context situations must be related to the mathematical task that is proposed in a precise way .... Is the context necessary to perform the task? If the task proposed does not resolve the challenges of the context or if it could be dispensed with in order to carry it out, or if its results do not contribute something significant to the context, what is pursued with the disciplinary axis is not achieved. The key must be the search for intellectual involvement of the student through situations that will allow him/her to develop his/her mathematical skills and abilities (p. 74).

We understand a mathematical problem with active contextualization as one in which the context is essential to solve the problem, i.e., it provides meaning to the mathematical concepts and contents involved in the solution and is realistic or imaginable. In other words, the problem in which the context is necessary to understand its formulation and to propose its resolution.

2.3. Meaningful contextualization

D'Ambrosio (2008) argues that "contextualizing mathematics is essential for everyone" (p. 92) [6]. Ethnomathematics as a research programme allows "understanding mathematical knowledge/doing throughout human history, contextualized in different interest groups, communities, peoples and nations" (p. 17). If mathematics is developed in the culture of a given social group, its study must be based on the reality of the group where the teaching and learning process takes place. In this line, a contextualized mathematical problem, from the point of view of Ethnomathematics, is one that responds to a situation that arises in the culture or reality close to the learner. In other words, the problem statement is somehow necessary for the reality in which it is framed.

The notion of meaningful contextualization is also based on the notion of task authenticity [25]. An authentic task is one that starts from: 1) an event that has occurred or has a high probability of occurring, 2) a question that matches a context outside the school, 3) information and data that are realistic or imaginable. In Albanese *et al.* (2017) from an ethnomathematical perspective, the contextualization of tasks becomes meaningful when the situation created for the problem is analogous to one that may actually arise in a real context (which coincides with our interpretation of the authenticity criterion), and the mathematical concepts and procedures are put into practice in a way similar to how people facing that problem in reality would do it [14].

We consider a problem with meaningful contextualization as one that arises from a real context, where the mathematical concepts and procedures are put into practice in a similar way to how the group facing the problem in reality would do it. This definition goes beyond authenticity as the problem statement and its solution must have meaning for the suggested reality. According to Freudenthal (2002), when contexts are familiar and close to the learner, they are starting points for their mathematical practice, thus promoting their common sense and cognitive strategies, until they move towards more formalized levels [6].

3. Methodology and methods

Our methodology is mixed, based on a qualitative content analysis supported by quantitative methods of data synthesis. With regard to content analysis, as Bardin (2012) we understand it as a "set of techniques for analyzing communications" (p. 23) [26], which leads to obtaining indicators through systematic procedures. The document analyzed is entitled *Programas de Estudio de Matemáticas* of MEP (2012) and focuses on specific indications [10].

Cabrera (2005) explains, within content analysis, the importance of establishing categories, pointing out that it is "one of the basic elements to take into account in the elaboration and distinction of topics from which information is collected and organized" (p. 64) [27]. For this reason, we synthesized the most relevant constructs, emanating from the bibliographic study, in order to elaborate a series of deductive categories that gave rise to subcategories for the analysis of each problem.

The system of two categories (context and contextualization) and subcategories was adjusted during the analysis with some eliminations due to insufficient information in the specific indications. We present the categories and subcategories that were used, indicating the documents from which they were extracted:

- Contexts: personal, educational/occupational, public, scientific [20,21,10]; urban, rural, indigenous (expanded from Aroca, 2013) [22].
- Contextualization: active, artificial [24,10]; meaningful and non-significant [25,14].

We analyzed the 141 problems in the punctual indications of the Mathematics Study Programme of the MEP (2012) of the III and IV cycle of formal education (ages between 12 and 17 years), in order to place them in subcategories. 35 are in the area of numbers, 40 in geometry, 35 in relations and algebra and 31 in statistics and probability, according to the document itself.

Within the 'context' category, we have the subcategories 'personal', 'educational/occupational', 'public' and 'scientific', as well as the subcategories 'urban', 'rural' and 'indigenous', and finally 'Costa Rican'. For the personal, educational/occupational and public context problems (scientific context problems are excluded, since, by definition, they are not linked to a context close to everyday life), we consider the category 'contextualization', analyzing whether it is 'active' or 'artificial'. To do so, we pose the following question: is the context provided necessary to solve the problem? If the answer is positive, we conclude that the problem is actively contextualized; if not, we conclude that it is artificially contextualized. In turn, problems with active contextualization are classified according to the subcategory 'meaningful' or 'not meaningful'. Here we ask: is the problem encountered as such in real life, and does the problem question reflect a situation in reality? If the answers are positive, we conclude that the problem is of meaningful contextualization.

The definition of the categories was supported by the judgement of two experts involved in the Ethnomathematics Programme (María Elena Gavarrete and Natividad Adamuz-Povedano) who were asked about the system of subcategories. Thanks to their contributions: a) a subcategory that was not initially contemplated was created, the subcategory 'Costa Rican' for context; b) the subcategory 'simulated' for context was eliminated due to the lack of information in the problems analyzed to address this aspect; c) the definition of the subcategory 'meaningful' for contextualization was clarified, associating it with the notion of authentic task.

4. Results

Below we present the results of the content analysis and the quantitative count of the problems proposed in the mathematics syllabus prompts [10] according to our categories and subcategories. We provide examples of how the analyses were carried out.

4.1. Results on problem contexts

With respect to the classification given in PISA (OECD, 2004) [21], of the specific indications of the Mathematics Study Programmes in Costa Rica, the scientific context is the most present with 59.5% of the problems. Within the scientific context, two types of problems stand out: those that correspond to the application of a formula and those that develop mathematical knowledge within the discipline. In the first case, most of these problems allow linking mathematics with other areas of knowledge, such as physics, chemistry, biology or finance. **Figure 1** shows the absolute values of the classification of the problems analyzed.

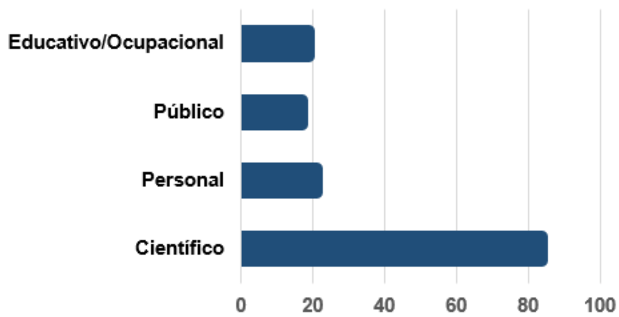
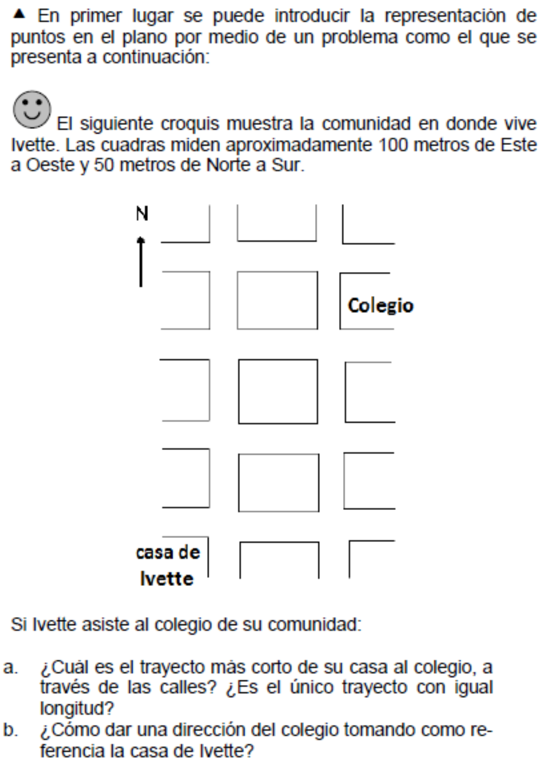


Figure 1. Absolute distribution of context type according to PISA

An example of a problem with a scientific context is presented in the area of relations and algebra: “The temperature in degrees Fahrenheit is a function of the temperature in degrees Celsius and is modelled by the equation $F(C) = 9/5C + 32$. Express C as a function of F” (MEP, 2012, p. 413) [10]. Here a connection to physics is noted. A second example is related to the Pythagorean theorem, where knowledge such as distance between points and classification of triangles according to measures of angles and sides are integrated: “Given the following coordinates of the vertices of a triangle A(2,1), B(6,5) and C(9,2), classify the triangle according to the measure of its angles and the measure of its sides” (MEP, 2012, p. 317) [10].

Within the personal contexts, there are problems of shopping, borrowing, recreational activities and recipes, especially in the area of numbers. On the other hand, public contexts are more present in the area of statistics and probability, where examples that promote analysis, decision-making and encourage discussion of civic, health and current issues stand out. With respect to occupational contexts, the problems correspond to the calculation of prices, earnings and situations that may arise in some occupations. In the educational context, the problems are mostly statistics and probability.

Of the 141 problems, 22 have been classified according to whether the context is urban, rural or indigenous. In others, despite the fact that a setting close to the learner, such as gambling, is involved, it was not possible to establish a geographical area. Of these 22 problems, none corresponds to a rural or indigenous environment. The reality suggested in the PSM examples always corresponds to urban contexts. These specific indications may be distant from the reality experienced in non-urban areas. In rural areas it is not common to have a layout of streets and avenues, or the use of “blocks” to name 100 meters of distance, so a problem like the one in **Figure 2** is not close to a learner living in the countryside. Even in most urban areas, it is difficult to have a road and housing layout like the one in the sketch.


▲ En primer lugar se puede introducir la representación de puntos en el plano por medio de un problema como el que se presenta a continuación:

El siguiente croquis muestra la comunidad en donde vive Ivette. Las cuadras miden aproximadamente 100 metros de Este a Oeste y 50 metros de Norte a Sur.

Si Ivette asiste al colegio de su comunidad:

a. ¿Cuál es el trayecto más corto de su casa al colegio, a través de las calles? ¿Es el único trayecto con igual longitud?
b. ¿Cómo dar una dirección del colegio tomando como referencia la casa de Ivette?

Figure 2. Problem with public and urban context. Source: MEP (2012, p. 306) [10]

Another example is that of a problem at a “farmer’s market” (a market for vegetables and fruit usually on the streets at weekends), where a list of products for sale is presented with their respective prices and the amount of money spent on certain purchases is requested (MEP, 2012, p 276) [10]. This is close enough for some urban dwellers, who from a young age go with their families to buy vegetables and fruits and make calculations to verify that they have enough money. But it is not close enough for inhabitants of indigenous areas, where the use of money is almost non-existent and barter is still used, this being a differential element between indigenous and non-indigenous rural realities.

Of the total number of problems, 26 (18.43%) have information that can be categorized according to the Costa Rican reality. Of these, 21 mention Costa Rica in some way. On the other hand, a group of problems is characterized by providing data from local institutions such as the Costa Rican Petroleum Refinery, the Costa Rican Social Security Fund or the National Production Council. Recreational activities, public services or demographic data are also mentioned. In four problems (out of the 21 referring to Costa Rica), although the country is mentioned, the situation described does not correspond to the Costa Rican reality. For example, in the following problem, Costa Rica is mentioned, but in order to solve the problem it must be assumed that the roads are straight, which is not the case in Costa Rica.

> Carolina leaves her house and goes to her mother's home 2 km south of hers. After greeting her and talking to her, they inform her that her brother Andrés (who is studying abroad and has not visited his family for more than 5 years) has arrived in Costa Rica and is in his house, 750 m north of her mother's house, so they go to welcome him. Considering Carolina's house as a reference point: a. Determine its current location in metres. b. Determine the distance in metres (MEP, 2012, p. 282) [10].

Another example problem contains information about animals that live outside Costa Rica, so the local population is not familiar with this context.

> The yak is an animal that lives in the mountains of Tibet at about 5000 m above sea level and the sperm whale lives 5900 m lower. Determine the altitude at which the sperm whale usually lives (MEP, 2012, p. 280) [10].

4.2. Results on the contextualization of problems

In the theoretical foundations of the Mathematics Study Programmes of the MEP (2012), the importance of working on problems associated with real, physical, social and cultural environments is reiterated (p. 13) [10] and "an active contextualization that stimulates student action, which requires the significant use of models of the reality close to them" (p. 36) [10] is proposed. From the above, it is important to analyze whether the problems presented have an active contextualization or whether, on the contrary, they are artificially contextualized. To classify the problem according to an active contextualization, one must ask whether the context is necessary to solve the problem, as established by the MEP (2012) [10] and Ruiz (2017) [24]. If the answer is negative, it is an artificial or forced contextualization.

Once those with a mathematical/scientific context were discarded, 57 problems were analyzed according to active or artificial contextualization. Of these, 45 were actively contextualized and 12 were artificially contextualized. In other words, just over 21% of the problems have a forced context, which is not necessary to give meaning to the situation posed, nor to provide a solution to it.

The problems in **Figure 3** and **Figure 4** provide a necessary context for their resolution, so there is active contextualization. The information on Cocos Island (shape and dimensions to be consulted on a map) is essential to approximate its area. On the other hand, the context is necessary to solve the problem where it is necessary to know the relationship between the dimensions of the room and the tiles, so that the learner can apply the division algorithm.

Calcule el área aproximada de la Isla del Coco, utilizando algún mapa de Costa Rica.

La idea es que se visualice la Isla del Coco como un cuadrilátero (por ejemplo: rectángulo) y, tomando en cuenta la escala del mapa, se aproxime su área. También, para una mejor estimación se podría dividir el mapa en varias figuras de áreas conocidas (triángulos, trapecios, cuadrados, rectángulos, etc.) y comparar los diferentes resultados del grupo. Con este ejercicio se estimula la creatividad.

▲ Se puede trabajar en subgrupos de la clase y comparar las medidas para ver quiénes dan la mejor aproximación.
Nota: La isla del Coco tiene aproximadamente 7,6 km de largo y 4,4 km de ancho, por lo tanto su área es aproximadamente 33,44 km^2.

Figure 3. Problem with active contextualization. Source: MEP (2012, p. 305) [10]

Don Manuel va a poner losetas en el piso de una habitación que mide 4 metros por 3 metros, las losetas miden 30 cm por 15 cm. Se van a colocar de forma análoga a lo que se ve en la figura, con el lado mayor de la loseta paralelo al lado mayor de la habitación.

Las losetas pueden cortarse para que encajen en los extremos de cada fila de ellas. Don Manuel le dio las dimensiones a su hijo y éste compró 135 losetas. Si no se quiebra ninguna, ¿le alcanzarán estas losetas a don Manuel?, ¿le sobrarán?, si es así, ¿cuántas? ¿Cuántas filas de losetas habrá que colocar?, ¿cuántas losetas por fila?

Figure 4. Problem with active contextualization. Source: MEP (2012, p. 277) [10]

Problems with artificial contextualization are those that present descriptive data (related to other sciences or geographical data), which are not necessary for their resolution. If these descriptive data are removed, the problem could be presented with a mathematical context. The following problem is an example:

> Mount Everest (the highest mountain in the world) is 5,413 metres higher than the Irazú volcano (one of the highest points in Costa Rica). If the sum of their heights is 12,283 metres, state an equation to calculate the height of each of them. (MEP, 2012, p. 335) [10]

The ability to pose and solve problems in real contexts must be answered here. It can be seen that the context is limited to indicating that there is a known quantity (5,413) and an unknown quantity, the sum of which is 12,283. The context could be left out and it could be worded as follows: the sum of two numbers is 12,283 and one of them is 5,413, what is the other number? The contextualization is therefore artificial.

A further point of note highlights a possible connection between linear equations, art and history, while "confirming the usefulness of mathematics in various areas of life" (MEP, 2012, p. 336) [10]. However, historical and artistic data are not essential to solve the problem.

> A very famous painting is the Mona Lisa by the artist Leonardo da Vinci. This painting is located in the Louvre Museum in Paris, France. The painting is rectangular in shape and its height is 24 centimetres more than its width. The perimeter of the painting is 260 centimetres. Calculate the height and width of the painting (MEP, 2012, p. 336) [10].

The contextualization of the above problem is artificial, as it is a mathematical problem with contextual details unnecessary for its solution. It could be solved with wording such as: determine the height and base of a rectangle whose height is 24 cm greater than the length of its base.

A problem has meaningful contextualization for participants of a group if the situation can be presented in their daily or working life and if the question is in line with a problem that could be faced in reality. Of the 45 problems classified as having active contextualization, 6 have meaningful contextualization and the other 39 (86.6%) have no meaningful contextualization. In order to decide the problems with meaningful contextualization, we analyzed whether they would be presented and solved in this way in the reality of the group involved. For these problems the learner is expected to do some mathematical or modelling work [16]. **Table 1** summarizes the problems with meaningful contextualization found.

Table 1. Problems with significant contextualization

Knowledge	Subject	Problem summary
Combined operations	Buyer	A young woman goes to the farmer's market and performs various calculations to determine the total amount she needs to pay.
Order of integer numbers	Businesspeople	Through a profit and loss graph of a company, comparisons need to be made.
Trigonometric ratios	Mason	The measurement of a ramp needs to be determined in order to comply with Accessibility Law 7600.
Equation of a line	Entrepreneur	An entrepreneur is looking for the selling price of a product, knowing the production costs per unit and initial investment.
Measures of position	Student	A student has grades for the categories assessed in a course and needs their weighted average.
Random sampling	Health program organizer	An organization needs to randomly select 15 ninth-grade students for a health program.

Figure 5 illustrates a problem with meaningful contextualization. A learner may find himself/herself in a situation like the one described in the problem. Although the grades are given from 0 to 10 and not from 0 to 100 as in secondary school, in several universities in Costa Rica, this scale is used. For these analyses, it is important to place oneself in the position of the subject of the problem, in order to verify whether the subject will indeed use a mathematical method such as the one suggested; otherwise, it is not a meaningful contextualization. For example, **Figure 6** presents a situation that, while it may be familiar (a bathing resort), poses a problem that in the real context would not be of interest. A bather would not calculate, in the way proposed in the statement, the number of centimetres that must be lowered before the water covers his shoulders, especially as the proposed measurements are not available in a swimming pool.

Una estudiante de la universidad obtuvo las siguientes calificaciones en un curso de Matemática, para una calificación de 0 a 10:

Pruebas	Calificaciones
Primer examen corto	6,00
Segundo examen corto	5,50
Tercer examen corto	6,50
Proyecto	6,00
Primer parcial	7,50
Segundo parcial	8,50

a. Los exámenes cortos tenían un valor de 5% cada uno, el proyecto valía 15% y los exámenes parciales 35% cada uno. Si la nota mínima de aprobación es un 7,00, ¿la estudiante aprobó el curso?

Figure 5. Problem with meaningful contextualization. Source: MEP (2012, p. 434) [10]

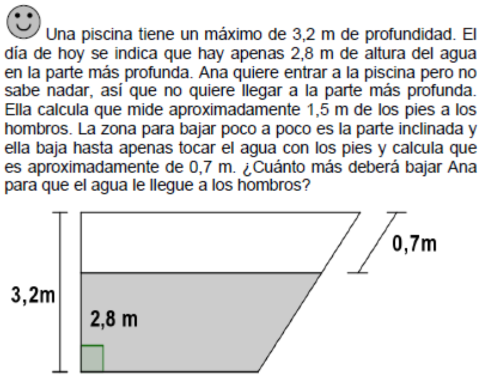


Figure 6. Problem with non-significant contextualization. Source: MEP (2012, p. 313) [10]

Another problem with non-significant contextualization is exemplified in the following excerpt, where he considers buying a ladder to climb up to the roof of his house.

> Diego needs to buy a ladder to climb onto the roof of his house. The roof is 97 inches high. In order to have good stability on the ladder when leaning against the wall, the legs of the ladder should be between 30 and 40 inches apart. What would be the approximate size of the ladder? (MEP, 2012, p. 315) [10]

As written in the associated skill, the idea is to introduce the Pythagorean theorem. The situation described has a real-life context; however, in everyday life, the Pythagorean theorem would not be used. Estimation would possibly be a more appropriate strategy. Moreover, one rarely buys a ladder to use it only in one situation. Another aspect is that the unit of measurement is not from the International System of Units of Measurement (where the metre is the unit of length), the one used in Costa Rica.

In another problem, a tabular representation of a linear function relating the number of kilometres travelled by a taxi to the fare to be paid for the service is requested, knowing that the first kilometre costs ₡550 and each additional kilometre is worth ₡200 (MEP, 2012, p. 331) [10]. In Costa Rica, to calculate a fare, in addition to the mileage, the time taken for the service is considered. Due to the complexity of the measurements, taxi drivers use a taximeter known as “Maria”, which performs this calculation. Neither the driver nor the user makes these calculations manually. Therefore, this problem is also one of non-meaningful contextualization.

We have detected a certain scarcity of problems with meaningful contextualization in the Mathematics Study Programmes of the MEP (2012), where it is explained that “using or applying mathematics within (well-selected) real contexts promotes contact with mathematical objects in their privileged relationship with the reality from which they emerged” (p. 28) [10]. **Figure 7** presents a summary of results.

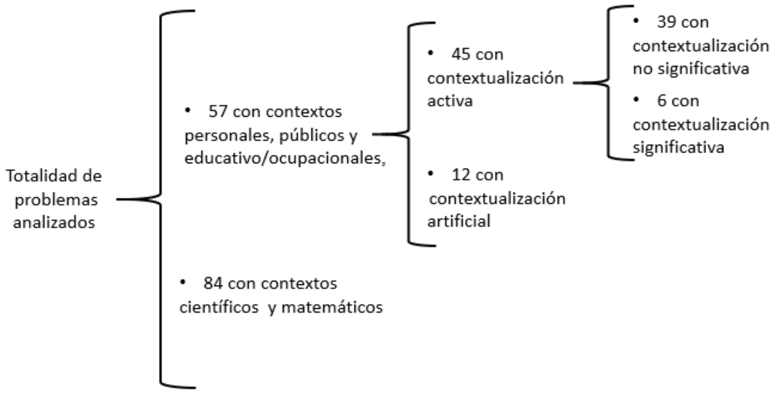


Figure 7. Ranking of problems in absolute values according to context-contextualization

5. Conclusions

Research in mathematics education has long established the importance of taking cultural aspects into account in the understanding and design of teaching and learning processes [2,3]. These efforts are embodied in the use of everyday contexts to teach mathematics. In this line of attention to cultural aspects, the theoretical foundations of the current mathematics curriculum in Costa Rica promote teaching with situations that are close to the learner, promoting the use of real contexts and modelling.

When analyzing the problems in the Mathematics Study Programmes of the MEP (2012), for the III and IV cycle, and going deeper into the active contextualization proposed in the theoretical foundations of this document, we nevertheless identify some mismatches. While the curriculum explains that learning should not be limited to the mastery of sophisticated techniques or abstract structures far removed from the environment, more than 59.5% of the problems belong to mathematical or scientific contexts that do not consider realities close to home. The theoretical foundations of the programme also explain the need to promote mathematics for all, but the examples do not consider problems that respond to the reality of rural or indigenous inhabitants. While it is true that the necessary adaptations must be made, depending on the teaching environment, providing problems with non-urban contexts would enrich the Mathematics Study Programmes in Costa Rica.

Regarding problems with significant contextualization, in the analyses we have shown that this type of contextualization is minimal; instead, there are many problems that are far removed from how they would be addressed in reality. Some problems, with minor modifications, can be transformed to make the contextualization meaningful, which would allow a closer experience of the mathematics of the environment. In others, it is necessary to construct new problems that respond to the skill to be developed, just as it would be used in a real situation. In this sense, it is advisable to encourage research that uses the contexts of the problems proposed by the MEP (2012) [10] as a basis in order to improve those with the potential to show mathematics as a human and social activity in a more natural way, as expressed by Freudenthal (2002) [4], Niss (1995) [16] and D'Ambrosio (2008) [6].

Our research has the limitation of not reporting on how the curriculum directions are put into practice in the classroom, as well as focusing on secondary school ages. The study could then be completed with observations of classrooms in different areas of the country and see how teachers reinterpret the curriculum directions in their teaching practice and with respect to more school ages. In any case, we believe that the current research provides mathematics didactics with a system of subcategories for context and contextualization that allows us to study the intended material curriculum, and can also serve as a guide to analyze problems in other documents such as textbooks and teaching units. The proposed categories and subcategories also provide indicators to support the selection and production of mathematical problems that are articulated through active and meaningful contextualization.

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Disclosure statement

The authors declare no conflict of interest.

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