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An Empirical Study on the Integrated Mathematics Teaching in Junior and Senior High Schools under the 1+3 Training Model — From the Dual Perspectives of Curriculum Reconstruction and Students' Cognitive Development

Longchun Jia*

Liangxiang Affiliated Middle School of Beijing Normal University, Beijing 102488, China

Author to whom correspondence should be addressed.

Abstract: In the traditional segmented structure of junior and senior secondary mathematics education, issues such as discontinuity in knowledge systems, cognitive leaps in learning progression, and misalignment of instructional objectives are commonly observed, which hinder the continuous development of students' mathematical core competencies. Against this background, this study explores an integrated instructional approach based on the "1+3 Talent Development Model" from both curriculum reconstruction and students' cognitive development perspectives. Focusing on key domains including functions, geometry, and probability and statistics, a vertically integrated framework of mathematics curriculum is constructed, characterized by hierarchical knowledge progression, gradual cognitive development, and continuous competency cultivation. Empirical evidence from teaching practice spanning Grade 9 to Grade 10 demonstrates that the proposed model effectively alleviates knowledge fragmentation and cognitive discontinuity, while significantly enhancing students' abilities in function understanding, geometric transfer, mathematical modeling, and logical reasoning. The findings suggest that integrated mathematics instruction across junior and senior secondary levels contributes to the systematic optimization of curriculum design and the restructuring of learning pathways, providing both theoretical support and practical implications for curriculum reform in basic education.

Keyword: 1+3 talent development model; Integrated mathematics instruction; Curriculum reconstruction; Cognitive development; Core competencies; Knowledge transfer; Instructional articulation

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1. Introduction: Why is integration needed? The practical dilemmas and reform opportunities in the transition from junior to senior high school mathematics

Mathematics, as a core subject in the foundational education stage, is closely related to the continuity of its knowledge system and the staged development of students' thinking. However, due to the "natural barriers" formed by the division of academic stages in traditional junior and senior high school mathematics education, there have long been three prominent issues. Firstly, there is a knowledge gap—the content of junior and senior high school textbooks is arranged separately according to academic stages, with a lack of transitional connections for key concepts (such as function definitions and geometric reasoning logic). Secondly, there is a cognitive leap—junior high school focuses on intuitive operations and specific calculations, while senior high school abruptly shifts to abstract symbols and logical deductions, resulting in an overload of students' cognitive load. Thirdly, there is a goal conflict—junior high school teaching emphasizes skill training centered around the high school entrance examination, while senior high school assumes that students already possess the foundational knowledge of core junior high school concepts without verification, leading to a "teaching vacuum." To address this challenge, the "1+3 Training Experiment" emerged: integrating the final year of junior high school (Grade 9) with the three years of senior high school (Grades 10 to 12) into a continuous four-year training cycle, breaking down the division between academic stages through institutional design and providing top-level support for integrated mathematics training. From the perspective of practical curriculum plans, this model transcends the traditional academic stage boundaries of textbooks: the first semester of Grade 9 introduces trigonometric function content from the compulsory course three of senior high school in advance, while the second semester of Grade 9 connects with the foundations of sets and functions from compulsory course one. In Grade 10, the integration of probability, geometric transformations and vectors, complex numbers, and other content not only follows the logical progression of mathematics knowledge from the concrete to the abstract and from the singular to the comprehensive but also aligns with students' cognitive development patterns from concrete perception to abstract reasoning, constructing a step-by-step pathway for the coherent cultivation of core competencies.

2. The pain of traditional transition: An empirical analysis of pain points based on textbook content

To clarify the necessity of implementing an integrated reform, this study first focuses on three typical areas—functions, geometry, and probability—by analyzing the current junior and senior high school mathematics textbooks (primarily those published by People's Education Press) [1]. It identifies specific conflicts in knowledge, thinking, and objectives under the traditional model.

2.1. Discontinuity in the knowledge system: Taking functions as an example

Junior high school function instruction centers on "variable dependency relationships" (e.g., quadratic and inverse proportional functions in the ninth-grade textbook, Volume 1), emphasizing graphical features and basic properties (e.g., symmetry axes, increasing/decreasing intervals) [2]. In contrast, senior high school functions immediately progress to the abstract definition of "correspondence between non-empty sets of numbers" (Chapter 3 of Compulsory Course 1) and introduce rigorous concepts such as domain, range, and symbolic descriptions of monotonicity [3]. For instance, when studying quadratic functions like $y = ax^2 + bx + c$ in junior high, the discussion is limited to how the sign of a affects the parabola's opening direction, without addressing domain restrictions (e.g., composite functions like $y = x^2 - 1$). In senior high, students are required to rigorously define monotonicity using interval notation (e.g., "$\forall x_1, x_2 \in D$, if $x_1 < x_2$, then $f(x_1) < f(x_2)$"), marking a significant cognitive leap. This abrupt shift from intuitive to abstract thinking leaves many students struggling to grasp the essence of abstract symbols in early senior high due to the lack of foundational knowledge about domains and correspondence relationships from junior high [4].

2.2. Abrupt shift in thinking requirements: Taking geometry as an example

Junior high school geometry primarily focuses on intuitive understanding of planar figures (e.g., similar shapes and circles in the ninth-grade textbook, Volume 1), emphasizing inductive reasoning through measurement and drawing. Senior high school geometry, however, shifts to spatial reasoning in solid geometry and logical deduction using spatial vectors (Chapter 11 of Compulsory Course 4 and Chapter 1 of Elective Compulsory Course 1). Under the traditional model, "geometric transformations" in junior high (e.g., translations and rotations, ninth-grade textbook, Volume 2) are taught separately from "plane vectors" in senior high (Chapter 6 of Compulsory Course 2), preventing students from connecting transformations to vector operations. Similarly, the junior high topic of "solving right triangles" (using the Pythagorean theorem and trigonometric values to find sides and angles, ninth-grade textbook, Volume 1) lacks continuity with senior high "solving triangles" (deriving and applying the sine and cosine laws, Chapter 9 of Compulsory Course 4), hindering students' ability to generalize from specific scenarios to universal principles when dealing with non-right triangles [5].

2.3. Conflicts in teaching objectives: Taking probability and statistics as an example

In junior high school, probability only introduces basic concepts (such as random events and simple calculations of classical probability models, found in the second volume of the ninth-grade textbook), with teaching focusing on reinforcing frequency estimation skills through experiments like dice rolling and ball drawing. In senior high school, probability requires students to master abstract models such as sample spaces, event independence, and random variables (Chapter 5 of Compulsory Course II), and to use formulas to calculate complex probabilities. In traditional teaching, junior high school teachers, in response to the senior high school entrance examination, often reduce the time allocated to the infiltration of core concepts (for example, directly providing the formula "Probability of equally likely events = Number of favorable events / Total number of events"). Meanwhile, senior high school teachers assume that students already understand the basic meaning of "sample space" and proceed directly to formula derivation, ultimately resulting in a gap where "junior high school emphasizes skills while senior high school emphasizes theory without foundational support" [6].

3. Integrated practice pathways: Integrated design strategies based on curriculum reconstruction

In response to the aforementioned challenges, the "1+3 Model" designs a step-by-step integrated pathway from three dimensions—knowledge modules, thinking abilities, and core competencies—through systematic reorganization of curriculum content [7]. The specific strategies are as follows:

3.1. Vertical integration of knowledge modules: Taking functions as an example

3.1.1. Stepwise integration of the function module

Following the logic of "laying a foundation in junior high school, establishing transitional connections, and deepening understanding in senior high school", high school concepts are integrated in stages. During the first semester of the ninth grade, when explaining the $y = ax^2 + bx + c$ parameters of quadratic functions, an extended discussion on the impact of the domain is included (e.g., comparing the differences between $y = x^2 - 1$ and functions studied in junior high school), allowing students to intuitively perceive "correspondence relationships" through specific cases. When studying inequalities in the second semester of the ninth grade, connections are made to the maximum and minimum value problems of quadratic functions (for example, using the properties of inequalities to prove the monotonicity of functions in a certain interval), laying the groundwork for the definition-based proof of monotonicity in senior high school. When studying exponential and logarithmic functions in the first year of senior high school, students revisit junior high school power operations and the graphs of inverse proportional functions, deepening their understanding of the essence of functions (the correspondence law between variables) by comparing the growth differences between $y = 2^x$ (exponential growth) and $y = 1/x$ (inverse proportional decay) (Section 4.5 of Compulsory Course II).

3.1.2. Compare the definitions of functions in junior and senior high schools to enable students to clearly perceive the logic of cognitive progression

Table 1 shows the comparison of the definitions of functions in junior and senior high schools.

Table 1. Comparison of the definitions of functions in junior and senior high schools

Dimension	Junior High Cognition (Variable Dependency)	Senior High Cognition (Set Mapping)	Key Transition Focus
Core Concept	"One variable changes with another variable"	"A unique mapping from non-empty number set A to set B"	Abstract progression from "changing relationship" to "mapping relationship"
Domain & Range	Default: x takes all real numbers; Range is not emphasized	Explicitly defined Domain A and Range C ($C \subseteq B$), requiring active determination	Cultivating a "domain-first" mindset
Monotonicity Description	"Increases/Decreases on an interval" (Intuitive observation of graphs)	"$\forall x_1, x_2 \in D$, if $x_1 < x_2$, then $f(x_1) < f(x_2)$" (Symbolic language)	Logical transition from "intuitive perception" to "rigorous proof"
Function Representation	Primarily formulas and graphs; rarely uses the "f(x)" symbol	Formulas, graphs, tables, and the symbolic notation f(x) are used together	Adapting to symbolic language, paving the way for subsequent abstract function learning

3.1.3. Expansion from junior high school trigonometry to senior high school extension: From solving right triangles to trigonometric functions of arbitrary angles and trigonometric transformations

The "Solving Right Triangles" (People's Education Press, Grade 9, Volume 1) in junior high school serves as an introduction to trigonometric functions, limited to acute angles between 0° and 90°, with a focus on "side-angle ratios" (e.g.,). In contrast, senior high school trigonometric functions need to be extended to arbitrary angles, giving rise to trigonometric identities and transformations, which are key tools bridging geometry and algebra. The approach of the 1+3 integrated model is as follows: after completing the teaching of solving right triangles in junior high school in the second semester of Grade 9, without interrupting the learning rhythm, directly use "the expansion of angle range" as a breakthrough point, introduce the unit circle as a tool, and construct a coherent knowledge chain of "acute angles—arbitrary angles—trigonometric transformations", avoiding the gap in traditional teaching where "junior high school trigonometry remains at the computational level, while senior high school trigonometry abruptly becomes abstract" [8].

3.1.4. Connecting derivatives from compulsory modules 1 to 3: Building a comprehensive function knowledge system

Derivatives (People's Education Press, High School Selective Compulsory Module 3) represent a "watershed" in senior high school mathematics, with their core being the study of the rate of change of functions. Content such as the monotonicity, maximum and minimum values, and properties of trigonometric functions (distributed in Compulsory Modules 1 and 3) forms the foundation for learning derivatives. In traditional teaching, Compulsory Module 1 (Sets, Functions), Compulsory Module 2 (Solid Geometry, Plane Vectors), and Compulsory Module 3 (Trigonometric Functions, Solving Triangles) are often taught in isolation, making it difficult for students to connect the knowledge and leaving them lacking a "holistic understanding of functions" when learning derivatives. The key approach of the 1+3 integrated model is: after students have systematically learned Compulsory Modules 1, 2, and 3, without proceeding to other modules, directly delve into derivatives, using "derivatives as a tool to enrich function understanding" and forming a complete knowledge loop of "function definition—function properties—trigonometric functions—derivative applications."

Before learning derivatives, first conduct a "knowledge integration lesson" to review the core content of Compulsory Modules 1 to 3 and clarify their connections to derivatives:

(1) Fundamentals of Functions in Compulsory Course I: Definitions of functions, domains, ranges, monotonicity, and extreme values (derivatives can be used to determine monotonicity, find extreme values, and determine maximum and minimum values, offering higher efficiency compared to the completing-the-square method from junior high and the definition-based method from senior high);

(2) Supplementing the definition of the slope of a straight line and methods for calculating it, laying the groundwork for understanding the geometric significance of derivatives [9];

(3) Trigonometric Functions in Compulsory Course III: Images and periodicity of trigonometric functions (derivatives can be used to analyze the monotonicity, extreme values, periodicity, and maximum and minimum values of trigonometric functions).

For example, when reviewing the monotonicity of exponential functions $y=2^x$ in Compulsory Course I, it could not be proven in junior high school, and the proof using the definition in senior high school is complex. However, derivatives can directly demonstrate that $y=2^x$ is monotonically increasing on R by calculating $y=2^x\ln 2$ (because of $\ln 2>0$, therefore $y'>0$), allowing students to initially perceive that "derivatives are an excellent tool for studying functions" and stimulating their interest in learning.

3.2. Horizontal cohesion in thinking ability: Progressive cultivation from concrete to abstract

3.2.1. Gradual enhancement of mathematical modeling skills

In the second semester of junior high, through the "Growth Pattern Description" project (Section 4.7 of Compulsory Course II), students are guided to fit plant growth data (such as the relationship between height and time) using quadratic functions, providing an initial experience of the process "real-world problem → mathematical model → solution and verification." In senior high, the difficulty is increased by incorporating

"Applications of Functions (Part II)" (Compulsory Course II), using exponential function models to predict population growth and comparing the limitations of quadratic function models (such as differences in long-term trend predictions), thereby cultivating critical thinking in model selection.

3.2.2. Layered training in logical reasoning skills

In the "Similar Figures" proof in junior high (ninth grade, Volume I), the terminology of "sufficient condition" and "necessary condition" from senior high is introduced (Chapter 1 of Compulsory Course I). For example, when proving that "two triangles are similar if their corresponding angles are equal", it is emphasized that "corresponding angles being equal" is a sufficient condition for "similarity", thus instilling the norms of logical terminology. In the "Trigonometric Identity Transformations" teaching in senior high (Chapter 8 of Compulsory Course III), students are required to write the derivation process in a "syllogism" format (such as proving $\sin(\alpha+\beta)=\sin\alpha\cos\beta+\cos\alpha\sin\beta$, clearly stating the basis for each step, including the cosine formula for the sum of two angles and the induction formula), thereby reinforcing the rigor of reasoning.

3.3. Coherent cultivation of core competencies: Taking data analysis as an example

In the teaching of "Probability" in the third year of junior high school (Volume II, Grade Nine), students are guided through dice-throwing experiments (e.g., recording the frequency of "rolling a six" in 100 trials) to observe how closely the experimental frequency aligns with the theoretical probability (1/6), thereby fostering an initial understanding of the concept that "frequency stabilizes around probability." This approach is extended in the "Statistics and Probability" module of the first year of senior high school (Compulsory Course II, Section 5.2), where students engage in investigative activities such as "estimating the total population from a numbered sample" (e.g., sampling to estimate the total quantity of a batch of products). By comparing the advantages and disadvantages of simple random sampling in junior high school with stratified sampling in senior high school, students deepen their data analysis competencies through sustained practice.

4. Practical outcomes and validation through typical cases

To evaluate the effectiveness of the integrated model, this study analyzed three consecutive years of teaching data from an experimental school (covering students from the third year of junior high school to the first year of senior high school), presenting key findings through pre- and post-test comparisons and case studies.

Case Study: Integrated Teaching Outcomes for Trigonometry — According to the curriculum plan, trigonometry is taught in three stages: introducing "Trigonometric Functions of Arbitrary Angles" (Chapter 7, Compulsory Course III) in the first semester of the third year of junior high school, reviewing and expanding to "Trigonometric Identities" (Chapter 8) in the first semester of the first year of senior high school, and deepening "Solving Triangles" (Chapter 9) in the second semester of the first year of senior high school. Practical data reveals that after exposure to the concept of arbitrary angles in junior high school, senior high school students demonstrated a 35% increase in accuracy when understanding "radian measure" and "trigonometric function lines" during their study of trigonometric identities. Furthermore, when deriving the sine theorem (extending from right triangles to oblique triangles), 82% of students were able to independently complete analogical reasoning, significantly surpassing the approximately 40% transfer rate observed under traditional teaching models. This evidence underscores the positive impact of knowledge integration on students' transfer abilities.

5. Reflection and optimization: Current challenges and directions for improvement

Despite the positive outcomes of the integrated model, two key challenges remain in practice that require optimization:

5.1. Existing issues

5.1.1. Progress discrepancies

Some students with weak foundations encounter difficulties when exposed to senior high school content (such as the descriptive notation for sets) in their third year of junior high school. They require additional supplementary examples (e.g., using descriptive notation to represent the set of points $\{(x,y) \mid y=ax^2+bx+c\}$ on the graph of a quadratic function).

5.1.2. Insufficient teacher collaboration

Junior and senior high school teachers have varying levels of understanding regarding the depth of teaching for transitional knowledge points (e.g., whether to teach the monotonicity of functions in the third year of junior high school through intuitive descriptions only or to introduce symbolic language). There is a lack of unified standards to guide them [10].

5.2. Optimization suggestions

5.2.1. Tiered instructional design

For students with weak foundations, reinforce the application of core junior high school concepts during the third year (e.g., modeling real-world problems involving the maximum and minimum values of quadratic functions). For high-achieving students, incorporate extension content (e.g., comparing the graphical relationships between power functions and fractional functions).

5.2.2. Establish a collaborative research mechanism

Organize junior and senior high school teachers to jointly compile a "Teaching Guide for Transitional Knowledge Points", clearly defining key transitional points (e.g., the differences between "circles" in the ninth-grade textbook and "spheres" and "spherical surfaces" in high school "spatial geometry"). Conduct regular joint teaching research activities to ensure consistency in teaching objectives.

6. Conclusion

The integration of junior and senior high school mathematics under the 1+3 training model essentially achieves an organic fusion of "junior high school foundations" and "senior high school extensions" through the systematic reorganization of textbook content and the graduated cultivation of thinking abilities. Empirical research indicates that this model effectively addresses issues of knowledge gaps, cognitive leaps, and conflicting objectives in traditional transitions, providing institutional and practical support for the coherent development of students' core mathematical competencies. In the future, it is necessary to further refine the evaluation system (e.g., focusing on process-oriented assessments of knowledge transfer abilities and cognitive coherence) to promote the shift from “formal integration” to “substantive fusion” in integrated training, offering replicable reference paths for subject transition reforms in the basic education stage.

Disclosure statement

The author declares no conflict of interest.

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