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Application of Nonlinear Triangular Fuzzy Fault Tree Algorithm in Predicting Lithium Battery Air Transport Accidents

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Abstract: The traditional triangular fuzzy fault tree prediction model adopts the linear approximation method. Therefore, the accident prediction error is large. Based on the analysis of the error sources and the fuzzy set, the precise calculation method of the event at the top of the fault tree is given. By using the numerical calculation software, an accurate calculation method of nonlinear triangular fuzzy accident prediction was adopted to predict lithium battery air transport fire accidents, and the fuzzy importance of the cause event was calculated.

Keywords: Nonlinear triangular fuzzy number; Fault tree; Lithium batteries; Air transport

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1. Introduction

Lithium batteries are classified as Class 9 dangerous goods, namely miscellaneous dangerous goods, when transported by civil aviation. There are dangers in the air transport of lithium batteries, and unsafe incidents, which threaten the safety of air transport, often occur. Therefore, it is of great significance to predict lithium battery air transport accidents for air transport safety. The fault tree analysis method has been widely used in production accident prediction, but when the fault tree analysis method is used to analyze the actual problem, the probability of the basic event of the fault tree is often difficult to determine. Therefore, Tanaka *et al.* [1] introduced fuzzy mathematics into fault tree analysis; they were the first to apply triangular fuzzy mathematics and trapezoidal fuzzy mathematics to calculate the probability of event occurrence at the top of the fault tree. On this basis, the theory of fault tree and fuzzy fault tree has been applied in the transport of dangerous goods by air. Du and Chen [2] have analyzed the causes of accidents in the transport of dangerous goods by air using the fault tree method. Du and Wang [3] have constructed a lithium battery air transport fire fault tree and made a quantitative analysis of the fault tree based on the principle of triangular fuzzy number and the expert scoring method. However, the aforementioned studies generally involve qualitative analyses of the fault tree, making a linear approximation of the result of fuzzy number multiplication in the operation process, which has a large error. In this paper, the nonlinear triangular fuzzy fault tree algorithm is studied and applied to lithium battery air transport accidents to obtain more accurate fuzzy calculation results.

2. Traditional accident prediction model and error analysis

2.1. Definition of triangular fuzzy number

(1) Definition of fuzzy sets

If A represents the mapping from X to [0,1], and X is the theoretical domain, represented by mathematical symbols as $A:X \to [0,1]$, $x \to A$; then, A is called fuzzy set, and $\mu_a(x)$ is called membership function.

(2) L-R type fuzzy function

The definition is as follows:

$$\mu_a(x) = \begin{cases} L(\frac{m-x}{l}) & x < m \\ 1 & x \in [m,n] \\ R(\frac{x-n}{u}) & x > n \end{cases} \tag{1}$$

where L is a monotonically increasing function that satisfies $0 \le L(x) < 1, \lim\limits_{x \to -\infty} L(x) = 0$; R is a monotonically decreasing function that satisfies $0 \le R(x) < 1, \lim\limits_{x \to +\infty} R(x) = 0$; m is the probability corresponding to the membership degree 1 of the left half of the function; n is the probability corresponding to the membership degree 1 of the right half of the function; and $l+u$ represents the degree of blindness.

(3) Triangular fuzzy function

Trigonometric fuzzy function is one of type functions. The trigonometric fuzzy number function curve is shown in **Figure 1**. In **Figure 1**, m represents the core; $l+u$ represents the degree of A's blindness and is denoted as $A = (m-l, m, m+u)$; and Z is a point on the x-axis. Assuming that the membership function curve of the triangular fuzzy number is represented by the area of the graph enclosed by the axis, the area of the graph is divided into two parts by the line perpendicular to the axis.

(4) α-cut of fuzzy sets

Let A be a fuzzy set. For a given $\alpha \in [0,1]$, the definition of α-cut is $A_\alpha = \{x | \mu_a(x) \ge \alpha\}, x \in X$, as where X is the universal set.

2.2. Calculation steps of fuzzy fault tree

(1) Construct the fault tree to determine the minimum cut set or minimum path set.

(2) The probability of occurrence of basic events is expressed by triangular fuzzy number. It can be obtained based on the interval algorithm of triangular fuzzy numbers. If the triangular fuzzy probabilities of q_1 and q_2 of the occurrence probabilities of basic events x_1 and x_2 are expressed as $(m_1 - l_1, m_1, m_1 + u_1)$ and $(m_2 - l_2, m_2, m_2 + u_2)$, then the arithmetic of the triangular fuzzy number is as follows:

$$q_1 + q_2 = ((m_1 - l_1) \pm (m_2 - l_2), m_1 \pm m_2, (m_1 + u_1) \pm (m_2 + u_2)) \tag{2}$$

$$q_1 \times q_2 = ((m_1 - l_1) \times (m_2 - l_2), m_1 \times m_2, (m_1 + u_1) \times (m_2 + u_2)) \tag{3}$$

$$C \times q_2 = (C(m_1 - l_1), Cm_1, C(m_1 + u_1)) \tag{4}$$

(3) Determine the triangular fuzzy probability of the top event. The fuzzy occurrence probability of the top event can be processed by fuzzy logic AND gate and fuzzy logic OR gate. The calculation formula of fuzzy logic AND gate and fuzzy logic OR gate is as follows:

$$q_{AND} = (\prod_{i=1}^{n}(m_i - l_i), \prod_{i=1}^{n} m_i, \prod_{i=1}^{n}(m_i + u_i)) \tag{5}$$

$$q_{OR} = ((1-\prod_{i=1}^{n}(1-(m_i - l_i))), (1-\prod_{i=1}^{n}(1-m_i)), (1-\prod_{i=1}^{n}(1-(m_i + u_i)))) \tag{6}$$

(4) The median method is used to calculate the fuzzy significance of each basic event. In **Figure 1**, the area of the graph enclosed by the membership function curve of the triangular fuzzy function and the *x*-axis is represented by *S*. If there is a point Z on the *x*-axis, a vertical line perpendicular to the *x*-axis is made through the point, and the entire closed area *S* is divided into S_1 and S_2. If $S_1 = S_2$ is satisfied, the point is called the median of triangular fuzzy numbers. The rules for fuzzy significance comparison by using the median of triangular fuzzy numbers are as follows: the fuzzy medians of two triangular fuzzy numbers A_1 and A_2 are Z_1 and Z_2, respectively; if $Z_1 < Z_2$, $A_1 < A_2$; if $Z_1 > Z_2$, $A_1 > A_2$; if $Z_1 = Z_2$, then $A_1 = A_2$. For a fault tree consisting of *n* basic events, the structure function is $\phi(x_1, x_2, ..., x_n)$, and the fuzzy median of the triangular fuzzy numbers is Z; the structure function of the fault tree without the i th basic event is $\phi(x_1, x_2, x_{i-1}, 0, x_{i+1}, ..., x_n)$, with Z_i as the fuzzy median; the expression of fuzzy importance of the i th basic event is as follows:

$$M_i = Z - Z_i \tag{7}$$

2.3. Error analysis

In calculation step (3) of the traditional triangular fuzzy prediction model, the result multiplied by *n* triangular fuzzy number is a new triangular fuzzy number. The membership function curve of the triangular fuzzy number is by the left endpoint of the triangular fuzzy number, the right endpoints and the intermediate point, and its corresponding membership values of the coordinates connected by a straight line. The straight line connects a triangle formed by three points: the left endpoint of triangle fuzzy number, 0; the right endpoint of triangular fuzzy number, 0; and triangle fuzzy number, 1 (**Figure 2**). In calculation step (4) of the fuzzy fault tree, the basic events of the fault tree can be represented by fuzzy numbers, and then the cut set of each fuzzy number is taken. According to the algorithm of the fault tree and the algorithm of the cut set, the result of multiplying A of *n* triangular fuzzy numbers is α-cut to the power of n; When $n > 1$, the line between the two ends and the vertex is a curve. Only when $n = 1$, it is appropriate for the two ends and the vertex to be connected by a straight line. Therefore, it is wrong to connect the left endpoint and vertex of the fuzzy membership degree at the top of the traditional triangular fuzzy fault tree directly with a straight line.

This paper calculates the product of *n* triangular fuzzy numbers based on the α-cut method to accurately calculate the traditional triangular fuzzy probability. Based on the numerical calculation software, the membership function graph of the improved triangular fuzzy number multiplication is drawn to determine the exact fuzzy importance value of each basic event.

3. Nonlinear triangular fuzzy fault tree algorithm

In this section, the product operation of *n* triangular fuzzy numbers is derived based on the α-cut of fuzzy set.

3.1. α-cut of triangle fuzzy number

For *L*-*R* type fuzzy numbers, assuming that both *L* and *R* are reversible functions, α-cut can be described as follows:

$$\alpha = L(\frac{m-x}{l}) \Rightarrow \frac{m-x}{l} = L^{-1}(\alpha) \Rightarrow A_\alpha^L = x = m - lL^{-1}(\alpha) \tag{8}$$

$$\alpha = R(\frac{x-m}{u}) \Rightarrow \frac{x-m}{u} = R^{-1}(\alpha) \Rightarrow A_\alpha^R = x = m + uL^{-1}(\alpha) \tag{9}$$

Therefore, α-cut of fuzzy set *A* can be expressed as follows: $A_\alpha = [A_\alpha^L, A_\alpha^R]$ (**Figure 1**).

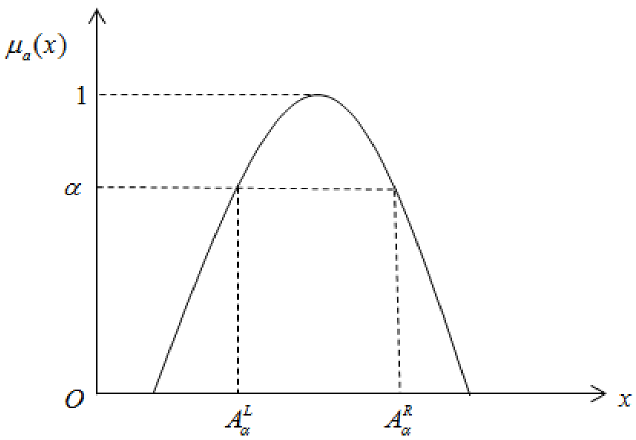

Figure 1. α-cut of *L*-*R* fuzzy number

For triangular fuzzy number, its α-cut can be represented by the following method. Suppose that $A = [m-l, m, m+u]$ represents a triangular fuzzy number, where A_α represents its α-cut ,

$$\alpha = \frac{x-(m-l)}{l} \Rightarrow A_\alpha^L = x = m - (1-\alpha)l \tag{10}$$

$$\alpha = \frac{m+u-x}{u} \Rightarrow A_\alpha^R = x = m + (1-\alpha)u \tag{11}$$

where, $A_\alpha = [A_\alpha^L, A_\alpha^R]$ denotes the related α-cut , as shown in **Figure 2**.

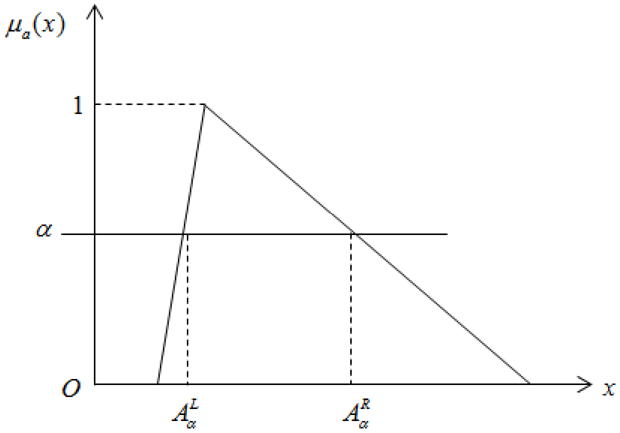

Figure 2. α-cut of triangular fuzzy number

3.2. Fuzzy product operator

Here, α-cut is used to compute the product of fuzzy numbers. The interval $[0,1]$ is divided into n identical subintervals, where each endpoint on the interval is represented as $\alpha_0 = 0$, $\alpha_i = \alpha_{i-1} + \Delta\alpha_i$, $i = 0,1,2,...,n$.

Suppose there are two fuzzy numbers A, B, the relevant α-cut expression is as follows: $A_\alpha = [A_\alpha^L, A_\alpha^R]$, $B_\alpha = [B_\alpha^L, B_\alpha^R]$. The product of α-cut donated as $A_\alpha \times B_\alpha = [L, R]$, where $L = \min(A_\alpha^L B_\alpha^L, A_\alpha^L B_\alpha^R, A_\alpha^R B_\alpha^L, A_\alpha^R B_\alpha^R)$, $R = \max(A_\alpha^L B_\alpha^L, A_\alpha^L B_\alpha^R, A_\alpha^R B_\alpha^L, A_\alpha^R B_\alpha^R)$.

Based on the numerical calculation method, n α_i-cut is calculated. When n is enough, the membership function curve of the product of two fuzzy numbers is obtained.

In the case of N fuzzy numbers, it can be calculated recursively based on the product of two fuzzy numbers. Suppose there are N fuzzy numbers $A_i, i = 1,2,...,N$, the relevant α-cut expression is as follows: $A_\alpha^i = [A_\alpha^{iL}, A_\alpha^{iR}], i = 1,2,...,N$. The α-cut product of A_α^0 and A_α^1 is expressed as $\tilde{A}_\alpha^1 = A_\alpha^0 \times A_\alpha^1 = [\tilde{A}_\alpha^{1L}, \tilde{A}_\alpha^{1R}]$, where

$$\tilde{A}_\alpha^{1L} = \min(A_\alpha^{0L} A_\alpha^{1L}, A_\alpha^{0L} A_\alpha^{1R}, A_\alpha^{0R} A_\alpha^{1L}, A_\alpha^{0R} A_\alpha^{1R}),\ \tilde{A}_\alpha^{1L} = \max(A_\alpha^{0L} A_\alpha^{1L}, A_\alpha^{0L} A_\alpha^{1R}, A_\alpha^{0R} A_\alpha^{1L}, A_\alpha^{0R} A_\alpha^{1R}). \quad (12)$$

The α-cut product of $\tilde{A}_\alpha^1$ and A_α^2 is $\tilde{A}_\alpha^2 = \tilde{A}_\alpha^1 \times A_\alpha^2 = [\tilde{A}_\alpha^{2L}, \tilde{A}_\alpha^{2R}]$, where

$$\tilde{A}_\alpha^{2L} = \min(\tilde{A}_\alpha^{1L} A_\alpha^{2L}, \tilde{A}_\alpha^{1L} A_\alpha^{2R}, \tilde{A}_\alpha^{1R} A_\alpha^{2L}, \tilde{A}_\alpha^{1R} A_\alpha^{2R}),\ \tilde{A}_\alpha^{2R} = \max(\tilde{A}_\alpha^{1L} A_\alpha^{2L}, \tilde{A}_\alpha^{1L} A_\alpha^{2R}, \tilde{A}_\alpha^{1R} A_\alpha^{2L}, \tilde{A}_\alpha^{1R} A_\alpha^{2R}). \quad (13)$$

The α-cut product of $\tilde{A}_\alpha^{N-1}$ and A_α^N is $\tilde{A}_\alpha^N = \tilde{A}_\alpha^{N-1} \times A_\alpha^N = [\tilde{A}_\alpha^{NL}, \tilde{A}_\alpha^{NR}]$, where

$$\begin{aligned} \tilde{A}_\alpha^{NL} &= \min(\tilde{A}_\alpha^{(N-1)L} A_\alpha^{NL}, \tilde{A}_\alpha^{(N-1)L} A_\alpha^{NR}, \tilde{A}_\alpha^{(N-1)R} A_\alpha^{NL}, \tilde{A}_\alpha^{(N-1)R} A_\alpha^{NR}), \\ \tilde{A}_\alpha^{NR} &= \max(\tilde{A}_\alpha^{(N-1)L} A_\alpha^{NL}, \tilde{A}_\alpha^{(N-1)L} A_\alpha^{NR}, \tilde{A}_\alpha^{(N-1)R} A_\alpha^{NL}, \tilde{A}_\alpha^{(N-1)R} A_\alpha^{NR}). \end{aligned} \quad (14)$$

4. Application in the safety of lithium battery air transport

With the continuous development of lithium battery and its wide application, there is a surge in demand for lithium battery air transport. However, lithium batteries are dangerous in air transport. When transported by civil aviation, they are classified as Class 9 dangerous goods, namely miscellaneous dangerous goods. Unsafe incidents involving lithium batteries may occur during civil aviation transport and cargo transport, in which the consequences of these incidents are relatively serious. Although laws and regulations related to the transport of dangerous goods by air have made strict provisions on the transport of lithium batteries, safety risks still exist in actual transport, thus threatening the safety of air transport. From January 23, 2006, to June 1, 2022, the Federal Aviation Administration (FAA) has recorded 380 air transport unsafe incidents involving lithium batteries [4]. As shown in **Table 1**, the major lithium battery air transport events include overheating (heat but no flame), fire explosion (including smoke), melting, *etc*. Among them, fire accidents, such as smoldering, sparking, smoking, burning, and other fire accidents of different degrees, account for 85.79% of all lithium battery accidents, with 326 cases in total.

Table 1. Statistical table of air transport unsafe incidents involving lithium batteries included by the Federal Aviation Administration

Event	Overheating	Fire explosion	Melting	Expansion
Number of events	27	326	12	15
Proportion of events	7.11%	85.79%	3.16%	3.95%

4.1. Fault tree analysis

"Lithium battery air transport fire accident" is selected as the top event of the fault tree [5], and the nonlinear modular fault tree algorithm is adopted to analyze its application (**Figure 3**).

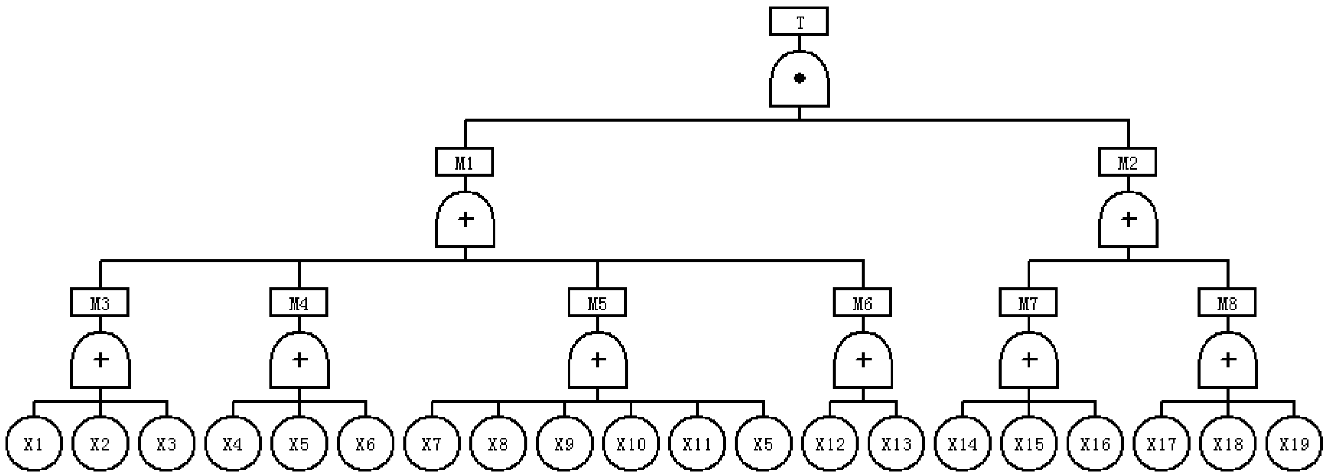

Figure 3. Lithium battery air transport fire fault tree (T, lithium battery air transport fire accident; M1, lithium batteries smoke and catch fire; M2, not in time to save; M3, production quality problem; M4, improper loading and unloading; M5, improper packing; M6, complex transportation environment; M7, not found in time; M8, non-functioning fire extinguishing facilities; X1, design flaw; X2, fake and inferior products; X3, disorganized production quality management; X4, failure to declare or improper declaration, leading to improper loading and unloading; X5, undertrained or untrained personnel; X6, rough handling; X7, improper or no anti-movement measures; X8, improper or no short-circuit prevention measures; X9, improper or no anti-accident measures; X10, packaging container or packaging does not comply with regulations; X11, failure to declare or improper declaration, leading to non-compliance to packaging regulations; X12, the lithium battery reached its maximum pressure or temperature; X13, bump, shock, or impact; X14, crew lacked sufficient information; X15, fire detector or indicator light failure; X16, passengers lack understanding of lithium batteries; X17, unfamiliar with fire extinguishing methods; X18, accidents beyond control; X19, fire extinguisher does not put out fire effectively)

4.2. Median and fuzzy significance calculation

Since there are no accurate records or statistics concerning the probability of occurrence of each basic event of lithium battery air transport fire accident, the probability of occurrence can be obtained after expert scoring and fuzzy processing of each basic event [6]. Using the nonlinear triangular fuzzy fault tree algorithm and MATLAB programming, the median fuzzy number of the top event Z = 0.35433, as well as the median value and fuzzy importance of each basic event can be obtained (see **Table 2** for specific data). The greater the fuzzy importance of the basic event, the greater the influence on the occurrence of the top event. Otherwise, it indicates that the influence degree of the top event is smaller. The fuzzy importance ranking of each basic event is as follows: $X_4 = X_{11} > X_3 > X_5 > X_{10} > X_{16} > X_6 = X_7 = X_8 = X_9 > X_2 > X_1 = X_{13} > X_{19} > X_{17} > X_{12} = X_{14} = X_{15} = X_{18}$.

Table 2. Basic event fuzzy processing data table

Number	Event name	Event probability	Median fuzzy	Significance
X_1	Design flaw	(0.012, 0.015, 0.012)	0.34529	0.00904
X_2	Fake and inferior products	(0.024, 0.020, 0.024)	0.34268	0.01165
X_3	Disorganized production quality management	(0.029, 0.050, 0.029)	0.32227	0.03206
X_4	Failure to declare or improper declaration, leading to improper loading and unloading	(0.024, 0.060, 0.024)	0.31478	0.03955
X_5	Undertrained or untrained personnel	(0.012, 0.025, 0.012)	0.32242	0.03191
X_6	Rough handling	(0.012, 0.020, 0.012)	0.34195	0.01238
X_7	Improper or no anti-movement measures	(0.012, 0.020, 0.012)	0.34195	0.01238
X_8	Improper or no short-circuit prevention measures	(0.012, 0.020, 0.012)	0.34195	0.01238
X_9	Improper or no anti-accident measures	(0.012, 0.020, 0.012)	0.34195	0.01238
X_{10}	Packaging container or packaging does not comply with regulations	(0.032, 0.035, 0.032)	0.33293	0.0214
X_{11}	Failure to declare or improper declaration, leading to non-compliance to packaging regulations	(0.024, 0.060, 0.024)	0.31478	0.03955
X_{12}	Lithium battery reached its maximum pressure or temperature	(0.002, 0.002, 0.002)	0.35318	0.00115
X_{13}	Bump, shock, or impact	(0.012, 0.015, 0.012)	0.34529	0.00904
X_{14}	Crew lacked sufficient information	(0.002, 0.002, 0.002)	0.35318	0.00115
X_{15}	Fire detector or indicator light failure	(0.002, 0.002, 0.002)	0.35318	0.00115
X_{16}	Passengers lack understanding of lithium batteries	(0.024, 0.030, 0.024)	0.33591	0.01842
X_{17}	Unfamiliar with fire extinguishing method	(0.004, 0.006, 0.004)	0.35071	0.00362
X_{18}	Accidents beyond control	(0.002, 0.002, 0.002)	0.35318	0.00115
X_{19}	Fire extinguisher does not put out fire effectively	(0.022, 0.015, 0.022)	0.34588	0.00845

Basic events X_4, X_{11}, X_3, X_5, X_{10}, and X_{16} have the greatest impact on the occurrence of the top event. These basic events are the most important factors leading to fire accidents in lithium battery air transport. Among them, failure to declare or improper declaration is the key factor. The occurrence of the top event is also influenced by basic events X_6, X_7, X_8, X_9, X_2, X_1, and X_{13}. These basic events have the second impact on the lithium battery air transport fire accidents. The control and management of these risk factors will effectively prevent the occurrence of lithium battery air transport fire accidents. Finally, the probability of occurrence of basic events like lithium battery air transport environment, fire detector or indicator light failure, fire extinguisher failure, insufficient information obtained by the crew, and accidents beyond control is very small, and their impact on lithium battery air transport fire accidents is also relatively small.

5. Conclusion

Based on the analysis of the cut set of each fuzzy number, we find that when the number of any basic event is two or more, the triangular fuzzy membership curve of the top event is nonlinear. As a result, there is a large error in linear prediction using the traditional triangular fuzzy fault tree. The product of triangular fuzzy numbers is calculated by the recursion method and fuzzy set, and the nonlinear membership curve of the triangular fuzzy fault tree top event and the fuzzy importance of each basic event are obtained. The nonlinear triangular fuzzy fault tree prediction algorithm was used in this study to analyze lithium battery air transport fire accidents and calculate the precise fuzzy importance of each basic event.

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Disclosure statement

The authors declare no conflict of interest.

References

[1] Tanaka H, Fan LT, Lai FS, et al., 1983, Fault-Tree Analysis by Fuzzy Probability. IEEE Transactions on Reliability, 32(5): 453–457.

[2] Du J, Chen Y, 2008, Analysis of Causes of Dangerous Goods Air Transport Accidents by Fault Tree Method. Journal of Civil Aviation University of China, 2008(04): 62–64.

[3] Du J, Wang R, 2014, Fire fault tree Analysis of Lithium Battery in Air Transportation Based on Triangular Fuzzy Number. Traffic Information and Safety, 32(03): 119–122 + 127.

[4] FAA, 2022, Lithium Battery Incident Chart, viewed June 1, 2022, https://www.faa.gov/hazmat/resources/lithium-battery-incident-chart

[5] Wang R, 2014, Risk Analysis and Control Measures of Lithium Battery Fire Accident in Air Transport, thesis, Civil Aviation University of China.