Improved Research on the Transformer-Inductor Simulation Model of Magnetics

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Abstract: Transformer-inductor simulation model not only reflects the characteristics of magnetic path and circuit, but also brings in magnetic components that reflected the parasitic capacitance. There are further research, strict derivation and magnetic circuit equivalent for the model in this article. Under the condition of considering hysteresis, saturation effect we can conclude a new modeling and its equivalent, which can make the magnetic curve and characteristic get better fitting. It shows that the transformer-inductance simulation model is easy to spread and use.

Key words: Mcomponents; Simulation model; Nonlinear; Hysteresis B-H curve

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1 Introduction

At present, based on the magnetic dual circuit method, the mathematical model of magnetic device can be divided into two categories: magnetic circuit-dual circuit equivalent model and gyroscope-capacitance equivalent circuit model. In the two models, the former cannot reflect the magnetic path parameters, and the latter can reflect the electric circuit and the magnetic circuit characteristics of magnetic component, but because it is a magnetic guide with capacitive simulation, it is easy to be confused with the concept of the stray capacitance of capacitance simulation in high frequency. In this paper, a new model - transformer-inductance simulation model (T-I model) is studied, and this model not only utilizes the magnetic circuit-dual circuit transform method, which can fully and clearly reflect the magnetic circuit of magnetic component and the magnetic circuit characteristics, but also the parameters that reflect the parasitic effect can be conveniently joined. In this paper, the magnetic dual circuit method is adopted to conduct modeling and theory conclusion on the simulation of the magnetic hysteresis effect of magnetic component and saturation effect on T-I model. [1]

2 The Improved T-I Model of Magnetic Component

2.1 The Improved T-I Model of Magnetic Component

The Figure 1 is the improved T-I model. According to the magnetic circuit-dual circuit analogous relation, the magnetic potential in the magnetic component can be compared with partial electric current of magnetic circuit in the model, and the magnetic conductance of magnetic component can be compared with the partial nonlinear resistance of magnetic circuit in the model. The characteristics of magnetic hysteresis in the model are simulated by the series connection (getting nonlinear resistance) of the fixed resistance R and the controlled voltage source. Nonlinear characteristics are simulated by the parallel connection of the fixed inductance L and controlled current source; Saturation characteristics are simulated by the parallel connection of the fixed conductance G and controlled current source. [2]
2.1.1 Nonlinear Inductance T-I Model Considering Magnetic Hysteresis

On the simulation of the characteristics of magnetic hysteresis of magnetic component, nonlinear controlled voltage source (by $\dot{\phi}_c$ control) $\dot{\phi}_c$ and $R$ the series connection are led in the model for the purpose of reducing the influence of excitation source size on the hysteresis loop width, as shown in Figure 2\textsuperscript{[3-5]}. The functional equation of controlled voltage source is shown in the following formula, which is the result of the simplified equation in reference\textsuperscript{[6]}.

$$\phi_c(\dot{\phi}_c) = c(\dot{\phi}_c)^n = c(RF_c)^n \quad (1)$$

When the hysteresis property is considered only, the following relation can be obtained from Figure 2.

$$\dot{\phi} = c(RF_c)^n + RF_c \quad (2)$$

Therefore, the equivalent nonlinear resistance obtaining $F_f$ branch can be worked out:

$$R_{eq} = \frac{\phi_f}{F_c} = cR^n F_c^{n-1} + R \quad (3)$$

Simulating characteristics of magnetic hysteresis of the magnetic component by resistance conforms to the physical mechanism. Because the resistance voltage $\dot{\phi}_c$ and $\dot{\phi}$ meet the relative reference direction, the partial pressure of the resistance will hinder the change of the flux, which is shown as the magnetic hysteresis characteristic.

2.1.2 Nonlinear Inductance T-I Model Considering Magnetic Saturation

On the verification of the saturation characteristics of the model, the change of incentive magnetic potential $F_r$ is random, so the nonlinear controlled current source $F_{r1}$ controlled by $\phi_g$ (different from $F$) and the conductance $G$ parallel are stimulated in this paper, as shown in Figure 3\textsuperscript{[6]}.

$$F_r = \phi_g F_{r1} = cR^n F_r^{n-1} + R \quad (3)$$
The nonlinear controlled current source can be described in the following formula:

$$F_{i1}(\phi_g) = \lambda(\phi_g)^n = \lambda(F_g/G)^n \quad (4)$$

The following relation can be obtained from Figure 3:

$$F_g = \lambda(F_g/G)^n + F_g \quad (5)$$

So the equivalent nonlinear conductance of $F_g$ branch is

$$G_{eq} = F_g/\phi = \lambda(F_g/G)^{n-1} + G \quad (6)$$

When the voltage value of the both ends of the conductance $G$ is less and the flux is less, the controlled current source is correspondingly small, and the magnetic potential is almost always on the conductance $G$, so the equivalent magnetic guide is $G_{eq}$. Naturally, when the voltage value of the both ends of the conductance is more and the flux is more, the figure of $F_{i1}(\phi_g)$ will increase exponentially. Because the magnetic potential is mainly provided by the controlled source, if the $\dot{\phi}$ is approximately zero, the magnetic component is at the saturation state.

### 3 The Calculation of the Parameters in the Model

The input parameters of the model can be divided into two types: the material parameters and geometric parameters of magnetic core. The material parameters include saturation magnetic flux density $B_{sat}$, saturation magnetic field intensity $H_{sat}$, residual flux density $B_r$, coercive force $H_c$, relative permeability $\mu_r$ curve and $B-H$ so on. And the geometrical parameters include the area $A_e$ of the effective magnetic circuit, and the length $l_e$ of the effective magnetic circuit. The following parameters are used to conduct the model parameters, which include the parameter influencing magnetic hysteresis characteristics $R$, $c$, $m$ and the parameter influencing the saturation characteristics $G$, $\lambda$, $n$.\[6\]

#### 3.1 The Simulation of Input Parameters

As mentioned above, a nonlinear controlled current source $F_i$ is used to simulate the nonlinear characteristics of the magnetic component model. The $b_i \quad (i = 3, 5, 7, \ldots)$ of can adopt the calculation method of data fitting. Within a certain range of precision, the formula can be simplified to the formula (7), and that is, a certain higher term is solely used to simulate the nonlinear of magnetic core. Namely,

$$F = a_1f + b_nA^n B^n \quad (7)$$

The practical simulation of this controlled source will be discussed below to show that there is little difference using formula (7) to conduct the simplified analysis. According to the magnetization curve standard in the magnetic core manual, the formula (7) is converted to the relational formula, between $H$ and $B$ which is as follows:

$$H = \frac{a_1A}{l_e} B + \frac{b_nA^n B^n}{l_e} \quad (8)$$

In the formula, $A_e$ is the effective magnetic conductive area and the $l_e$ is the effective magnetic circuit length.\[3\]

#### 3.2 The Conclusion of Magnetic Hysteresis Parameters

Because the $\dot{\phi}$ and $F$ follow the circuit theory in the model, the equation and the solving differential equation among the $\dot{\phi}$ and each branch $F$ can be listed drawing on Ampere’s rule, the relationship between $\dot{\phi}$ and $F$ can be worked out, and the magnetic hysteresis parameters can be calculated by combining with the points $(B_i, H_i)$, $(B_s, H_s)$, the corresponding $(\phi_i, F_i)$, and $(\phi_s, F_s)$, of the fundamental magnetization curve.

#### 3.2.1 Calculation R

From the Figure 4 simulating inductance magnetic hysteresis effect:

$$F = F_i + F_j + F_k \quad (9)$$

Namely,
When the magnetic core is unsaturated, it satisfies the following formula:

\[ F_e = \phi / R \]  

(11)

Without loss of generality, has

\[ F = F_s \sin wt \]

(12)

Because the non-odd nonlinear differential equation solving is trivial, the numerical value \( \phi \) is not easy to obtain, and it's also because when the hysteresis effect is simulated, the main parameters are the remanence \( B_r \) (corresponding \( \phi_r \)) and the coercive force \( H_c \) (corresponding \( F_c \)), and at the same time, because the higher power of the nonlinear equation is very small in the vicinity of zero, it can be approximately believed \( \phi \) and \( F_e \) is only influenced by \( F_e = a_1 f \) when the hysteresis effect is simulated. Therefore, the expression can be obtained:

\[ \dot{\phi} / R + a_1 \phi = F_s \sin \omega t \]

(13)

Solve the differential equation, and the result is shown in equation (14):

\[ \phi = RF_c(a_1 R \sin \omega t - \omega \cos \omega t) \]

\[ a_1^2 R^2 + \omega^2 \]

(14)

Therefore, the magnetic potential \( F \) and flux \( \phi \) have established a parametric equation with the parameter \( t \), and if \( t \) is removed, the equation expressing the relationship between \( F \) and \( \phi \) can be obtained

\[ (a_1 R^2 F - a_1^2 R \phi - \omega^2 \phi)^2 + F_s^2 = 1 \]

(15)

Substitute equation (13):

\[ F_e = F_s \sin \omega t - a_1 \phi \]

(16)

The following equation can be obtained by removing the intermediate variable \( t \) from equation (14) and equation (22):

\[ \frac{(a_1 R^2 F - \alpha^2 \phi)^2 + (F_s + a_1 \phi)^2}{\omega^2 R^2 F_s^2} = 1 \]

(17)

Then, the equivalent nonlinear resistance \( R_{eq} \) in the branch is used to replace \( R \) in equation (17) and the value of \( R \) is determined by \( F_e(\phi = \phi_r) = 0 \).

\[ R = \omega \phi_r / \sqrt{F_s^2 - a_1^2 \phi_r^2} \]

(18)

3.2.2 Select \( m \)

As can be seen from equation (3), the three parameters of \( R \), \( c \) and \( m \) all influence the value of \( F_e \). To ensure that the value of \( R_{eq} \) is not negative, the value of \( m \) shall be the odd number. From the above analysis, as \( m \) increases, the impact of stimulated magnetic hysteresis of magnetic pieces will be smaller, so the selection of \( m \) shall be as large as possible.\[5\]

3.2.3 Calculate \( c \)

Replace \( R \) in equation (17) with the equivalent nonlinear resistance \( R_{eq} \) of \( F_e \), and the value of \( c \) is determined through \( F_e \big|_{\phi=0} = F_c \).

\[ c = \frac{\omega \sqrt{\omega^2 \phi_r^2 + a_1^2 R^2 \phi_r^2 - R^2 F_s^2 - R}}{a_1 R^{n+1} F_e} \]

(19)

3.3 Calculation of Parameters of Saturation

3.3.1 Calculation of \( G \)

The following magnetic potential relationship can be deduced from the introduction of the nonlinear conductance T-I model (Figure 3):

\[ F = F_a + F_f + F_L \]

(20)

Among them, \( F_a \) represents the sum of the current flowing by the conductance and the controlled current source in parallel therewith. If linear conductance is used for simulation calculation, then

\[ \dot{\phi} G + a_i \phi = F_s \sin \omega t \]

(21)

Equation (21) is a differential equation, and the following can be obtained by solving it:
\[
\phi = \left(\frac{a}{G}\sin\omega t - \omega \cos\omega t\right) \times G \times \frac{F_s}{\omega^2 + \omega^2 G^2} \quad (22)
\]

Magnetic potential \( F \) and \( \phi \) constitute a set of parametric equations
\[
\begin{cases}
F = F_s \sin \omega t \\
\phi = \left(\frac{a}{G}\sin\omega t - \omega \cos\omega t\right) \times G \times \frac{F_s}{\omega^2 + \omega^2 G^2} 
\end{cases} \quad (23)
\]

The following can be obtained by eliminating the parameter \( t \) from the above equation
\[
\left[\frac{a_t(F_s + a_t\phi) - \phi(a_t + a_t G^2)}{G F_s^2}ight]^2 \times \frac{1}{\omega^2} \cdot \frac{(F_s + a_t\phi)^2}{F_s^2} = 1 \quad (24)
\]

The following can be obtained from \( F_s(\phi = \phi^*) = 0 \)
\[
G = a_t F_s \sqrt{F_s^2 - F_s^2} \quad (25)
\]

### 3.3.2 Selection of \( n \)

As can be seen from equation (6), as flux increases, \( F_s(\phi^*) \) increases exponentially, and when reaching a certain degree, the magnetic potential will mainly fall on the controlled source, while the magnetic pieces will be saturated. From calculation, we can see that to ensure that the value of \( G_{eq} \) is non-negative, \( n \) shall take the odd number.\[7\]

### 3.3.3 Calculation of \( \lambda \)

Use the equivalent nonlinear resistance \( G_{eq} \) of \( F_s \) branch to replace \( G \) in equation (23), and the value of \( \lambda \) can be calculated from \( F_{G_{eq}} = F_s \). Therefore

\[
F_s = \pm \omega F \sqrt{\frac{\omega^2 + \omega^2 G^2}{(\frac{F_s}{\omega^2 + \omega^2 G^2} + G)^2}} \quad (26)
\]

The expression of parameter \( c \) can be obtained from equation (26)
\[
\lambda = \left(\frac{\omega F_c c}{\sqrt{\omega^2 F_s^2 - \omega^2 F_c^2}} - G\right) \times \frac{G_{eq}^m - 1}{F_c} \quad (27)
\]

### 4 Simulation Realization and Results of the Model

#### 4.1 Simulation of Nonlinear Inductance Model

Use the relationship between magnetic field intensity \( H \) and magnetic induction intensity \( B \) in the above equation (8) to carry out theoretical simulation. To facilitate analysis, take effective magnetic conductivity area \( A_e = 1 \text{m}^2 \) (corresponding to \( m_e = 2000 \)), \( b_n = 3.2 \times 10^{-9} \) (here, \( b_n \) is a given value) to conduct quantitative analysis, and the basic magnetic curve is shown in Figure 4 (a).\[8\]

Using the equivalent circuit in Figure 3 to simulate, and analog parameters are set as follows: \( F(t) = 500\sin(314t) \) A., \( L_i = 2.513\text{mH} \). The basic magnetic curve obtained by model analog is shown in Figure 4 (b), showing that analog results are consistent with the theoretical analysis.
4.2 Hysteresis Model Simulation

The selected simulation parameters are as follows: \( \phi_0 = 0.4 \text{Wb} \), \( F_s = 500 \text{A} \), \( F_c = 20 \), \( m = 7 \) and other parameters are the same as before. Through calculation, it can be obtained that resistance \( R = 0.265 \Omega \), \( c = 3.356 \times 10^{-3} \) and values of amplitude of excitation \( F \) are 500, 750 and 1000. The hysteresis effect using nonlinear resistance simulation and the results corresponding to the analog hysteresis curve are shown in Figure 5. It can be seen from the figure that, with the introduction of the T-I model of non-linear resistance, the simulation coercivity \( F_c \) and residual magnetism \( \phi_0 \) will not be greatly affected by \( F_m \).\(^{[9]}\)

Different \( F_c \) are given, the value of \( c \) will be calculated according to equation (19) for simulation according to the calculated \( c \) and simulation results are shown in Table 1. By comparing theoretical and analog values, it can be seen that the analog forward magnetic value is slightly larger than the theoretical value while the reverse magnetic value is slightly smaller than the theoretical value. Therefore, it can be proved that analog results are basically consistent with the theoretical analysis results. Therefore, parameter \( c \) can be fully determined by using equation (19).

Table 1 Comparison between theoretical and simulation value of \( F_c \) under various parameter \( c \)

<table>
<thead>
<tr>
<th>Given ( F_c )</th>
<th>Calculated ( C )</th>
<th>Simulated ( F_c )</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>0.429</td>
<td>-9.925</td>
</tr>
<tr>
<td></td>
<td></td>
<td>10.113</td>
</tr>
<tr>
<td></td>
<td></td>
<td>3.356\times10^{-3}</td>
</tr>
<tr>
<td>20</td>
<td>-19.981</td>
<td>20.129</td>
</tr>
<tr>
<td>30</td>
<td>1.962\times10^{-4}</td>
<td>29.932</td>
</tr>
<tr>
<td>40</td>
<td>2.615\times10^{-5}</td>
<td>39.954</td>
</tr>
<tr>
<td>50</td>
<td>5.475\times10^{-6}</td>
<td>49.915</td>
</tr>
<tr>
<td>60</td>
<td>1.524\times10^{-6}</td>
<td>59.931</td>
</tr>
<tr>
<td>70</td>
<td>5.169\times10^{-7}</td>
<td>69.971</td>
</tr>
<tr>
<td></td>
<td></td>
<td>70.296</td>
</tr>
</tbody>
</table>

Figure 5 Simulation hysteresis curves under various excitations

Another important parameter that reflects the characteristic of the hysteresis loop is the residual magnetism \( B_r \) (i.e. the corresponding \( \phi_r \)). With further
analysis of the synthetic hysteresis curve and in terms of non-rectangular magnetic materials, the parameter \( a_i \) can be determined by the following equation

\[
 a_i = F_i / \phi_i
\]

(28)

As for rectangular hysteresis material, as the magnetic permeability is very large, it can be directly determined according to \( \mu \)

\[
a_i = l_s / \mu_s A_i
\]

(29)

By referring to the \( B - H \) curve in the magnetic core data manual, turning points exist from the linear to saturation segments. Select a value of magnetic flux density \( B_i \) at turning points, which corresponds to two magnetic field intensity \( H_{\text{min}} \) and \( H_{\text{max}} \), and the average value \( H_i \) of the two are taken as the magnetic field intensity in the basic magnetic curve, therefore

\[
 H_i = (H_{\text{min}} + H_{\text{max}})/2
\]

(30)

\( B_i \) and \( H_i \) are equivalent to \( \phi_i \) and \( F_i \), which will be substituted into equation (7) along with \( \phi_s \) and \( F_s \) to obtain values of parameters \( n \) and \( b_n \).

\[
 \begin{align*}
 n &= \frac{\ln(F_s - a_i \phi_s) - \ln(F_i - a_i \phi_i)}{\ln \phi_s - \ln \phi_i} \\
 b_n &= \frac{F_s - a_i \phi_s}{a_i^2 \phi_i^2}
\end{align*}
\]

(31)

4.3 Characteristic of Saturation Simulation

By referring to the magnetic core manual, use the slope \( \phi \) and \( F \) corresponding to variables of the \( B - H \) curve to simulate the characteristic of saturation. During calculation, the taken parameters are the same as before, take the derivative of the parameter \( F \) in equation (23), and the following can be obtained

\[
 \frac{d \phi}{d F} = \left( \frac{a_i}{G} \pm \frac{\omega F}{\sqrt{F_i^2 - F^2}} \right) \frac{G}{a_i^2 + \omega^2 G^2}
\]

(32)

Set the above equation \( d \phi/d F = 0 \), and saturated magnetic potential \( F_{\text{sat}} \) and magnetic induction intensity \( H_{\text{sat}} \) can be obtained.

\[
 \begin{align*}
 \phi_{\text{sat}} &= \left( \pm \frac{a_i G \omega F_i}{a_i^2 + G^2 \omega^2} \pm \frac{F_i a_i^2}{\sqrt{a_i^4 + G^2 \omega^2}} \right) / a_i^2 + G^2 \omega^2 \\
 F_{\text{sat}} &= \pm \frac{F_i a_i^2}{\sqrt{a_i^4 + G^2 \omega^2}}
\end{align*}
\]

(33)

Figure 6 Simulation of saturation model

In equation (33), \( \pm F_i a_i^2 / \sqrt{a_i^4 + G^2 \omega^2} \) is much smaller than the previous value, which can be omitted in calculation. Calculated values of the saturated magnetic potential and saturated magnetic induction intensity can be deduced, and it can be seen that theoretical and calculated values are very close.\(^{10}\)

\[
 \begin{align*}
 \phi_{\text{sat}} &= 0.5(\text{NegativeRounding}) \\
 F_{\text{sat}} &= \pm 497.6
\end{align*}
\]

(34)

When verifying, the equation can be regarded as the quadratic equation with \( G \) as the unknown element.
\[
\frac{d\phi}{dF} \omega^2 G^2 \pm \frac{\omega F}{\sqrt{F_x^2 - F^2}} G + \frac{d\phi}{dF} a_i^2 - a_i = 0 \quad (35)
\]

As \( G \neq 0 \), it can be known that in the equation, \( \geq \Delta_i \), and the corresponding relationship between \( d\phi/dF \) and \( F \) can be obtained.

\[
\Delta = \frac{\omega^2 F^2}{F_x^2 - F^2} - 4 \frac{d\phi}{dF} \omega \left( \frac{d\phi}{dF} a_i^2 - a_i \right) \geq 0 \quad (36)
\]

Set \( \Delta = \Delta_0 \), and in this situation, \( G \) has the unique answer,

\[
\frac{d\phi}{dF} = \pm \sqrt{1 - \frac{F^2}{F_x^2 - F^2}} \quad (37)
\]

\[
\frac{d\phi}{dF} = -\left(0.2 \times 10^{-4} F^2 + 5\right) \pm \sqrt{3 \times 10^{-4} F^2 + S^2} \quad (38)
\]

By using the above parameters of \( a_i = 399 \) and \( F_i = 500 \), \( G = 0.387 \), so \( d\phi/dF \) and \( F \) can be obtained. Compared it with curve values in the magnetic core data manual to prove that this model is practical and applicable.[11]

<table>
<thead>
<tr>
<th>( F ) theoretical value</th>
<th>100</th>
<th>125</th>
<th>150</th>
<th>200</th>
</tr>
</thead>
<tbody>
<tr>
<td>( d\phi/dF ) theoretical value</td>
<td>0.002739</td>
<td>0.002877</td>
<td>0.003053</td>
<td>0.003545</td>
</tr>
<tr>
<td>( \phi ) theoretical value</td>
<td>0.3969</td>
<td>0.4189</td>
<td>0.4398</td>
<td>0.4653</td>
</tr>
<tr>
<td>( F ) manual value</td>
<td>100</td>
<td>125</td>
<td>150</td>
<td>200</td>
</tr>
<tr>
<td>( d\phi/dF ) manual value</td>
<td>0.00137</td>
<td>0.000974</td>
<td>0.000707</td>
<td>0.0002969</td>
</tr>
<tr>
<td>( \phi ) manual value</td>
<td>0.4</td>
<td>0.428</td>
<td>0.45</td>
<td>0.475</td>
</tr>
</tbody>
</table>

5 Conclusion

The experimental platform is a four-phase (two-phase high) VRM test platform, its control chip is ISL6558EVAL1Z, the input voltage is 12VDC, the output voltage is 1.5VDC, the control chip is ISL6558, and the switch tube MOSFET selects HUF76143S3S; the inductance of the integrated magnet pieces is \( L_1 = 100.4 \mu H \) and \( L_2 = 101.2 \mu H \), the iron core selects spiral film magnetic pieces, and its winding air gap is \( g_c = 0.4 \text{mm} \); the amount of leakage inductance is about \( 41.2 \mu H \). After verification, several results in the following figures can be obtained. The Figure 7 is the trigger waveform of switch tube, Figure 8 is the inductance current of single channel, Figure 9 is the current waveform of two-phase inductance and Figure 10 is the total output current.[12]

Figure 7 The trigger waveform of switch tube  Figure 8 The inductance current of single channel
6 Concluding Remarks

In this article, a new T-I simulation model of magnetic pieces is deduced theoretically, which solves the problem that effect degree of the excitation on the width of hysteresis curve when the linear resistance simulates hysteresis. At the same time, the methods of determining the model and its parameters are given. The B-H simulation curve is obtained by giving certain excitation and material parameters, and be compared with the data manual curve. The results show the accuracy of the model and model parameters.

References